## Election in Arbitrary Networks

Mega-Merger

Yo-Yo

Some Considerations



## Election in Arbitrary Networks
(Gallager, Humblet, Spira '84)

### The Mega-Merger

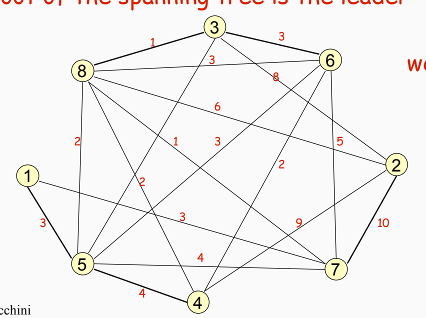
In general networks, the election problem and the spanning tree construction problem are equivalent.



## The Mega-Merger

Minimum spanning tree construction algorithm.

The root of the spanning tree is the leader



weights



## The Mega-Merger

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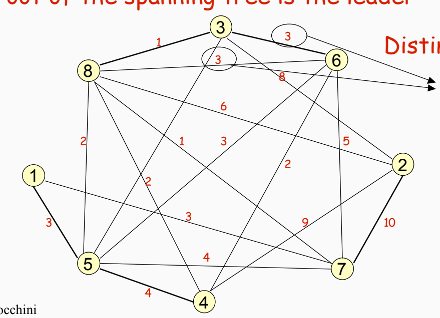

Distinct weights

## The Mega-Merger

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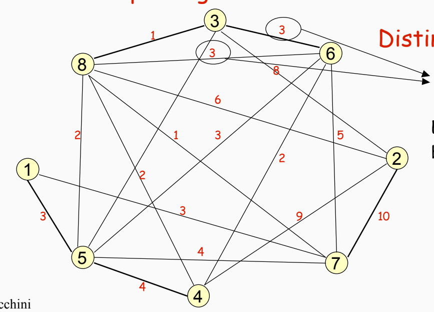


Distinct weights

(3,6)

(8,6)

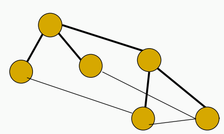
Equal weights can be made distinct



So, from now on we assume that

edges have distinct weigths





node = village with a name

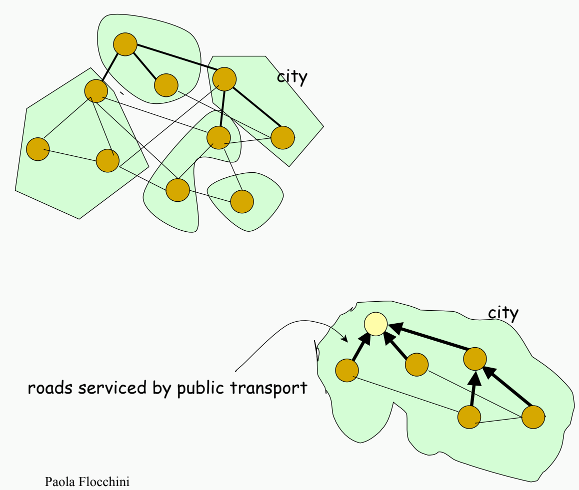
edge = road with a distance

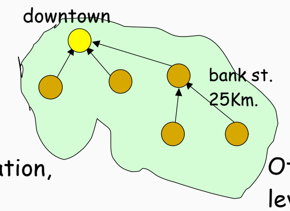
names and distances are different

**The goal**

to merge all the villages into one mega-city








1) City is a subgraph,
its spanning tree
has public transportation,
root is downtown

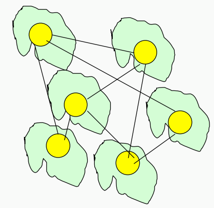
Ottawa
level=i

2) a city has a unique **name** and has a **level**
all districts eventually know the name of the city

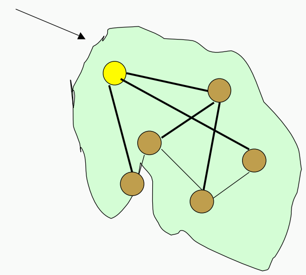
3) Edges are roads with a distinct **name** and **distance**

4) **Initially**: each node is a city with just one district and no roads. All cities are at the same level





Cities are merged into bigger cities until only ONE city is left.



