Concurrent Actions and Interacting Effects

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Abstract

Recently, several authors have proposed extensions to the language of the Situation Calculus in order to deal with concurrent actions. Two important problems have been recognized when modeling these actions. First, the *precondition interaction problem*, which arises when some action can or cannot be performed depending upon what other actions, if any, are performed concurrently. The second problem, the *effects interaction problem*, arises when a set of concurrently executed actions have effects that are different from the conjunction of the effects of each individual action. In this article, we discuss the second of these problems; namely, the *effects interaction problem*. First, we show that straightforward axiomatizations are not amenable to solutions based on policies of minimal change, due to the presence of unwanted minimal models. Then, we discuss alternative approaches to solve this problem. We advocate for an approach in which the interaction is decomposed along the temporal dimension, allowing the use minimal change and the elimination of spurious models.

1 INTRODUCTION

We explore the problem of modeling concurrency in theories of action within the framework of the Situation Calculus (McCarthy and Hayes 1969). Our work is based upon the premise that primitive actions are instantaneous. Actions with *durations* can be modeled as pairs of actions. Thus, the action "pour a cup of coffee" would be modeled with the start and the end of the pouring action. With each such action, one associates a fluent that is true while the action is in progress. As a consequence of treating actions with duration in this manner, problems like Allen's *door-latch* problem (Allen 1991) can be represented in terms of sequential actions. However, this way of modeling actions with durations, in a branching time language, introduces problems of its own. In particular, assume that one wants to model an action with duration that has an upper bound on its duration. To do this properly, one must ensure that in no path (in the branching structure) the action is started and not ended, or ended at a time that violates the duration upper bound. This problem is addressed in (Baier and Pinto 1998).

Primitive, instantaneous actions, when executed concurrently, may interact in two different ways. Firstly, there is *precondition interaction*, that arises when actions can or cannot be performed, depending upon whether other actions are performed concurrently. In general, this interaction arises when the actions require resources that might not be available if the actions are executed in parallel. For example, if there is one paint brush and two painters, then the actions of each painter painting interact, and they cannot be performed simultaneously.

Secondly, the effects of concurrently executed actions may also interact. In turn, this interaction can arise in two ways, as *cancellation* or as *synergy* (Pinto 1994). Cancellation is the phenomenon by which two actions cancel each other's effects when performed concurrently. For example, if a door is pushed and pulled at the same time and with the same force. Synergy arises when actions that are performed concurrently produce effects that neither action would have if performed in isolation. For example, lifting both ends of a table simultaneously has the effect of raising the table.

In this article, we focus our attention on the second type of interaction. A possible approach to deal with the *precondition interaction* problem is currently under development.

The article is organized as follows. In section 2 we present the formal foundations of the concurrent Situation Calculus. In section 3, we present the general structure of theories of action with concurrency. This structure is a straightforward extension of the style of axiomatization proposed by Reiter for theories without concurrency (Reiter 1991). In section 4 we define, in precise terms, the notion of *interacting effects* within the models of a theory of action. Later, in section 5, we show that the minimal change models of theories of action (as defined in section 3), in which there are interacting effects, do not have a single minimal model, leading to spurious minimal models. In section 6 we discuss approaches to solve this problem. In particular, we advocate for a solution in which the interacting effects are decomposed along the temporal dimension. This leads to axiomatizations in which *trigger* actions are used as intermediary agents of change. Finally, in section 8 we present our conclusions.

2 FORMAL FOUNDATIONS

We assume familiarity with the basics of the Situation Calculus (McCarthy and Hayes 1969), and with Reiter's monotonic solution to the frame problem (Reiter 1991). Our work is based on the concurrent Situation Calculus (Pinto 1994; Reiter 1996), which extends the Situation Calculus with a sort for concurrent actions. Concurrent actions are treated as sets of atomic actions.

The concurrent Situation Calculus is a second order language with sorts $\mathcal{A}$, $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{D}$ for atomic actions, concurrent actions, situations and domain objects. The sort of the variables and constants used in the examples should be obvious from the context. There is a distinguished initial situation denoted with the constant S_0. The function *do* takes a concurrent action and a situation and yields another situation. Fluents are predicates that take one argument of type situation, and represent properties that are static within a situation, but that may change between situations. Also, we have the predicate *Poss*, which takes a concurrent action and a situation, and should hold when the concurrent action is executable in the situation. The foundational axioms for the concurrent Situation Calculus are:

$$(\forall\varphi).[\varphi(S_0) \wedge (\forall s,c)\ (\varphi(s) \supset \varphi(do(c,s)))] \supset (\forall s)\ \varphi(s), \tag{1}$$

$$do(c_1,s_1) = do(c_2,s_2) \supset c_1 = c_2 \wedge s_1 = s_2, \tag{2}$$

$$\neg s < S_0, \tag{3}$$

$$s < do(c,s') \equiv Poss(c,s') \wedge s \leq s'. \tag{4}$$

Concurrent actions are treated as sets of primitive actions. Thus, $\in$ is used as a relation between atomic actions and concurrent actions. We write $a \in c$ to mean that action a is part of the concurrent action c.

As discussed later, a domain axiomatization only includes axioms describing preconditions for atomic actions. For non-atomic actions, we have:

$$Poss(c,s) \equiv (\forall a)[a \in c \supset Poss(\{a\},s)]. \tag{5}$$

This axiom embodies the assumption that actions can be performed concurrently, if and only if all the component actions can be performed individually. In general, this is not correct, since there are actions that can be performed concurrently, but not individually and vice-versa.

In the following section we describe how to write theories of action based on the language of the concurrent Situation Calculus. Our approach is based upon Reiter's monotonic solution to the frame problem.

