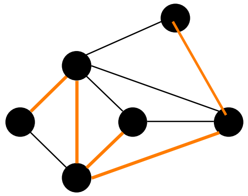
Spanning Tree Construction

A spanning tree T of a graph G = (V,E) is an acyclic subgraph of G such that T=(V,E') and E' ⊂ E.

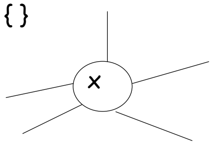
Assumptions:

single initiator
bidirectional links
total reliability
G connected



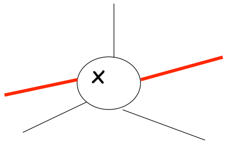
Protocol SHOUT

Initially: ∀ x, Tree-neighbors(x) = { }

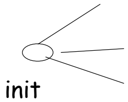


At the end:

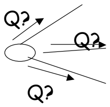
∀ x, Tree-neighbors(x) = {links that belong to the spanning tree }



1\.

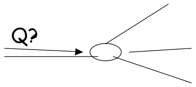


init

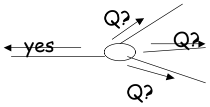


Q? = do you want to be my neighbour in the spanning tree ?

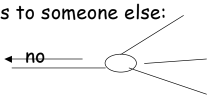
2\.



If it is the first time:



If I have already answerd yes to someone else:



Example

States S={INITIATOR, IDLE, ACTIVE, DONE}
Sinit = {INITIATOR, IDLE}
Sterm = {DONE}

```
INITIATOR
Spontaneusly
        root:= true
        Tree-neighbours := { }
        send(Q) to N(x)
        counter:=0
        become ACTIVE
```

```
IDLE
receiving(Q)
        root:= false
        parent := sender
        Tree-neighbours := {sender}
        send(yes) to sender
        counter := 1
        if counter = |N(x)| then
                become DONE
        else
                send(Q) to N(x) - {sender}
                become ACTIVE
```

```
ACTIVE

receiving(Q)
        send(no) to sender

receiving(yes)
        Tree-neighbours:=
                Tree-neighbours ∪ sender
        counter := counter +1
        if counter = |N(x)|
                become DONE

receiving(no)
        counter := counter +1
        if counter = |N(x)|
                become DONE
```

Note: SHOUT = FLOOD +REPLY

Correctness and Termination

- If x is in Tree-neighbours of y, y is in Tree-neighbours of x
- If x send YES to y, then x is in Tree-neighbour of y and is connected to the initiator by a chain of YES
- Every x (except the initiator) sends exactly one YES

⇒

The spanning graph defined by the Tree-neighbour relation is connected and contains all the entities

Note: local termination

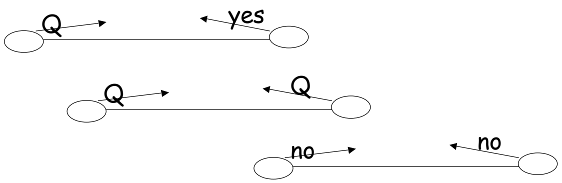
Message Complexity

SHOUT = FLOOD + REPLY

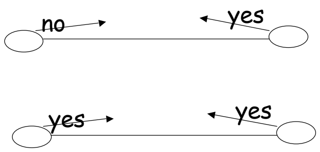
⇒

Messages(SHOUT) = 2 M(FLOOD)

Possible situations



Impossible situations



Message Complexity - worst case

Total n. of Q:

(n-1)

only one Q on the ST links

m -(n-1)

on the other links

Total: 2(m -(n-1)) + (n-1)
= 2m -n +1

Message Complexity - worst case

Total n. of NO:

as many as Q---Q

2(m - (n-1))

Total n. of YES:

Exactly: (n-1)

Message Complexity - worst case

$2m - n + 1 + 2(m - (n-1)) + n-1$
$= 2m - n + 1 + 2m - 2n + 2 + n - 1$
$= 4m - 2n + 2$

$$\boxed{\text{Messages(SHOUT)} = 4m - 2n + 2}$$

In fact: M(SHOUT) = 2 M(FLOOD) = 2(2m-n+1)

$\Omega(m)$ is a lower bound also in this case

Spanning Tree Construction

Without “NO”

