Skiplists

COMP2402
Carleton University
Winter 2018
Skiplists

- SinglyLinkedList augmented with 'skips'
- Randomly generated structure
- Faster searches than linked lists

Reading:
- ODS Sections 4.1-4.3
Theorem:

The *expected* length of the search path for any node, $u$, in $L_0$ is at most $2 \log(n) + O(1) = O(\log n)$. 

Search Path
SSet Interface

**find**(x): find the smallest value in the set that is greater than or equal to x

**add**(x): add the item x into the set in sorted order, ignoring duplicates

**remove**(x): remove the item from the set if it is there
Theorem:

A **SkiplistSSet** implements the **SSet** interface.
A SkiplistSSet supports the operations:

- `add(x)`, `remove(x)`, and `find(x)` in $O(\log n)$ expected time per operation
<table>
<thead>
<tr>
<th>Implementation</th>
<th>get/set</th>
<th>add/remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrays</td>
<td>O(1)</td>
<td>O(1 + min{i, n-i})</td>
</tr>
<tr>
<td>LinkedLists</td>
<td>O(1 + min{i, n-i})</td>
<td>O(1)**</td>
</tr>
<tr>
<td>Skiplists</td>
<td>O(log n)</td>
<td>O(log n)</td>
</tr>
</tbody>
</table>

** given a pointer to location
Theorem:

A **SkiplistList** implements the **List** interface. A SkiplistList supports the operations:

- `get(i)`, `set(i,x)`, `add(i,x)`, and `remove(i)` in $O(\log n)$ expected time per operation.
Definitions

Random Variable: A random sample from a group of values.

Expected Value: Average value of a random variable.

Indicator Variables: Random variable with values 0 or 1.

Linearity of Expectation: The expected value of a sum is equal to the sum of expected values.
Skiplist Analysis

Theorem:

The expected length of the search path for any node, \( u \), in \( L_0 \) is at most \( 2\log(n) + O(1) = O(\log n) \).

Outline:

• Expected number of coin tosses before Heads
• Total size of skiplists
• Search Path length
  – Height & width