3 DOMAIN AXIOMATIZATION

A domain axiomatization is divided in several sets of axioms. First, we have a set T_d of axioms that only mention terms of sort $\mathcal{D}$. Also, we have a suitable set of unique names axioms (T_{una}, which we omit) for terms of sorts $\mathcal{A}$ and $\mathcal{D}$. The other sets of axioms are described below.

Action Precondition Axioms

The set T_{prec} of action precondition axioms for atomic actions, each of the form[1]:

$$Poss(\{a_l(\underline{\mathbf{x}})\},s) \supset \Phi_{a_l}(\underline{\mathbf{x}},s), \tag{6}$$

where $a_l(\underline{\mathbf{x}})$ is an $\mathcal{A}$ term, and $\Phi_{a_l}(\underline{\mathbf{x}},s)$ is a formula simple on s. A formula is simple on a situation term s_t, if it does not mention any situation terms, other than s_t, and does not quantify over s_t (Reiter 1991). Thus, preconditions for actions are formulated as necessary conditions for *Poss*. If we assume, as we will, that there are no *qualification state constraints* (Lin and Reiter 1994), we can apply Clark's completion to obtain a complete characterization of *Poss*.

[1] We write $\underline{\mathbf{x}}$ to denote a tuple of domain variables (i.e., variables of sort $\mathcal{D}$).

Example: Let us consider the following variant of the soup bowl example from (Gelfond, Lifschitz, and Rabinov 1991): There is a bowl filled with soup, the bowl can be lifted from either of two sides (left and right). If the bowl is lifted from one side only, then the soup spills. If the bowl is lifted from both sides simultaneously, then there is no spillage. To model this example, we have the action constants $Lift_l$, and $Lift_r$. There are three fluents $lifted_l$, $lifted_r$, and *spilling*.

The action precondition axioms for the actions in this domain are:

$$Poss(\{Lift_l\},s) \supset \neg lifted_l(s), \quad (7)$$
$$Poss(\{Lift_r\},s) \supset \neg lifted_r(s), \quad (8)$$

The assumption that these preconditions for performing actions are not only necessary, but also sufficient, leads to transforming each implication (7)-(8) into an equivalence (Clark's completion). A more general treatment of preconditions goes beyond the scope of this article[2].

Direct Effect Axioms

The set T_{eff} of effect axioms. These axioms are restricted to specify only the effects that are a direct consequence of the action, regardless of what other actions, if any, are performed concurrently.

Since *Poss* only takes concurrent actions as first parameter, these axioms have a slightly different syntactic form from Reiter's effect axioms (Reiter 1991). A positive direct effect axiom is of the syntactic form (9), while a negative direct effect axiom is of the form (10).

$$Poss(c,s) \wedge A \in c \wedge G_f^+(\underline{\mathbf{x}},A,s) \supset f(\underline{\mathbf{x}},do(c,s)), \quad (9)$$
$$Poss(c,s) \wedge A \in c \wedge G_f^-(\underline{\mathbf{x}},A,s) \supset \neg f(\underline{\mathbf{x}},do(c,s)). \quad (10)$$

Here, G_f^+ and G_f^- denote qualifications for the effects of an action on the fluent f.

Now, all the effect axioms can be compiled together in one positive and one negative general direct effect axiom of the forms (11) and (12).

$$Poss(c,s) \wedge \gamma_f^+(\underline{\mathbf{x}},c,s) \supset f(\underline{\mathbf{x}},do(c,s)), \quad (11)$$
$$Poss(c,s) \wedge \gamma_f^-(\underline{\mathbf{x}},c,s) \supset \neg f(\underline{\mathbf{x}},do(c,s)). \quad (12)$$

If there are no positive (or negative) effect axioms for a fluent f, then the formula $\gamma_f^+(\underline{\mathbf{x}},c,s)$ (or $\gamma_f^-(\underline{\mathbf{x}},c,s)$ for the negative case) is the atom *False*.

[2] For a more general treatment of qualifications for actions see (Lin and Reiter 1994).

Example (cont.): For the example, we have:

$$Poss(c,s) \wedge Lift_l \in c \supset lifted_l(do(c,s)),$$
$$Poss(c,s) \wedge Lift_r \in c \supset lifted_r(do(c,s)).$$

The Initial Situation

A set of axioms, T_{S_0}, characterizing the initial situation S_0.

Example (cont.): The initial situation for the example is characterized with:

$$\neg lifted_l(S_0) \wedge \neg lifted_r(S_0) \wedge \neg spilling(S_0).$$

Successor State Axioms

Reiter's monotonic solution to the frame problem leads to the derivation of a set of successor state axioms (one per fluent in the language). In the absence of state constraints (i.e., if T_{rc}, below, is empty), the successor state axioms for a fluent f would be derived from (11)-(12) along with the *explanation closure assumption* (Schubert 1990):

$$Poss(c,s) \supset [f(\underline{\mathbf{x}},do(c,s)) \equiv \gamma_f^+(\underline{\mathbf{x}},c,s) \vee f(\underline{\mathbf{x}},s) \wedge \neg\gamma_f^-(\underline{\mathbf{x}},c,s)]. \quad (13)$$

These axioms state necessary and sufficient conditions for fluents to change after some possible action is performed. This solution was originally proposed for theories of actions without concurrency and without state constraints. This work was extended to theories in which state constraints are present (Lin and Reiter 1994; Pinto 1994). These results show that Reiter's solution is not always applicable. In particular, in (Pinto 1994), it is shown that the approach is not directly applicable when there are constraints such as (15), below. If Σ denotes a theory formed with the previous sets of axioms, we will write $\mathcal{R}(\Sigma)$ to denote the theory Σ extended with the successor state axioms.

