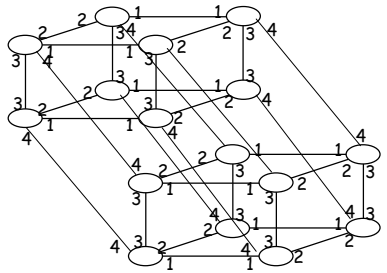
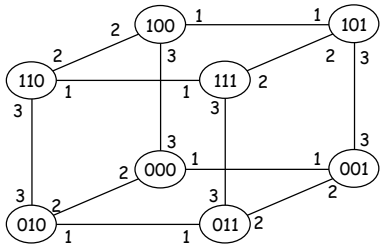
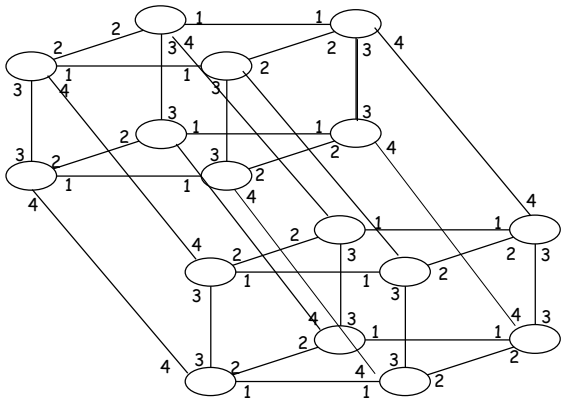
Election in the Hypercube

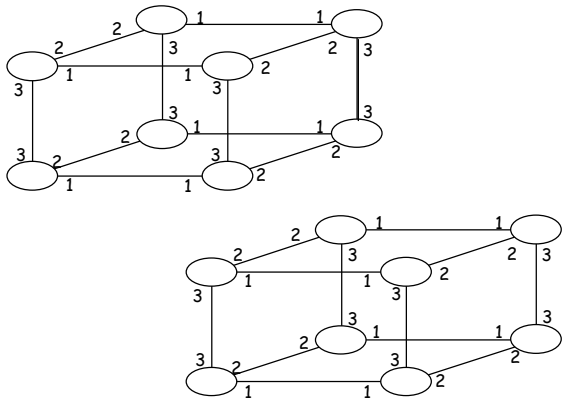


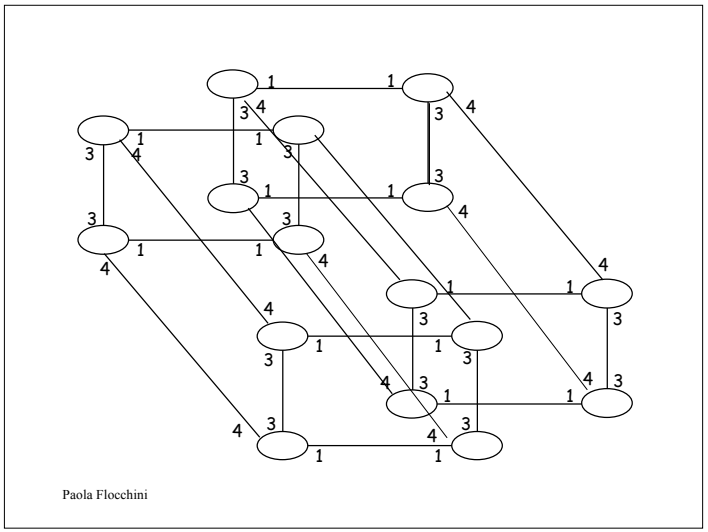
Dimension-Based Sense of Direction

Each link between two nodes is labeled by the dimension of the bit by which the nodes' identities differ.





