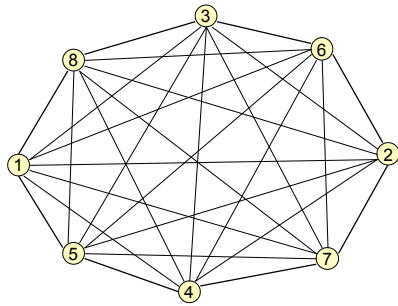


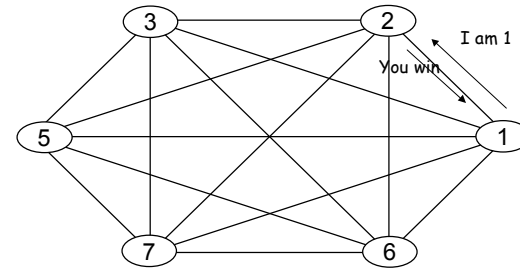
Election in the Complete Graph



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Trivial Algorithm.

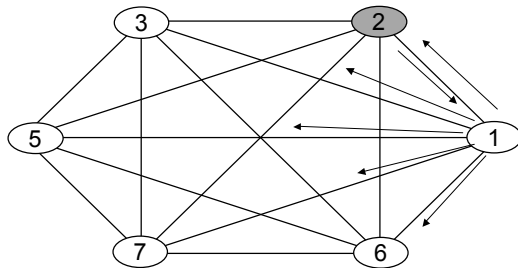
Ask neighbours one at a time



Paola Flocchini

Trivial Algorithm.

Ask neighbours one at a time



Paola Flocchini

Message Complexity

$$O(n^2)$$

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### Better Algorithm

Ideas:

- In stages
- Territory acquisition (capture neighbours) ensuring that a node is captured by at most one candidate in the same stage
- Disjoint territories

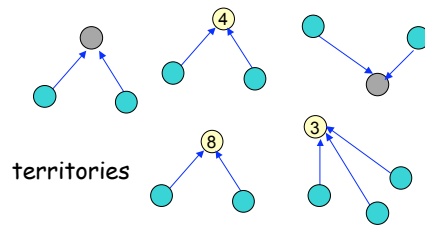
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A node attacks another node, if successful it captures the node increasing the **size of its territory (= stage number)**

Defeated nodes become captured (belonging to a owner) and stop attacking

- **CANDIDATE**: still playing trying to increase the territory
- **PASSIVE**: transitional phase, will not attack anymore, will eventually become captured
- **CAPTURED**: belong to a territory, owned by a candidate

An attack could reach **CANDIDATE PASSIVE CAPTURED**

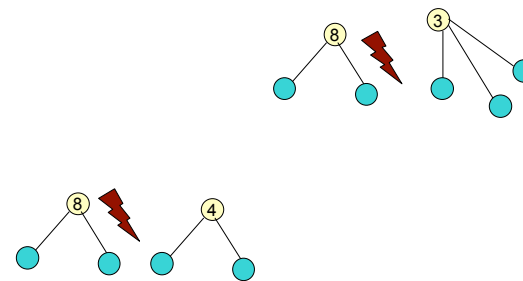


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### The Attack

Bigger territories win over smaller ones (i.e. higher stages)

In case of tie, smaller Ids win



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### The Attack

Bigger territories win over smaller ones (i.e. higher stages)

In case of tie, smaller Ids win

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### Attacking a CANDIDATE

Send capture message to one neighbour

Increase territory and stage (+1)

If still candidate, 3 becomes passive

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### Attacking a PASSIVE

yes

Increase territory and stage (+1)