Ramification State Constraints

A set T_{rc} of ramification state constraints (Lin and Reiter 1994). In general, ramification constraints are of the form:

$$\Psi(\underline{\mathbf{x}},s), \quad (14)$$

where $\Psi(\underline{\mathbf{x}},s)$ is a simple formula. We assume that constraints are written as clauses. Also, it might be convenient to write these clauses as implications.

Example (cont.): For this domain we have:

$$spilling(s) \equiv [\neg lifted_l(s) \wedge lifted_r(s) \vee lifted_l(s) \wedge \neg lifted_r(s)] \tag{15}$$

Thus, this constraint states that *spilling* holds if and only if a single side of the bowl is lifted.

4 INTERACTING EFFECTS

In the following discussion, we assume a theory of action Σ_{rc}, composed of axioms T_d, T_{una}, T_{prec}, T_{eff}, T_{rc} and T_{S_0}, as in the previous section, in addition to the foundational axioms of section 2.

At an informal level, we say that there is interaction between the effects of actions when there is some concurrent action that either:

- has effects that are not present when the sub-actions are performed individually (synergy), or
- does not have effects that some of its sub-actions have when performed individually (cancellation).

The example with the bowl of soup is a case where there is cancellation of effects. Performing $Lift_l$ together with $Lift_r$ cancels the *spilling* effect that each action would have, if performed by itself.

In order to simplify the following discussion, we assume that fluents are propositional (i.e., they only take a situation argument). However, it is straightforward to extend the analysis below to the more general case.

In what follows, we use σ_a, σ_c and σ_s to denote assignment functions from free variables of sort $\mathcal{A}$, $\mathcal{C}$ and $\mathcal{S}$ to objects of the corresponding sorts. Before we can formalize the notion of effect interaction, we need the following:

Definition 4.1 (Model Similarity) *Let Σ_{rc} be a theory of action, and σ_s be an assignment function from free situation variables to situations. We say that two models, $\mathcal{M}$ and $\mathcal{M}'$ of Σ_{rc} are similar with respect to $\sigma_s(s)$ iff:*

- *$\mathcal{M}$, $\mathcal{M}'$ have the same universe, and differ only in their interpretation of fluents,*
- *for any fluent f:*

$$\mathcal{M}, \sigma_s \models f(s)$$

if and only if

$$\mathcal{M}', \sigma_s \models f(s).$$

Definition 4.2 (Synergy) *Synergistic effects arise in a set $\mathbf{M}_{\Sigma_{rc}}$ of models of a theory of action Σ_{rc}, with respect to a fluent f, when there is a model $\mathcal{M}$ in $\mathbf{M}_{\Sigma_{rc}}$, and assignment functions σ_s and σ_c such that:*

1. *In the model $\mathcal{M}$, there is a concurrent action that makes the fluent f change in some situation:*

$$\mathcal{M}, \sigma_s, \sigma_c \models Poss(c,s) \wedge ab(f,s,c).$$

where $ab(f,s,c)$ is a shorthand for:

$$f(s) \equiv \neg f(do(c,s)).$$

2. *For any model $\mathcal{M}'$ in $\mathbf{M}_{\Sigma_{rc}}$, similar to $\mathcal{M}$ with respect to $\sigma_s(s)$,*

$$\mathcal{M}', \sigma_s, \sigma_c \models ab(f,s,c).$$

3. *For some model $\mathcal{M}''$ in $\mathbf{M}_{\Sigma_{rc}}$ similar to $\mathcal{M}$ with respect to $\sigma_s(s)$, there is no assignment σ_a, such that:*

$$\mathcal{M}'', \sigma_s, \sigma_c, \sigma_a \models a \in c \wedge ab(f,s,\{a\}).$$

Intuitively, condition 1 states that there is a model and situation $\sigma_s(s)$ in which f changes when the action $\sigma_c(c)$ is performed. Furthermore, condition 2 states that f must change in the models in $\mathbf{M}_{\Sigma_{rc}}$. I.e., there is no model in $\mathbf{M}_{\Sigma_{rc}}$, similar to the first one with respect to $\sigma_s(s)$, that does not change f when the concurrent action is performed. Condition 3 states that synergy arises. I.e., the change arises with the concurrent action $\sigma_c(c)$, but that there is no single action in $\sigma_c(c)$ that forces f to change.

Definition 4.3 (Cancellation of Effects) *Cancellation of effects arise in a set $\mathbf{M}_{\Sigma_{rc}}$ of models of a theory of action Σ_{rc}, with respect to a fluent f, when there is a model $\mathcal{M}$ in $\mathbf{M}_{\Sigma_{rc}}$, and assignment functions σ_s, σ_a and σ_c such that:*

1. *In the model $\mathcal{M}$, the fluent f does not change when some concurrent action is performed:*

$$\mathcal{M}, \sigma_s, \sigma_c \models Poss(c,s) \wedge \neg ab(f,s,c).$$

2. *For any model $\mathcal{M}'$ in $\mathbf{M}_{\Sigma_{rc}}$, similar to $\mathcal{M}$ with respect to $\sigma_s(s)$,*

$$\mathcal{M}', \sigma_s, \sigma_c \models \neg ab(f,s,c).$$

3. *For some model $\mathcal{M}''$ in $\mathbf{M}_{\Sigma_{rc}}$ similar to $\mathcal{M}$ with respect to $\sigma_s(s)$:*

$$\mathcal{M}'', \sigma_s, \sigma_c, \sigma_a \models a \in c \wedge ab(f,s,\{a\}).$$

Intuitively, condition 1 states that there is a model in $\mathbf{M}_{\Sigma_{rc}}$ and situation in which f does not change when some specific concurrent action is performed. On the other hand 2, ensures that f must not change. I.e., there is no model in $\mathbf{M}_{\Sigma_{rc}}$, similar to the first one with respect to $\sigma_s(s)$, that changes f when the concurrent action is performed. Condition 3 states that an effect has been canceled. I.e., there is a primitive action, that is included in the concurrent action, mentioned above, after which the change arises.

