Introduction to Number Systems

COMP 1002/1402

Objectives

- Introduce basics of numbering systems
- Use decimal, octal, binary and hexadecimal
- Convert: standard notation expanded numbers
- Add octal, binary & hexadecimal numbers

Decimal Numbers

Q: How do we *interpret* the decimal number 1921?

Ans: Weigh each digit by its position

1000	***100***	***10***	***1***
10^3	10^2	10^1	10^0
1	***9***	***2***	***1***

$1920 = 1 * 10^3 + 9 * 10^2 + 2 * 10^1 + 1 * 10^0$

Fractional Decimals?

Q: How do we *interpret* the number ***0.1921***?

Ans: Weigh each digit by its position

0.1	***0.01***	***0.001***	***0.0001***
10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	***9***	***2***	***1***

$0.1920 = 1 * 10^{-1} + 9 * 10^{-2} + 2 * 10^{-3} + 1 * 10^{-4}$

An Arbitrary Base (*b*)

- A sequence of digits represents a number
- Each location has a weight
- Positions are labelled ***0-n*** from right
- i^{th} position weight = b^i

An Arbitrary Base

A number in any “base” ***b*** is written:

$d_n\ d_{n-1}\ d_{n-2} \ldots\ d_2\ d_1\ d_0$

where $0 < d_i < b-1$
and d_0 is the Least Significant Digit.

The number represents the value:

$d_n * b^n + d_{n-1} * b^{n-1} + \ldots + d_1 * b^1 + d_0 * b^0$

Fractions in an Arbitrary Base

Extending fractional notation yields:

0. $d_{-1}\, d_{-2}\, d_{-3}$ **represents the value**

$d_{-1}*b^{-1} + d_{-2}*b^{-2} + d_{-3}*b^{-3}$

Notation

Q: How do you know the base?

Ans: Label using a subscript,
if not labelled assume decimal:
e.g.: $(d_3 d_2 d_1 d_0)_b$

Note: $(10)_b = b$

Binary Number System

Only two digits required ***{1,0}***

2^1	2^0	Base 10 Equivalent
	0	0
	1	1
1	0	2
1	1	3

Binary Numbers

What is the following number?

$\boldsymbol{(101101)_2}$

$\boldsymbol{=(1*64+0*32+1*16+1*8+1*4+0*2+1*1)_{10}}$

$\boldsymbol{=(93)_{10}}$

Binary Numbers

What is the following number?

$\boldsymbol{(11011001)_2}$

Fractions in Base 2

What is the following fraction?

$\boldsymbol{(0.1101)_2}$

$\boldsymbol{=(1*.5 \; +1*.25 \; +0*.125 +1*.0625)_{10}}$

$\boldsymbol{=(0.8125)_{10}}$

Fractions in Base 2?

What is the following fraction?

$\boldsymbol{(0.011001)_2}$

Addition in Base 2

Addition Table

+	0	1
0	0	1
1	1	10

Example:

```
  1 1 1
+ 1 1 1
-------
1 1 1 0
```

Multiplication in Base 2

Multiplication Table

*	0	1
0	0	0
1	0	1

Example:

```
        1 1 1
*       1 1 1
-------------
        1 1 1
      1 1 1
    1 1 1
-------------
  1 1 0 0 0 1
-------------
```

Division in Binary

Don’t worry about it.

It is in one of the notes but we will not test it.

Decimal to Binary Conversion

Algorithm generates binary digits from ***0*** to ***n***:

```
Q = decimal number
While Q is not equal to 0 do the
  following
       Binary digit = Q mod 2
       Q = Q / 2    (quotient)
End While
```

Conversion Example

Convert $(58)_{10}$ to $(?)_2$

58 mod 2 = 0   ***29***
29 mod 2 = 1   ***14***
14 mod 2 = 0   ***7***
7 mod 2 = 1   ***3***
3 mod 2 = 1   ***1***
1 mod 2 = 1   ***1***   Ans: $(111010)_2$

Conversion:

Convert $\boldsymbol{(271)_{10}}$ to $\boldsymbol{(?)_2}$

Binary, Octal and Hexadecimal

Binary easily corresponds to the on/off state of electronics

Octal and Hexadecimal are often used due to ease of conversion to and from binary

Hexadecimal

In hexadecimal the base is $\boldsymbol{16=2^4}$

16 digits are: ***{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}***

$\boldsymbol{(1AF)_{16}}$
$\boldsymbol{=(1*16^2+ 10 * 16^1+15* 16^0)_{10}}$
$\boldsymbol{=(431)_{10}}$
What is: $\boldsymbol{(3A1F)_{16?}}$

Hex to Binary

Converting from Hex to Binary is trivial:

Every hex digit becomes 4 binary digits

$\boldsymbol{(1AF5)_{16}}$
$\boldsymbol{=(0001\ \ 1010\ \ 1111\ \ 0101)_2}$
$\boldsymbol{=(1101011110101)_2}$

Binary to Hex

Just as simple, reverse to process

$(111001010101011101)_2$
$=(0011\ \ 1001\ \ 0101\ \ 0101\ \ 1101)_2$
$=(3955D)_{16}$

Octal

In octal the base is $8=2^3$

8 digits are: $\{0,1,2,3,4,5,6,7\}$

$(153)_8$
$=(1*8^2 + 5 * 8^1 + 3* 8^0)_{10}$
$=(107)_{10}$

Octal to Binary

Converting from Octal to Binary is trivial:

Every octal digit becomes 3 binary digits

$(175)_8$
$=(001\ \ 111\ \ 101)_2$
$=(1111101)_2$

Octal to Hex

Just as simple, reverse to process

$(11001010101011101)_2$
$=(011\ \ 001\ \ 010\ \ 101\ \ 011\ 101)_2$
$=(312535)_8$