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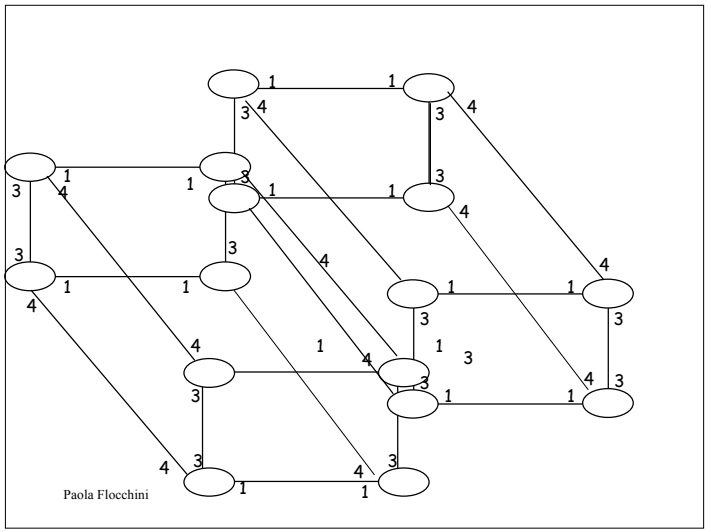
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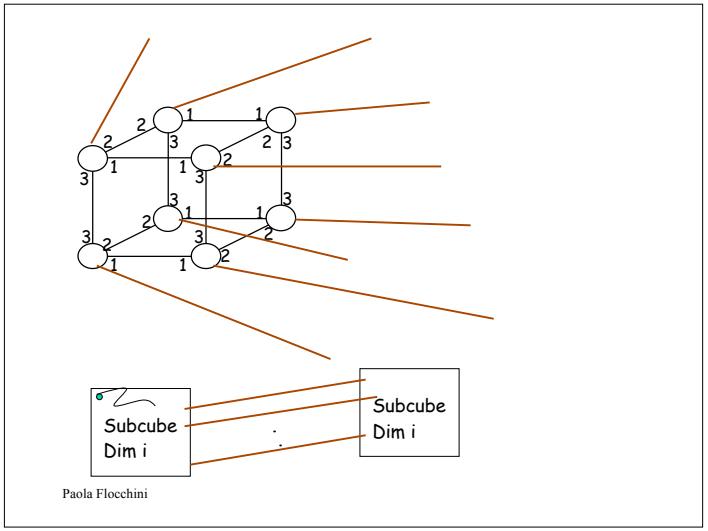
Subcube
Dim i
Subcube
Dim i
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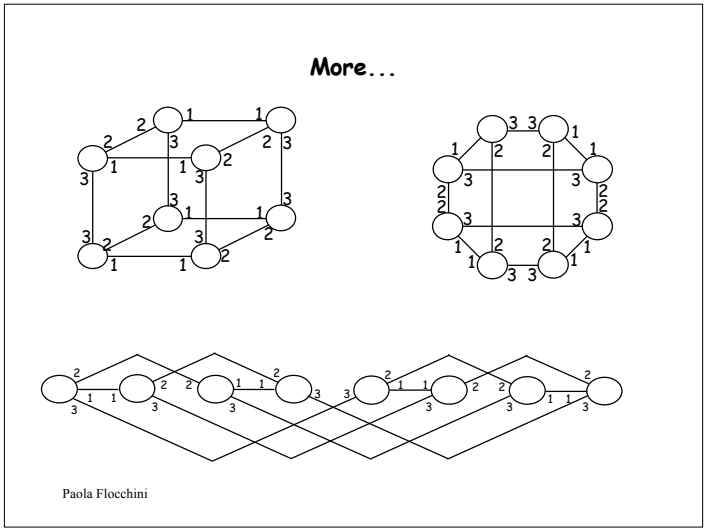
More...
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Idea

Tournament:

- Initially: All node are queens.
- Termination: All nodes except one have become citizens.

During each phase, a queen challenges its neighbor according to the dimension given by the step number.

- log n = d steps (one per dimension.)

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Hand-shake at level K:

- If a queen holding the smallest value is attacked by its neighbor, that queen wins.
  It becomes a level *k*+1 queen.
- If a queen holding a larger value is attacked, that queen loses. It becomes citizen (of level k) for the next fights and keeps the path to the queen who defeated it.

Initially: n level-0 queens
At the end: One level-log n queen

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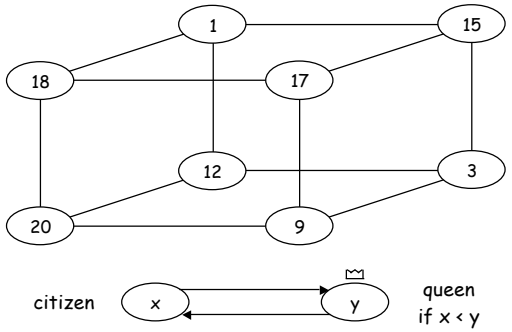
Initially...