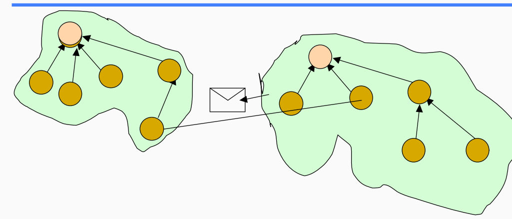


**Issues to consider when merging two cities:**

**How to name the new city**
will depend on several factors

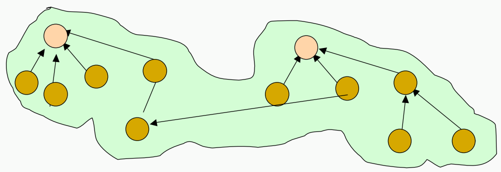
**which roads of a city will be serviced by public transports**
[the roads serviced in the two cities plus a connecting road]





5) A city must merge with the closest neighboring city.
To request a merge, it sends a *let-us-merge* message on the shortest road connecting the cities

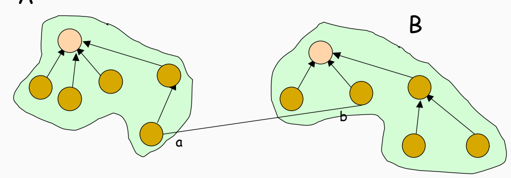
6)The decision to request a merge must come from downtown.
There cannot be more than one request at a time



7) When the merge occurs, the roads of the new city serviced by buses will be the road of the two cities + the connecting road.
The new downtown will depend on several factors.



A: city
D(A): downtown
level(A): level of city A
e(A) = (a,b): merge link with closest city (let it be B)

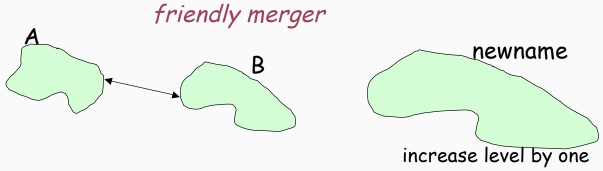



When the request arrives:
- the two cities have the same level
- the two cities have different levels

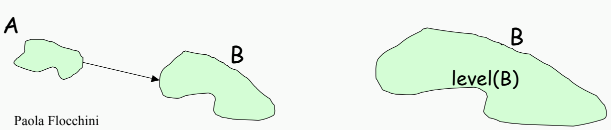


## Let A send the *let-us-merge* message to B

8) If level(A) = level(B) AND the link chosen by A is the same as the one chosen by B (e(A)=e(B)), then:
*friendly merger*



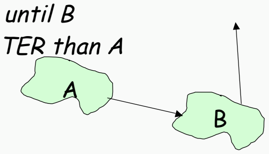

9) If level(A) < level(B) A is absorbed in B



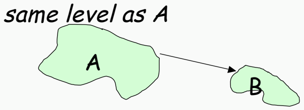


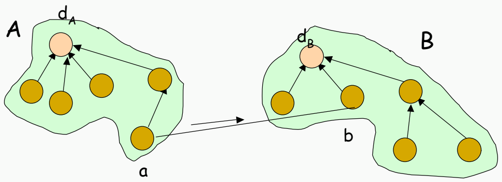

In the other cases the decision is postponed

10) If level(A) = level(B) BUT e(A) ≠ e( B), then:
*the merge is suspended until B arrives at a level GREATER than A*




11) If level(A) > level(B) then:
*the merge is suspended until B arrives at the same level as A*






**Absorption (rule 9)**

level(A) < level(B)

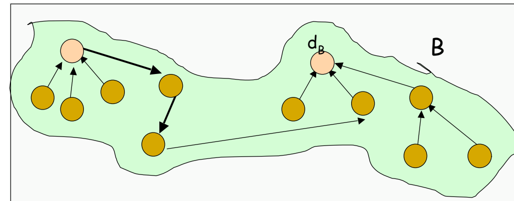
A will be absorbed by B

*b* notifies *a* about the absorption (putting B's name in the message)

a broadcast the info in A

flip all logical link direction to point to the new downtown



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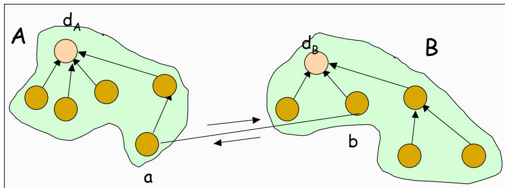
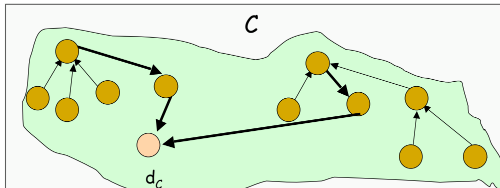