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### Attacking a CAPTURED

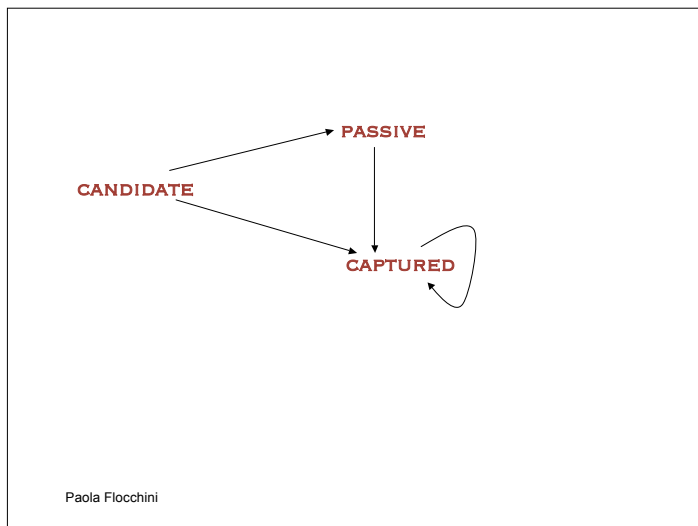
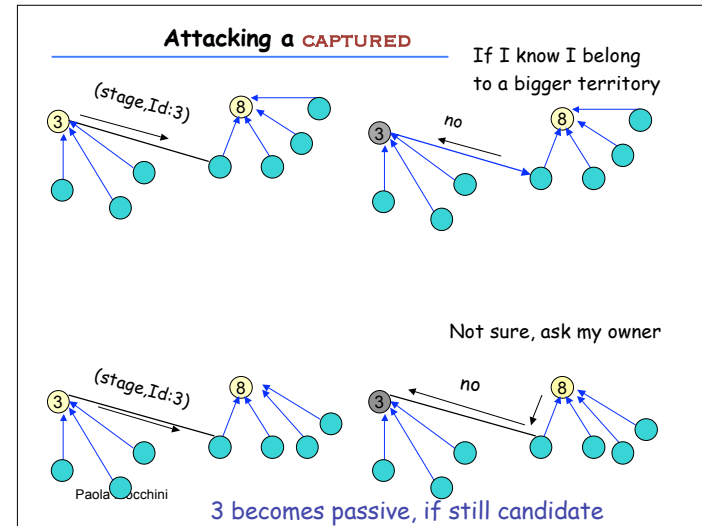
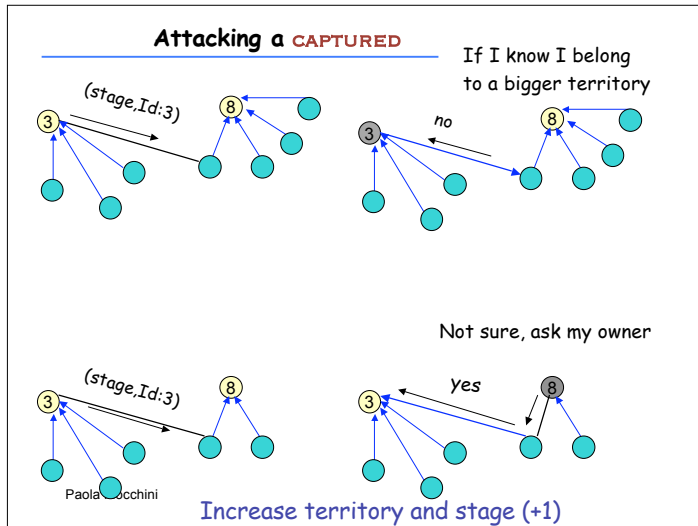
If I know I belong to a bigger territory

no

3 becomes passive, if still candidate

Not sure, ask my owner

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**When to terminate ?**

When does a candidate become a leader ?

When it captures more than  $n/2$  nodes

If a candidate has captured more than  $n/2$  nodes nobody else can become leader

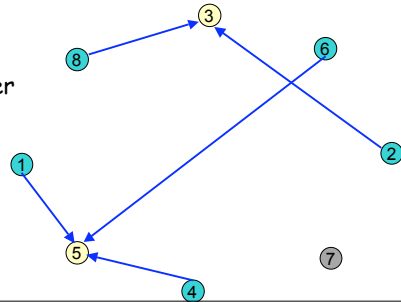
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**Important**

The territories of any two candidates are disjoint

Because at any time, any node has only ONE owner.

Each territory is rooted in its owner



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We need: number of stages and messages per stage

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**Messages per attack**

candidate --- candidate 2 msgs

candidate --- passive 2 msgs

candidate --- captured 4 msgs



At most 4 messages per attack

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Number of stages

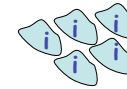
A candidate with  $n/2 + 1$  captured nodes becomes leader and notify

$n/2 + 1$  stages

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How many candidates in each stage ?

Stage  $i \rightarrow$  territory of size  $i$   
With disjoint territories



There cannot be more than  $n/i$  candidates in stage  $i$

$\rightarrow n_i \leq n/i$

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Message Complexity

$n_i \leq n/i$

At most 4 messages per attack

Messages in stage  $i \leq 4 n/i$

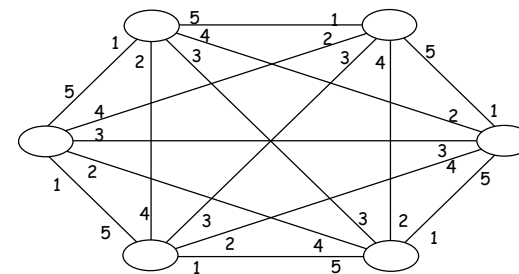
Harmonic number  $H_{n/2}$

$$O\left(\sum_1^{n/2} 4 n/i\right) = O\left(4 n \sum_1^{n/2} 1/i\right) = O(\log n)$$

$M(\text{completeElect}) = O(n \log n)$

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Election in a Complete Graph with Chordal Sense of Direction



Any ring algorithm

**IDEA:** Put information in messages. At the next step, use a smaller ring.