Protocol SHOUT+

States S={INITIATOR, IDLE, ACTIVE, DONE}
Sinit = {INITIATOR, IDLE}
Sterm = {DONE}

```
INITIATOR
Spontaneusly
        root:= true
        Tree-neighbours := { }
        send(Q) to N(x)
        counter:=0
        become ACTIVE
```

```
IDLE
receiving(Q)
        root:= false
        parent := sender
        Tree-neighbours := {sender}
        send(yes) to sender
        counter := 1
        if counter = |N(x)| then
                become DONE
        else
                send(Q) to N(x) – {sender}
                become ACTIVE
```

```
ACTIVE

receiving(Q) (to be interpreted as NO)

        counter := counter +1
        if counter = |N(x)|
                become DONE


receiving(yes)
        Tree-neighbours:=
                Tree-neighbours ∪ {sender}
        counter := counter +1
        if counter = |N(x)|
                become DONE
```

On each link there will be exactly 2 messages:

either

or

Messages(SHOUT+) = 2m

Much better than:

Messages(SHOUT) = 4m -2n + 2

Spanning Tree Construction

With Notification

States S={INITIATOR, IDLE, ACTIVE, DONE}
Sinit = {INITIATOR, IDLE}
Sterm = {DONE}

```
INITIATOR
Spontaneusly

        root:= true
        Tree-neighbours := { }
        send(Q) to N(x)
        counter:= 0
        ack-counter:= 0
        become ACTIVE
```

```
IDLE
receiving(Q)

        root:= false
        parent := sender
        Tree-neighbours := {sender}
        send(yes) to sender
        counter := 1
        ack-counter:= 0
        if counter = |N(x)| then
                CHECK
        else
                send(Q) to N(x) - {sender}
        become ACTIVE
```

```
ACTIVE

receiving(Q)
        counter := counter +1
        if counter = |N(x)| and not root then
                CHECK

receiving(yes)
        Tree-neighbours:=
                Tree-neighbours ∪ {sender}
        counter := counter +1
        if counter = |N(x)| and not root then
                CHECK
```

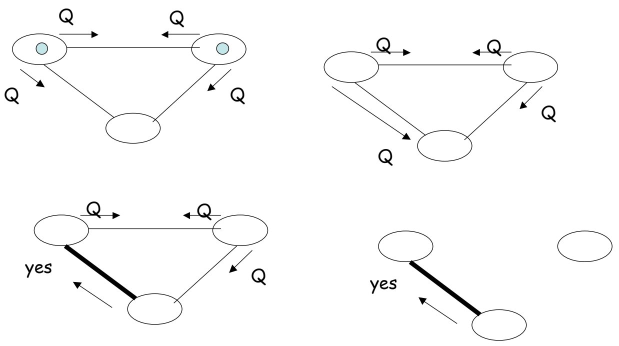
ACTIVE (cont)

```
receiving(Ack)
        ack-counter:= ack-counter +1
        if counter = |N(x)| /* indicate tree-neighbors is done
            if root then
                if ack-counter = |Tree-neighbours|
                send(Terminate) to Tree-neighbours
                become DONE
            else if ack-counter = |Tree-neighbours| - 1
                send(Ack) to parent
receiving(Terminate)
        send(Terminate) to Children
        become DONE
```

CHECK

```
If  I am a leaf
        (* that is:  Children:= Tree-neighbours – {parent}
                 if Children = emptyset   *)
 send(Ack) to parent
```

What happens if there are multiple initiators ?



An election is needed to have a unique initiator.

or

Another protocol has to be devised.