Example (cont.): As mentioned before, the bowl of soup example is a case of cancellation. Indeed, if Σ_{bowl} is the theory of action for that example:

$$\Sigma_{bowl} \models Poss(\{Lift_l, Lift_r\}, S_0) \wedge Lift_l \in \{Lift_l, Lift_r\},$$
$$\Sigma_{bowl} \not\models ab(spilling, S_0, \{Lift_l\}),$$
$$\Sigma_{bowl} \not\models \neg ab(spilling, S_0, \{Lift_l\}),$$

and

$$\Sigma_{bowl} \models \neg ab(spilling, S_0, \{Lift_l, Lift_r\}).$$

From here, it is straightforward to verify that the conditions for cancellation are present in the set of first order models of Σ_{bowl} (they are also present in the minimal change models of Σ_{bowl}.

Observations

The definitions for synergy and cancellation given above are based on comparing the results of performing a concurrent action c, and the results of performing some primitive sub-action a, where $a \in c$. However, more general definitions can be given by comparing the results of performing a concurrent action c and the results of performing some other concurrent action c', such that $c' \subsetneq c$. It is not hard to rewrite the definitions. But, we kept the definitions presented here for simplicity of exposition.

Notice that, as motivated by many examples, synergy and cancellation are properties that we would like our theories to exhibit. Therefore, after solving the frame problem, we would like the resulting theories to exhibit these properties as well. The result that we show, is that if the minimal models of a theory of action exhibit synergy or cancellation, then the theory must admit more than one minimal model. Thus, there might be spurious minimal models.

5 MINIMAL CHANGE POLICIES

In the previous section we presented a formal characterization of the interaction between the effects of actions. Up until now, we have assumed that the theory of action includes axioms describing necessary effects, along with axioms establishing ramification state constraints. It is well known that these axioms are not enough to characterize change, and that some approach to derive non-change should also be considered, in the form of *frame axioms* or by some other means. A common approach to complete the specification of the theory of action is to incorporate the assumption of minimal change. This minimal change policy can be formalized with the use of circumscription (McCarthy 1986), as in (Baker 1989; Shanahan 1997). Also, in some cases this assumption can be realized by using *explanation closure* (Schubert 1990), which leads to the formulation of successor state axioms (Reiter 1993). This latter approach results in a theory in which the effects of actions are completely characterized, given complete knowledge about the state in which they are performed.

Here, we argue that an approach based on minimal change cannot properly characterize change in theories of the form discussed in the previous section, in which there is interaction between the effects of actions.

The following definition (based on (Lin and Reiter 1994)) characterizes minimal change models:

Definition 5.1 (Minimal Model) *A model $\mathcal{M}$ of a theory of action Σ_{rc} is minimal relative to $\sigma_s(s)$ iff there is no other model $\mathcal{M}'$ of Σ_{rc}, similar to $\mathcal{M}$ with respect to $\sigma_s(s)$, such that:*

- *For any assignment σ_c, and any fluent f:*

$$\begin{array}{rl} \textit{if} & \mathcal{M}, \sigma_s, \sigma_c \models Poss(c,s) \wedge \neg ab(f,s,c), \\ \textit{then} & \mathcal{M}', \sigma_s, \sigma_c \models Poss(c,s) \wedge \neg ab(f,s,c). \end{array}$$

- *There is an assignment σ_c, and a fluent f, such that:*

$$\begin{array}{rl} & \mathcal{M}, \sigma_s, \sigma_c \models Poss(c,s) \wedge ab(f,s,c), \\ \textit{but} & \mathcal{M}', \sigma_s, \sigma_c \models Poss(c,s) \wedge \neg ab(f,s,c). \end{array}$$

Also, we say that a model is minimal, if it is a minimal model relative to situation $\sigma_s(s)$, for any assignment σ_s.

The following definition is inspired in the notion of well-foundedness defined in (Lifschitz 1994):

Definition 5.2 (Well Foundedness) *Let Σ_{rc} be a theory of action, we say that Σ_{rc} is well founded if for any model $\mathcal{M}'$ and assignment function σ_s there exists a model $\mathcal{M}$ similar to $\mathcal{M}'$ with respect to a situation $\sigma_s(s)$, which is also minimal relative to $\sigma_s(s)$.*

Theorem 5.1 *Let Σ_{rc} be a well founded theory of action, composed of the foundational axioms of section 2, along with axioms T_d, T_{una}, T_{prec}, T_{eff}, T_{rc} and T_{S_0}, as in section 3. If cancellation of effects or synergy arise in the set of minimal models of Σ_{rc}, then Σ_{rc} does not have a unique minimal model.*

Proof Let us assume that Σ_{rc} is a theory of action in which cancellation of effects arise (the proof for synergy is similar). Thus, let $\mathcal{M}''$, σ_s, σ_c and σ_a be as in definition 4.3, and let us assume that $\mathcal{M}''$ is minimal. In this model, we have:

$$\mathcal{M}'',\sigma_s,\sigma_c,\sigma_a \models Poss(c,s) \wedge a \in c \wedge ab(f,s,\{a\}),$$

and

$$\mathcal{M}'',\sigma_s,\sigma_c \models \neg ab(f,s,c).$$

Now, we can build a model $\mathcal{M}'$ similar to $\mathcal{M}''$ such that for any fluent g:

$$\begin{array}{rl} & \mathcal{M}',\sigma_s,\sigma_c,\sigma_a \models g(do(\{a\},s)), \\ \text{iff} & \mathcal{M}'',\sigma_s,\sigma_c,\sigma_a \models g(do(c,s)). \end{array}$$

This assignment satisfies the state constraints, otherwise $\mathcal{M}''$ would not satisfy the state constraints either. Given this construction:

$$\mathcal{M}',\sigma_s,\sigma_c,\sigma_a \models \neg ab(f,s,\{a\}).$$

From the well-foundedness of Σ_{rc}, it follows that there will be a minimal model $\mathcal{M}^{m\prime}$ relative to $\sigma_s(s)$, in which the abnormality is not present.