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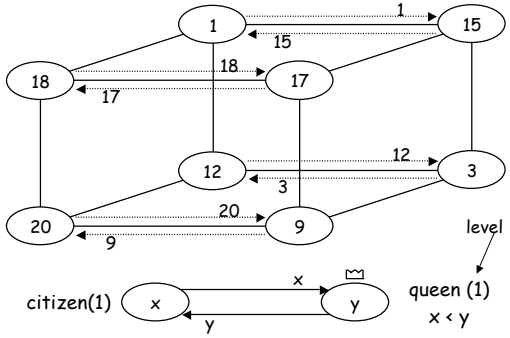
First Step



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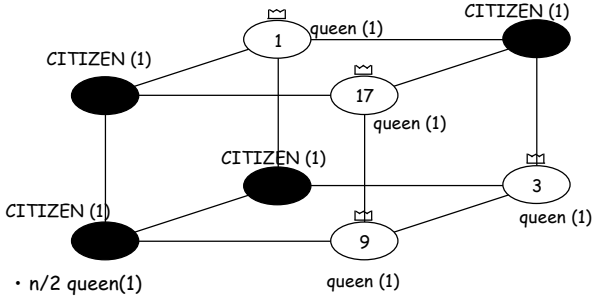
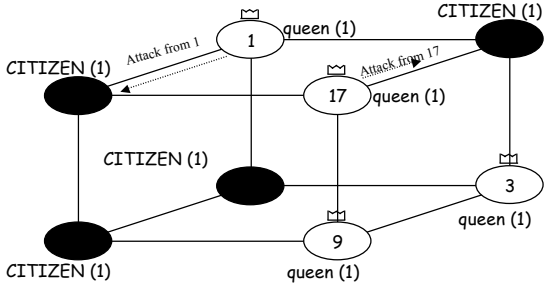
After First Step



- n/2 queen(1)
- n/2 citizen(1)

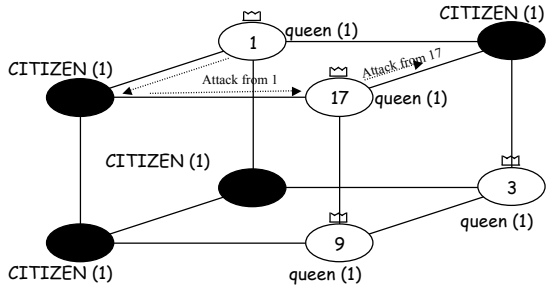
Each citizen knows where its queen is located.

Example of attack on Link 2



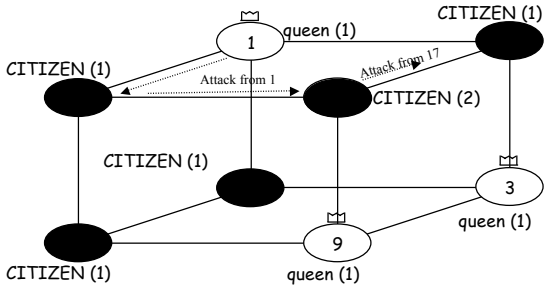
- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

Forward to my queen



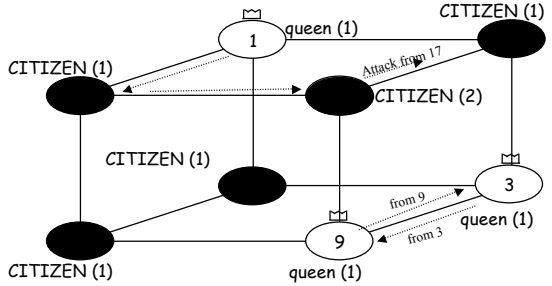
- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

My queen looses



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

Other attacks



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

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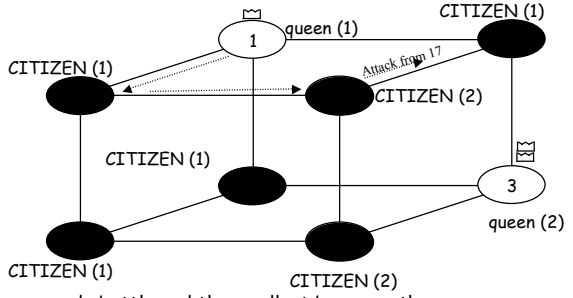
Winners and loosers



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

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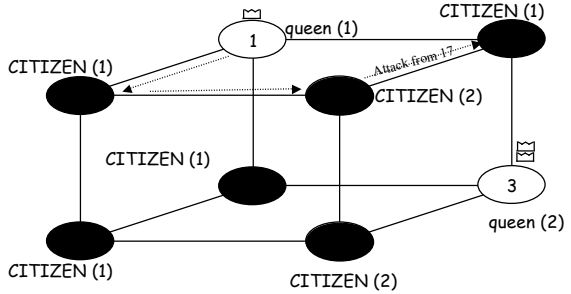
Attack from 17 arrives