**Friendly Merger (rule 8)**

level(A) = level(B)
e(a)= e(B)

new downtown, new name, new level
downtown = min{*a*,*b*}
newlevel = oldlevel + 1
new name = name of the road connecting *a* and *b*






let a < b

**Friendly Merger (rule 8)**

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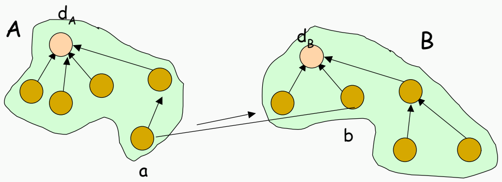
*a* and *b* compute the new info independently and broadcast
the appropriate links are flipped




**Suspension (rules 10,11)**

level(A) = level(B) BUT e(A) ≠ e( B)
or
level(A) > level(B)

*b* locally keeps the necessary info for later

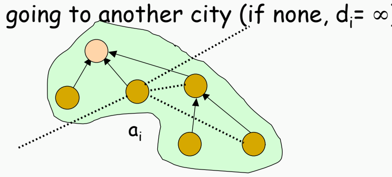
*NOTICE:* nobody in A knows about the suspension
no other request can be launched from A



## Choosing the merging link

$d_A$ needs to find the min length among all edges exiting the city

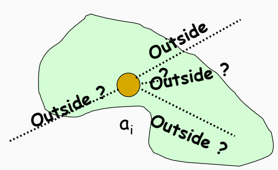
5.1) each district $a_i$ of A determines $d_i$ of the shortest road going to another city (if none, $d_i = \infty$)




5.2) $d_A$ finds the smallest
(min in a rooted tree)

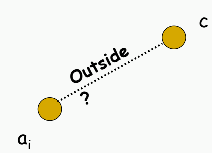


How to compute the shortest road going to another city ...




one at a time






if name(A) = name(C)
reply (internal)

if name(A) ≠ name(C)

the road is not necessarily external
(maybe C has been absorbed by A and c does not know :
in such a case level(C) < level(A))

so:

If **name(A) ≠ name(C)**
and **level(C) ≥ level(A)** then

*reply(external)*

If **name(A) ≠ name(C)**
and **level(C) < level(A)** then

*don't reply*

## More Details Discovering a friendly merger

level(A) = level(B) and e(A) = e( B)

To decide, b needs to know e(A) and e(B)

How does b know e(B) ?

e(B) is chosen by D(B), which will send the request through b

When receiving the request, b will know

So,

If e(A) =e(B), b will eventually know

If e(A) ≠ e(B), b is not the exit point, it will never know what e(B) is.



## More Details Discovering a friendly merger

level(A) = level(B) and e(A) = e( B)

**Receiving a let-us-merge:**

If b has already received a let-us-merge from D(B) to be sent to a

both b and a will know that this is a friendly merger

Otherwise

b waits

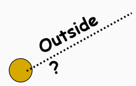
eventually, either it will know that it is a friendly merger
or its level will be increased (because of
requests from B to other cities)
and level(B) will become greater than level(A).
[absorption] [Note: A is waiting,its level cannot increase]



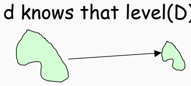
## More Details Deadlocks

waiting cases:

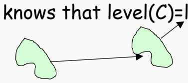
1) c send Outside? to d (level(D)< level(C)



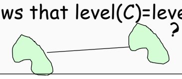

2) receiving let-us-merge on e(C)=(c,d), d knows that level(D) < level(C)



3) receiving let-us-merge on e(C)=(c,d), d knows that level(C)=level(D)
but it is not friendly



4) receiving let-us-merge on e(C)=(c,d), d knows that level(C)=level(D)
but does not know if it is friendly