An alternative to this proof considers the necessary existence of ramification constraints, responsible for the interaction between effects. For illustrative purposes, we provide a sketch for this alternative proof. We can differentiate the direct effects of actions from the indirect effects, derivable as ramifications of the direct effects and the state constraints in T_{rc}. Without the set T_{rc} of ramification constraints, there can be no interacting effects. Indeed, it is easy to see that, in this case, the change that is produced by a concurrent action is simply the conjunction of the changes produced by the individual actions. Therefore, if f is the fluent that changes due to synergistic effect interaction (the argument for cancellation is similar), then T_{rc} along with T_d and T_{una}, must entail a formula of the form:

$$l_1(s) \wedge \ldots \wedge l_n(s) \supset f(s),$$

where $l_1(s) \ldots l_n(s)$ denote fluent literals. Also, we have assumed that f changes positively. Therefore, after the concurrent action is performed, all l_i fluent literals are made true, and each l_i is a direct effect of a primitive action in the synergistic concurrent action. From here it follows that, given the definition of synergy, some action has some, but not all, the l_i's as direct effects. Therefore, this action would have a disjunction of literals as a ramification, without having any of its disjuncts as a direct effect. From here it follows that there will be several minimal models.

Example (cont.): According to the previous result, the soup bowl example cannot have a single minimal model. Indeed, one minimal model is the intended one. That is, the model in which $lifted_l$ is true in a situation $do(c,S_0)$ if and only if $Lift_l$ belongs to c. For instance, we would have a model in which:

$$spilling(do(\{Lift_l\},S_0) \wedge lifted_l(do(\{Lift_l\},S_0)) \wedge \\ \neg lifted_r(do(\{Lift_l\},S_0)).$$

However, there is also a minimal model in which:

$$\neg spilling(do(\{Lift_l\},S_0) \wedge lifted_l(do(\{Lift_l\},S_0)) \wedge \\ lifted_r(do(\{Lift_l\},S_0)).$$

Thus, in this second model, lifting the left side of the bowl has the effect of lifting both sides. It is not difficult to verify that there is a minimal model in which this holds. On the other hand,

$$\neg spilling(do(\{Lift_l,Lift_r\},S_0) \wedge \\ lifted_l(do(\{Lift_l,Lift_r\},S_0)) \wedge lifted_r(do(\{Lift_l,Lift_r\},S_0)).$$

is true in all minimal models.

6 DEALING WITH INTERACTING EFFECTS

Why is the presence of interacting effects a problem? If one wants to build a theory of actions in the presence of concurrency, it would be inconvenient to specify the direct effects of all possible concurrent actions; in particular, modularity would be lost if every new action that is incorporated in the model required a reformulation of the effect axioms for concurrent actions. Therefore, it would be desirable that these effects be derivable from the direct effects of the atomic actions that are performed as part of the concurrent action. However, as the previous result states, if the effects of actions interact, an axiomatization of the style proposed in section 3, along with a minimal model semantics, fails to characterize the intended models.

How can we proceed? In situations like the one illustrated by the soup bowl example, it is very clear which model is the intended one. At this point, there are a few alternatives. Clearly, one can argue that there is not enough information built into the theory in order to single out the *right* model. In fact, one can add *causal* information, and state that lifting one side of the bowl cannot make the other side move, but can cause a spill. This causal information can be used to filter unwanted models. Therefore, in this approach the theories of action would be extended with explicit *causal axioms*. This extension would need to be accompanied by a more sophisticated minimal model semantics. Approaches based on this idea have been used in order to solve the ramification problem. For instance, see (Lin 1995; Thielscher 1997).

A second alternative is to use a *categorization based* approach. For example, we can define a stratification for the fluents. In the example, we could define $lifted_l$ and $lifted_r$ to be in one stratum, and *spilling* can be placed in another stratum. Then, we can prefer models where persistence in one stratum is preferred to persistence in another. In this case, each stratum would correspond to a separate category. This categorization of fluents is also used to deal with the ramification problem. For example, in (Lifschitz 1990; Brewka and Hertzberg 1993; del Val and Shoham 1993).

A third approach, which we pursue here, is based on the observation that the proposed theory fails to characterize the nature of the reality being modeled. Thus, we propose an alternative approach to modeling situations in which the effects of the individual actions interact. This approach is based on the observation that indirect effects of actions may arise because of the existence of intervening events. For instance, if one side of the bowl of soup is lifted, then there is an event, *Spills*, which is triggered by the lifting action. The *Spills* event is triggered in an intermediate state in which only one side of the bowl is lifted, and yet *spilling* is not true. Therefore, modeling the problem in this form implies changes in the way the problem is axiomatized. In particular, because some of the state constraints originally proposed might be violated. This approach has the following advantages:

- In many cases, the nature of the problem that is modeled is such that the intermediate states exist, probably with a very short time span. For instance, in the soup bowl example, it is not unreasonable to consider the existence of an intervening state where *spilling* is not true and only one side of the bowl is lifted. This intervening state will have a very short duration and will trigger the event *Spills*. Therefore, introducing an intermediate state seems to be a more accurate portrayal of the phenomenon.
- Minimal change approaches are still applicable. The problem that was described in the previous section does not arise. In fact, constraints responsible for interaction are replaced by a triggering axiom, (see below) and new effect axioms. In this approach, minimization of change is only meant to deal with the frame problem.
- The conditions on triggered events are expressed in terms of the truth values of fluents, as opposed to the actions that are performed. For example, independently of what actions are involved, if only one side of the bowl is lifted, and *spilling* is not already true, then *Spills* will occur. This is independent of what other actions might be present in the model. Therefore, the axiomatization remains modular.