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

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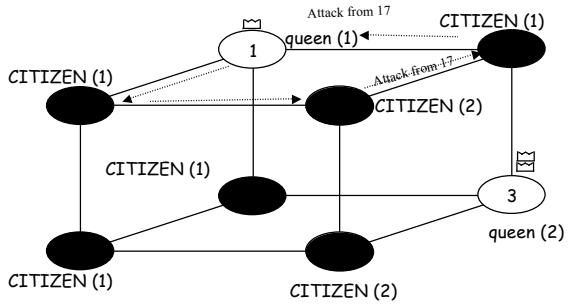
Forward to my queen



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

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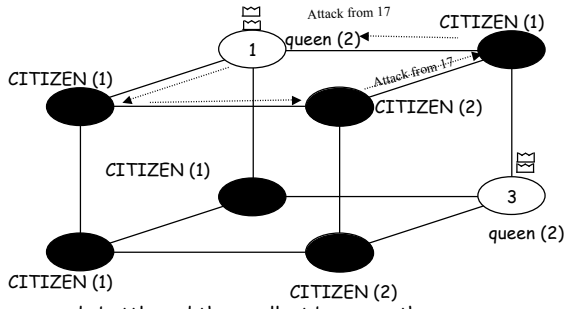
The queen is promoted



- queens do battle and the smallest becomes the new queen.
- If a citizen is attacked by a queen, it will forward the message to its queen.

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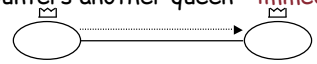
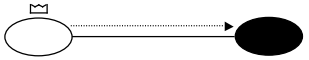
Situations

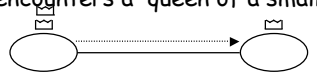
A queen encounters another queen - immediate fight



A queen encounters a citizen of the same level - THE CITIZEN forward the attack to its queen



A queen encounters a queen of a smaller level - WAIT



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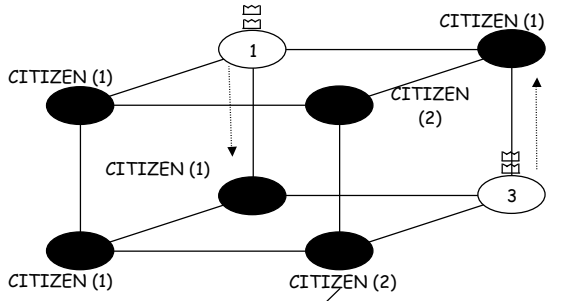
Note: in the example



WAS queen(1)
NOW CITIZEN(2)
Knows where queen(2) (the queen that defeated it) is.

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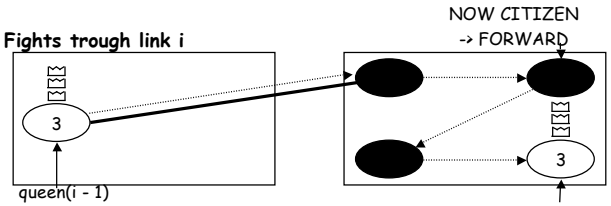
i-th Step



queen(i - 1) could meet:

- a queen(i - 1) --> FIGHT
- a queen(k), k < i - 1 --> WAIT

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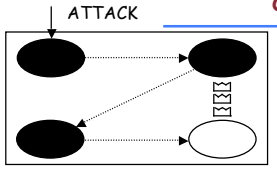
- a CITIZEN(k) --> FORWARD TO ITS queen

Complexity



What is the maximum distance that a citizen must travel to reach the queen?

Worst case: the ATTACK reaches a CITIZEN(1)

CITIZEN(1) forwards to its QUEEN(1)

but QUEEN(1) is now a CITIZEN(2)

CITIZEN(2) forward to its QUEEN(2)

but QUEEN(2) is now a CITIZEN(3)

...

Distance from CIT(1) to queen(1) = 1
Distance from CIT(2) to queen(2) = 2
...
Distance from CIT(i) to queen(i) = i

For a level-i attack:

$$\text{Max-dist} = 1 + \sum_{k=1}^{i-1} k = 1 + (i(i-1)) / 2$$

$O(i^2)$

How Many Steps?

1st Step	n/2 queen(1)
2nd Step	n /4 queen(2)
...	...
i^{th} Step	$n/2^i$ queen(i)
...	...

There is 1 queen (the LEADER) when:

$$n / (2^i) = 1$$

$$i = \log n$$

How Many Messages?

For all steps i>0: $n / 2^{i-1}$ entities send an attack

Number of steps = $\log n$

An attack can cross $1 + (i(i + 1))/2$ links.

$$\text{number of messages} = \sum_{i=1}^{\log n} (n / 2i-1)(1 + (i(i + 1) /2)$$

$= O(n)$

Correctness

Let id(x) be the smallest Id in one of the sub-cubes of dimension i,
Then x will be Queen of level i+1