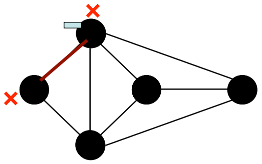
NOTE: Election is impossible if the nodes do not have distinct IDs

Traversal
Depth First Search

Assumptions

Single initiator
Bidirectional links
No faults
G connected

S = {INITIATOR, SLEEPING, ACTIVE, DONE}



One version

1) When first visited, remember who sent,
forward the token to one of the unvisited neighbours
wait for its reply

2) When neighbour receives,
if already visited, it will return the token saying it is a back edge
otherwise, will forward it (sequentially)
to all its unvisited neighbour before returning it

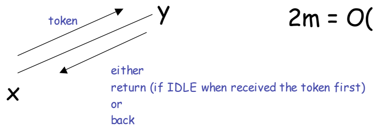
3) If there are no more unvisited neighbours, return the token (reply)
to the node from which it first received the token

4) Upon reception of reply, forward the token to another unvisited neighbour

Complexity

Message Complexity:

Type of messages: `token, back, return`



2m = O(m)

Time Complexity:
(ideal time)

2m = O(m)

Totally sequential

Ω(m) is also a lower bound

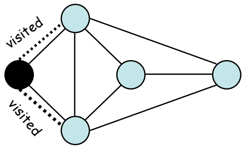
Note:
most messages are on Back Edges

---> most time is spent on Back Edges

Idea: avoid sending messages on back edges

How ?

DF+ Improving Time



DF+ Complexity — Message

Messages: **`Token`**, **`Return`**, **`Visited`**, **`Ack (ok)`**

Each entity (except init): receives 1 **`Token`**, sends 1 **`Return`**:

2(n-1)

Each entity:

1 **`Visited`** to all neighbours except the sender

Let s be the initiator

$$|N(s)| + \sum_{x \neq s} (|N(x)|-1)$$

$$= 2m - (n-1)$$

(same for **`Ack`**)

TOT: 4m

DF+ Complexity — Time (ideal time)

Token and Return are sent sequentially: 2(n-1)

Visited and Ack are done in parallel: 2n

TOT: 4n -2

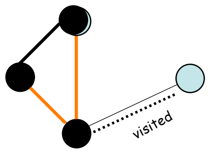
Summarizing:

DF Traversal

	Messages	Ideal Time
DF:	2m	2m
DF+:	4m	4n -2

DF++

Do not send the `Ack`
What happens ?



A token is sent to an already visited node (= back edge)

Both nodes will eventually understand the “mistake”

and pretend nothing happened

DF++ Complexity

In the worst case there is a “mistake” on each link except for the tree links

Messages = 4m -(n-1)

BUT when we measure ideal time:

“mistakes” will not happen

Time = 2(n-1)

Summary

	Messages	Ideal Time
DF:	2m	2m
DF+:	4m	4n -2
DF++	4m-n+1	2n+1

Observations

Time ...

Termination ...

An application:
access permission problems, e.g., Mutual Exclusion

Any Traversal does a Broadcast (not very efficient)
The reverse is not true.

Another Traversal: Smart Traversal

1- Build a Spanning Tree with SHOUT+

Messages = 2m

2- Perform DF Traversal

Messages = 2(n-1)

Total Messages = 2(m+n-1)

Another Traversal: Smart Traversal

1- Build a Spanning Tree with SHOUT+

Time ≤ d+1 d: diameter

2- Perform DF Traversal

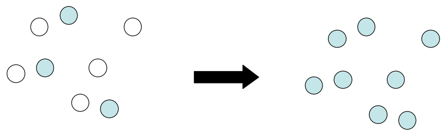
Time = 2(n-1)

Total Time ≤ 2n+d-1

Summary

	Messages	Ideal Time
DF:	2m	2m
DF+:	4m	4n -2
DF++	4m-n+1	2n+1
Smart	2m+2n-2	2n+d-1

Computations with Multiple initiator: WAKE-UP



FLOOD solves the problem.

General FLOOD algorithm: $O(m)$

More precisely: $2m - n + k^*$ WHY ?

$\searrow$ n. of initiators

1 init = broadcast = $2m - n + 1$

All init = $2m$

Computations with Multiple initiator: WAKE-UP

In special topologies ?

TREE

Flood is optimal $n + k^* - 2$

Computations with Multiple initiator: WAKE-UP

COMPLETE GRAPH

Broadcast		Wakeup	
Flood	Specific	Flood	Specific
$O(n^2)$	$O(n)$	$\Omega(n^2)$	

Need additional assumptions to reduce the complexity

HYPERCUBE

Broadcast		Wakeup	
Flood	Specific	Flood	Specific
$O(n \log n)$	$O(n)$	$\Omega(n \log n)$	