Natural Actions and Triggers

In order to handle triggers, we extend the language of the Situation Calculus with the notion of *natural actions* (Pinto 1994; Reiter 1996). Natural actions are not regular *agent actions*. Instead, they are actions that arise due to the nature of the world. One characteristic of these actions is that they must occur (i.e., they are triggered) whenever they are possible. Thus, we need:

$$(\forall a,s).\ Poss(\{a\},s) \land natural(a) \supset occurs(a,s). \tag{16}$$

I.e., if a natural action is possible, it must occur. There are several ways in which occurrences can be modeled in the Situation Calculus. One approach is to consider that there is a single branch or narrative line in the tree of situations, such that the actions that occur fall in that branch (Miller and Shanahan 1994; Pinto and Reiter 1993). An alternative is to consider occurrences as constraints that limit the situations that can be considered legal, as in (Pinto 1998). Here, we use a simplified version of the latter approach. We define the predicate *legal* for situations, such that:

$$\begin{aligned}(\forall s).\ legal(s)\ &\equiv (s \geq S_0 \land \\ &(\forall s',c,a)\ [s' < s \land occurs(a,s') \land \\ &\quad do(c,s') \leq s \supset a \in c]).\end{aligned} \tag{17}$$

Thus, a situation is considered legal if all the actions that lead from S_0 to it are possible (expressed with the $s \geq S_0$ literal), and if all occurrences within the branch $S_0 - s$ are present in the branch. Therefore, to reason with occurrences, we focus on the situations that are *legal*.

Notice that the predicate *occurs*, as well as the predicate *Poss*, is not a fluent, since it is not used to describe the *state* of a situation. Rather, it is used to constraint the dynamics of the world. Naturally, the state of a situation might determine the truth of *occurs*, affecting, indirectly, the dynamics of the world.

Later on, we will make use of the notion of situation stability. Situations in which natural actions are possible are not considered to be stable, thus, motivating the introduction of the predicate $stable \subseteq \mathcal{S}$ with the definition:

$$stable(s) \equiv \neg(\exists a).natural(a) \wedge Poss(\{a\}, s). \tag{18}$$

Furthermore, we introduce the predicate $leads \subseteq \mathcal{C} \times \mathcal{S} \times \mathcal{S}$ with the definition:

$$\begin{aligned} leads(c,s,s') \equiv\ & s < do(c,s) \leq s' \wedge stable(s') \wedge \\ & ((\forall s_i)\ s < s_i < s' \supset \neg stable(s_i)) \wedge \\ & ((\forall c_i, s_i, a_i)\ do(c,s) < do(c_i, s_i) \leq s' \wedge a \in c_i \supset \\ & \qquad natural(a)). \end{aligned} \tag{19}$$

This latter predicate relates a situation s and an action a to the *stable* situation that results after all natural actions triggered by a in s are performed. The third conjunct on the right side of the equivalence requires that all intermediate situations be unstable. The last conjunct establishes that the resulting stable situation be reached by performing only natural actions.

Triggers and interacting effects.

As mentioned in the sketch of the proof of theorem 5.1, the interactions between effects arise because of the presence of ramification constraints. Our proposal is based on the hypothesis that the ramification constraints can be decomposed along the temporal dimension. What this means is that some constraints hide a process that can be modeled at a higher level of detail. For instance, a constraint of the form:

$$l_1(s) \wedge l_2(s) \supset l(s). \tag{20}$$

where l_1, l_2 and l represent some fluent literals, might be used to model a causal relationship between these literals. Since causality cannot be captured with implication alone, problems arise. One possibility for decomposing this type of constraint is to introduce new events as intermediate *causes*. Thus, instead of writing a constraint like (20), we propose to introduce some intermediate natural action A_l in the language, and add the following axioms:

$$natural(A_l), \tag{21}$$

$$l_1(s) \wedge l_2(s) \wedge \neg l(s) \supset Poss(\{A_l\}, s), \tag{22}$$

$$Poss(c,s) \wedge A_l \in c \supset l(do(\{A_l\}, s)). \tag{23}$$

Furthermore, it is convenient to assume that the necessary conditions for the truth of *Poss* are also sufficient. Thus, converting sets of implications like (22) into equivalences.

Under these conditions, the interaction of effects, due to the original state constraints, is converted into a triggered effect. Synergy or cancellation due to state constraints can be handled in this manner in many important cases, as described in the next section.

Example (cont.): In the soup bowl example, we can replace the ramification constraints with the precondition axioms:

$$\neg lifted_l(s) \wedge lifted_r(s) \wedge \neg spilling(s) \supset Poss(\{Spills\}, s),$$
$$lifted_l(s) \wedge \neg lifted_r(s) \wedge \neg spilling(s) \supset Poss(\{Spills\}, s),$$
$$lifted_l(s) \wedge lifted_r(s) \wedge spilling(s) \supset Poss(\{Nspills\}, s),$$
$$\neg lifted_l(s) \wedge \neg lifted_r(s) \wedge spilling(s) \supset Poss(\{Nspills\}, s),$$

Along with the requirement:

$$natural(Spills) \wedge natural(Nspills),$$

and the effect axioms:

$$Poss(c,s) \wedge Spills \in c \supset spilling(do(c,s)),$$
$$Poss(c,s) \wedge Nspills \in c \supset \neg spilling(do(c,s)).$$

Here, we have added two different natural actions, one that results in spillage, and the other that stops it.

