Graphs

COMP2402
Carleton University
Winter 2018
Graphs

• A graph $G$, is a pair of sets $(V,E)$
  – $V =$ vertices/nodes
  – $E =$ edges
  – $n =$ # of vertices
  – $m =$ # of edges

• Reading:
  – ODS Chapter 12
Graph Interface

- `addEdge(i, j)` – adds the edge \((i, j)\) to \(E\)
- `removeEdge(i, j)` – removes the edge \((i, j)\) from \(E\)
- `hasEdge(i, j)` – returns true if \((i, j)\) is in \(E\), false otherwise
- `outEdges(i)` – returns a list of all vertices \(j\) where \((i, j)\) is in \(E\)
- `inEdges(i)` – returns a list of all vertices \(j\) where \((j, i)\) is in \(E\)`
## Graph Interface

<table>
<thead>
<tr>
<th></th>
<th>Adjacency Matrix</th>
<th>Adjacency List</th>
</tr>
</thead>
<tbody>
<tr>
<td>addEdge</td>
<td>$O(1)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>removeEdge</td>
<td>$O(1)$</td>
<td>$O(deg(i))$</td>
</tr>
<tr>
<td>hasEdge</td>
<td>$O(1)$</td>
<td>$O(deg(i))$</td>
</tr>
<tr>
<td>outEdges</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>inEdges</td>
<td>$O(n)$</td>
<td>$O(n + m)$</td>
</tr>
<tr>
<td>Space used</td>
<td>$O(n^2)$</td>
<td>$O(n + m)$</td>
</tr>
</tbody>
</table>
Graph Traversals

• Exploration of the entire graph
  – Using **breadth-first** or **depth-first** search order
  – Will find all vertices reachable from a given starting node

• Summary

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Adjacency Matrix</th>
<th>Adjacency List</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breadth First</td>
<td>$O(n^2)$</td>
<td>$O(n + m)$</td>
</tr>
<tr>
<td>Depth First</td>
<td>$O(n^2)$</td>
<td>$O(n + m)$</td>
</tr>
</tbody>
</table>