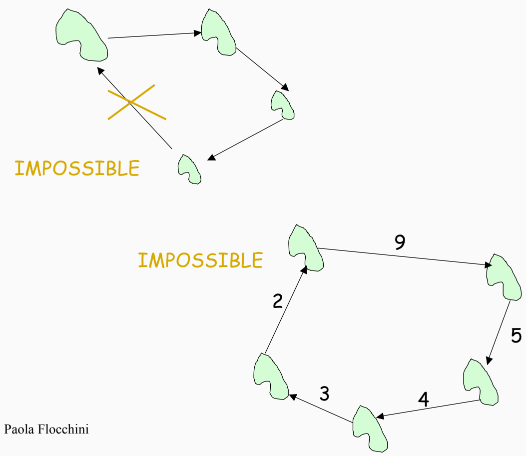

## Correctness

If a city of level l will not be suspended, its level will increase
(unless it is the mega-city)

Let city C at level l be suspended by a district d in D.
If the level of D becomes greater than l, C will no longer be suspended

No city in C will be suspended by a city of higher level

Protocol Mega-merger is deadlock-free

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## Termination

If A is the mega-city, there are no other cities.
All the unused links are internal

The minimum finding will return a special value ( $\infty$)

D(A) understands and broadcasts termination

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## Complexity

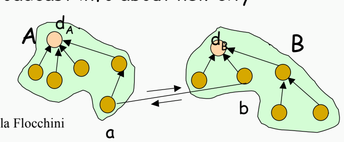
Number of messages per level: CITY A

**For each friendly merger from level i-1 to level i**

Computation of merge links: 2 (n(A)-1)

Forwarding of let-us-merge from D(A) to e(A): n(A)

Broadcast info about new city: n(A)-1




TOT: 4 n(A) - 3

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## Complexity

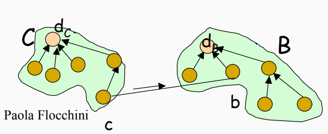
Number of messages per level : CITY C

**C absorbed at level i**

Computation of merge links: 2 (n(C)-1)

Forwarding of let-us-merge from D(C) to e(C): n(C)

Broadcast info about new city: n(C)




TOT: 4 n(C)-2

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## Complexity

disjoint cities, so: $\sum_{B \in City(i)} n(B) \le n$ $\quad City(i) = Merge(i) \cup Absorb(i)$

Number of messages per level

$$\sum_{A \in Merge(i)} (4n(A)-3) + \sum_{C \in Absorb(i)} 4n(C)-2 = 4\sum_{B \in City(i)} n(B) - \text{somethingsmall}$$

$$\le 4n$$

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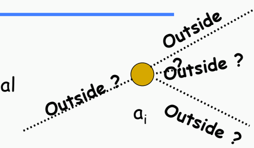
## Complexity

Outside? with answer external




$\le n$

one at a time until found the smallest

Total Cost(i) $\le$ 5n

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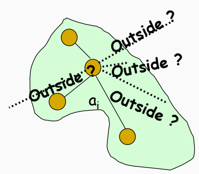
## Complexity

Useless messages Outside? with answer internal

A pair for each road that is not in the city

2(m - (n-1))

Broadcasting Termination: n-1

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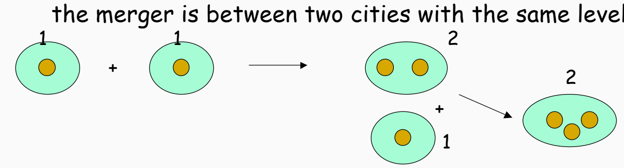
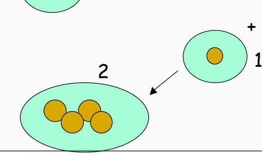
## Complexity

**How many levels ?**

The level is incremented only if
the merger is between two cities with the same level




Level 2 there are at least 2 nodes
(maybe MORE)

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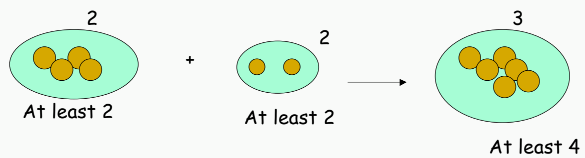


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## Complexity

In general, at Level i there are at least $2^i$ nodes (maybe MORE)

$$\text{Nodes at level } i \geq 2^i$$

$$n \geq 2^i$$

$$i \leq \log n$$

useless notification merges

**Total:** $\leq 2(m - (n-1)) + n-1 + 5n \log n$

$\leq 2m + 5\, n \log n + n+1$

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