Our proposal to decompose ramification state constraints in the temporal dimension, by the introduction of new natural actions with their derived triggered effects, is based on the following observations:

- In our everyday lives we function under the *common-sense* assumption that actions are deterministic. Non-determinism, in this context, arises because of lack of knowledge of the state of the world, or for lack of control regarding the actions that are performed.
- In physical domains, such as the bowl with soup example, one can accurately predict the direct effects of actions.
- Indirect effects of actions arise because the action creates the conditions for these indirect effects to arise. In general, these effects can be considered to arise subsequent to the performance of the action.

Our view is based on the last observation. Synergy or cancellation arises because of the indirect effects of one or more actions. Thus, this approach is applicable in settings where these observations hold.

The applicability of this approach can also be judged by comparing it to other approaches to formalize the same phenomena. In the following section, we propose the alternative of treating the set T_{rc} of ramification constraints as *causal rules*, inspired by the work of Lin (1995, 1998). Then, we show how these causal rules can be translated into natural actions and triggered effects in a mechanical manner.

7 CAUSAL RULES

7.1 PROPOSITIONAL FLUENTS

In this section we discuss some preliminary results regarding the applicability of this approach to handle effects interactions in theories with concurrent actions. For simplicity, we assume that all fluents are propositional. Thus, fluents are predicates that only take a single argument of sort $\mathcal{S}$.

We start with a theory of action Σ_{rc}, as before. The theory includes the set T_b of axioms (1)-(5), along with a set T_d of domain specific axioms (that only mention terms of type $\mathcal{D}$), a set T_{prec} of action precondition axioms of the form (6), a set T_{eff} of direct effect axioms like (11)-(12), a set T_{rc} of ramification state constraints of the form (14), and a set of axioms T_{S_0} that describe the initial situation (a simple formula applied to S_0).

As mentioned earlier, a set of state constraints are not enough to characterize change in the presence of concurrent actions and interaction between effects. What we need is more refined *causal* information. We will assume that each state constraint in the set T_{rc} is written as a *causal* implication.

Now, we take a theory of action Σ_{rc} and mechanically derive a new theory Σ_{nat}. The steps that are followed to derive this new theory are the following:

1. Each axiom in T_{rc} should be written as a *causal rule*, such as:

   $$l_1(s) \wedge \ldots \wedge l_n(s) \supset l(s). \qquad (24)$$

   The implication symbol will be interpreted as a *causality operator*, and not as logical implication. Therefore, the syntactic form of these axioms is very important. We should use another operator for causality, and rewrite the rules with this new operator. Given the space limitations, we are not able to pursue this in detail. Instead, we appeal to the reader's understanding.

   How this step is achieved depends crucially on the domain of application. All causal rules of the form (24), with the same literal on the right-hand side can be compiled together in a single causal rule of the form:

   $$\Phi_l(s) \supset l(s). \qquad (25)$$

   For notational convenience, we write $T_{rc}(s)$ as the conjunction of all the formulas in T_{rc}, leaving s free. We will assume that there are no axioms in T_{eff} in which the literal l is falsified.

   Our interpretation of these causal rules corresponds roughly to what Lin has called *dynamic causal rules* (Lin 1998).

   These causal rules will not be part of the resulting theory Σ_{nat}, but are used in the steps below.

2. The original language is extended with one new action constant per causal rule. The T_{prec} axioms are extended with action precondition axioms of the form (one axiom per causal rule (25)):

   $$\Phi_l(s) \supset Poss(\{A_l\}, s),$$

   where A_l is a new action constant. Let T^n_{prec} denote the extended set. Also, T_{una} is extended for these new action constants, leading to T^n_{una}

3. The axioms T_{eff} are extended with a new effect axiom for each new action A_l:

   $$Poss(c, s) \wedge A_l \in c \supset l(do(\{A_l\}, s)).$$

   Let T^n_{eff} denote the extended set of axioms.

4. Σ_{rc} is extended with a new set of natural axioms, for each new action symbol added to the language in the previous step, we add:

   $$natural(A_l).$$

   We will assume that these are the only natural actions. This assumption can be formalized by a minimization of the extension of this predicate. Given the simple form of these axioms, we can simply write: $natural(a) \equiv a = A_{l1} \vee a = A_{l2} \ldots$. Let T_{nat} be a singleton set with this complete specification for *natural*.

5. The axioms T_{S_0} are extended with a $T_{rc}(S_0)$, that is, for each state constraint $(\forall s)\ \Psi(s)$ in T_{rc}, we add $\Psi(S_0)$ as an axiom. Let $T^n_{S_0}$ denote the extended set of axioms about S_0.

6. The theory Σ_{nat} is given by the foundational axioms of the situation calculus, axioms about natural actions (16)-(19), and T_d, T^n_{una}, T^n_{eff}, T^n_{prec}, T_{nat} and $T^n_{S_0}$.

The theory Σ_{nat} is of a form such that we can apply Reiter's approach to derive a monotonic solution to the frame problem. Indeed, except for the axioms about natural actions, the structure of the axiomatization is a standard one. Furthermore, the axioms about natural actions do not interfere with the changes that are direct consequence of actions. Therefore, we can obtain successor state axioms for all fluents of the domain. Let $\mathcal{R}(\Sigma_{nat})$ be the theory obtained from Σ_{nat} following Reiter's approach.

