COMP 3400 Computational Logic
Winter 2018 Assignment 5

Instructions:

1. ## For your solution use the template file that was posted on the course news, and follow the instructions in it.

   In particular: (a) Include at the top of the first page: full name, student number, and email address. (b) Assignments have to be created with Latex, and submitted in pdf format. (c) Every problem solution MUST include the problem statement. The source file for this assignment is provided.

   Latex has to be used as such, not as you would use a text editor, such as Notepad. In particular, formulas have to be written using Latex's mathematical features, and then compiled.

2. Assignments are individual, no groups.

3. Submit by email to the instructor, with "Assignment "Number", CompLog" in the subject. **Include your last name in the file name!** For example, in the subject: "Assig. 5 CL". The file name: "bertossi-5.pdf".

   Only a single pdf file will be accepted as submission. No tar or zip files (or anything like that), please. Keep your Latex source files in case you are requested to show them.

4. Explain your solution very carefully, but still be succinct with your answers. No unnecessary verbose arguments, please. Go to the point.

   Make explicit all your assumptions.

5. **Not following the instructions above or the solution template file will make you lose points.**

The answer-set programs for the problems below have to be run all with the DLV system. The programs have to be general enough to handle different graphs; and those for problems 1 and 2, they should be easily generalizable to handle also different clique sizes. The whole assignment has to be submitted as a single PDF file, including the DLV programs and outputs

1. A clique in an undirected graph $G = (V, E)$ is a subset $V' \subseteq V$ of vertices, each pair of which is connected by an edge in E. The size of a clique is the number of vertices it contains. We want to determine whether a graph has a clique of certain sizes. (The problem of deciding if for a graph there is a clique of size at least K (part of the input) is an NP-complete problem.)

(a) Determine whether the graph in Figure 1 has a clique of size 3 using DLV, in such a way that a stable model will show if the graph has a clique of size 3. Use predicate `clique` for this. Do this by inspecting the stable model(s). [6 points]

(b) Solve previous item, not by inspecting the stable model(s), but by adding (or posing) a query to the program. [3 points]

(c) Use DLV to determine whether the graph in Figure 2 has a clique of size 5, explain your conclusion. [6 points]

(d) Do the same as in 2. above, but for the preceding item. [3 points]

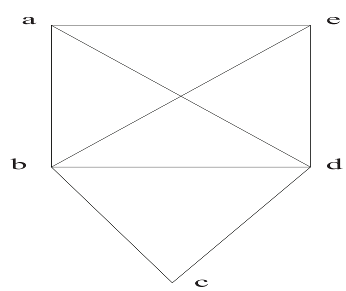

Figure 1:

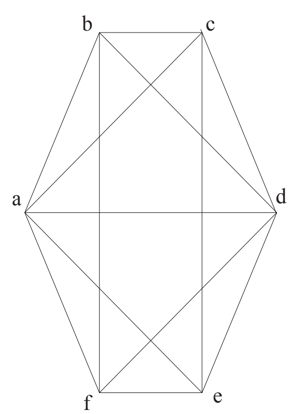

Figure 2:

2. A vertex cover of an undirected graph $G = (V, E)$ is a subset $V' \subseteq V$, such that if $(u, v) \in E$, then $u \in V'$ or $v \in V'$ (or both). We are interested in minimum vertex covers in size, where the size of a vertex cover is its number of vertices. Notice that a graph G has a vertex cover of size N if and only if its complement graph $G^c := (V, E^c)$ (containing all and only the edges that are not in G) has a clique of size $\mid V \mid - N$.

(a) Write a logic program to define the complement graph G^c of a graph G. For that introduce a new edge predicate, say *cedge* (for edge in complement graph). Use DLV to compute the complement graph for that in Figure 3. [6 points]

(b) Extend the program in the preceding item producing one that can be used to determine whether the graph in Figure 3 has a vertex cover of size 2. Run the program in DLV to answer the latter question, by inspecting the stable model(s). [4 points]

Hint: Use problem 1.

(c) Do the same as in the preceding item but through query answering. [3 points]

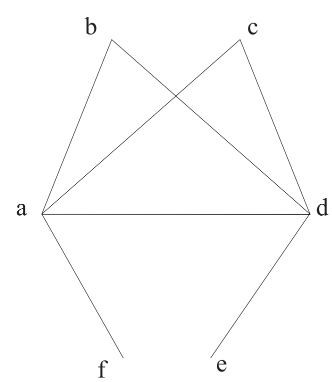

Figure 3:

3. (a) Write a answer-set program to solve the Hamiltonian-cycle problem (HCP), in the sense that the answer sets are in correspondence with the Hamiltonian cycles in a directed graph. You may assume the graph does not have self-loops. (Deciding if a graph has a Hamiltonian cycle, i.e. a closed

path that goes through all the nodes without repetitions, is an NP-complete problem.) [10 points]
Hint: This is not the only way, but you may represent a (simple) cycle by means of a set of atoms of the form: $\{in(a_1, a_2), \ldots, in(a_n, a_1)\}$ (here $n = 5$). A simple way to solve the problem uses the choice operator.

(b) Apply the general program above to solve the HCP for the graph in Figure 4. Do this by inspecting the stable model(s) (if any). [4 points]

(c) Do the same as in (b), but adding or posing a query to the program. [3 points]

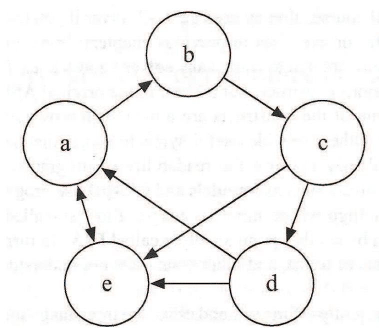

Figure 4:

Deadline: April 11, at 23:55