7.2 SOME PROPERTIES

Previously, we described how Σ_{rc} can be transformed into a theory $\mathcal{R}(\Sigma_{nat})$. What properties does the resulting theory have with respect to the original one? First, does the resulting theory entail the state constraints for legal situations? The answer is no, thus:

$$\mathcal{R}(\Sigma_{nat}) \not\models (\forall s).\ legal(s) \supset T_{rc}(s).$$

$T_{rc}(s)$ is the conjunction of the constraints (26)-(27) applied to s. For example, consider the theory Σ_{rc}:

$$T_{prec}: \quad Poss(\{A\},s),$$

$$T_{eff}: \quad Poss(\{A\},s) \supset f_1(do(\{A\},s)),$$

$$T_{rc}: \quad f_1(s) \supset f_2(s), \tag{26}$$

$$f_2(s) \supset f_3(s), \tag{27}$$

$$T_{S_0}: \quad \neg f_1(S_0) \wedge \neg f_2(S_0) \wedge \neg f_3(S_0).$$

The theory that results from applying the previous scheme leads to a new theory in which the new action constants A_{f2} and A_{f3} are introduced. After deriving the successor state axioms, we obtain a theory $\mathcal{R}(\Sigma_{nat})$ that includes the following axioms:

$$\begin{aligned}
&natural(a) \equiv a = A_{f2} \vee a = A_{f3},\\
&Poss(\{A\},s),\\
&Poss(\{A_{f2}\},s) \equiv f_1(s) \wedge \neg f_2(s),\\
&Poss(\{A_{f3}\},s) \equiv f_2(s) \wedge \neg f_3(s),\\
&Poss(c,s) \supset f_1(do(c,s)) \equiv A \in c \vee f_1(s),\\
&Poss(c,s) \supset f_2(do(c,s)) \equiv A_{f2} \in c \vee f_2(s),\\
&Poss(c,s) \supset f_3(do(c,s)) \equiv A_{f3} \in c \vee f_3(s),\\
&\neg f_1(S_0) \wedge \neg f_2(S_0) \wedge \neg f_3(S_0).
\end{aligned}$$

Clearly:

$$\mathcal{R}(\Sigma_{nat}) \models f_1(do(\{A\},S_0)) \wedge \neg f_2(do(\{A\},S_0)).$$

Thus, violating the original state constraint (26). However, we do have the following:

$$\mathcal{R}(\Sigma_{nat}) \models (\forall s).\ leads(\{A\},S_0,s) \supset T_{rc}(s).$$

In general:

Proposition 7.1 *Let Σ_{rc} be a theory of action, as in the previous sections, and let $\mathcal{R}(\Sigma_{nat})$ be the theory that results from the application of the steps described earlier. Then:*

$$\mathcal{R}(\Sigma_{nat}) \models (\forall s).\ stable(s) \wedge legal(s) \supset T_{rc}(s).$$

The proof of this proposition is fairly direct. If the state constraints were violated in a given situation, then the situation would not be a stable one, prompting for some natural action to occur in it.

If Σ_{rc} is a theory of action, and if one wants to know whether some fluent f holds after performing an action A in some situation S, one simply tries to find out whether:

$$\Sigma_{rc} \models S_0 < S \wedge f(do(\{A\},S)).$$

However, in theories with natural actions, such as $\mathcal{R}(\Sigma_{nat})$, this question becomes:

$$\mathcal{R}(\Sigma_{nat}) \models (\forall s).\ leads(\{A\},S,s) \supset f(do(\{A\},s)).$$

Thus, we are interested in *stable* changes. Now, we can define abnormality with respect to stable situations as follows:

$$\begin{aligned}
&ab^{+}(f,s,c) =_{\text{def}}\\
&\quad stable(s) \wedge leads(c,s,s') \wedge [f(s) \equiv \neg f(s')]
\end{aligned}$$

Another property of $\mathcal{R}(\Sigma_{nat})$, in relation to Σ_{rc}, is that there is a natural correspondence between the models of $\mathcal{R}(\Sigma_{nat})$ and the minimal models of Σ_{rc}. If $\mathcal{M}^{+}$ is a model of $\mathcal{R}(\Sigma_{nat})$, then there must be a minimal model $\mathcal{M}$ of Σ_{rc}, such that for each abnormality ab^{+} in $\mathcal{M}^{+}$ there is an abnormality ab in $\mathcal{M}$.

In order to know whether some fluent becomes true after performing some action in an stable situation, we must find an stable situation after the action is performed. If fluents are propositional and finite in number, and the theory is consistent, it is not hard to show that this stable situation must exist. However, this is not true for theories with non-propositional fluents.

7.3 NON-PROPOSITIONAL FLUENTS

If the language has non-propositional fluents, the existence of a stable situation, after an action is performed, is not guaranteed. Indeed, assume a theory Σ_{rc} in which T_{rc} has a state constraint such as:

$$f(x,s) \supset f(s(x),s),$$

along with an A and an effect axiom:

$$Poss(c,s) \wedge A \in c \supset f(x,do(c,s)).$$

and an initial situation specification with:

$$(\forall x)\ \neg f(x,s).$$

It is not difficult to build a theory Σ_{rc} with these axioms such that $\mathcal{R}(\Sigma_{nat})$ will have models in which $leads(A,S_0,s)$ is identically false.

However, there are many cases in which we can guarantee the existence of stable states after performing any action. For instance, we can take stratified sets of causal rules. The definition of stratification is similar to the one used in logic programming (Apt, Blair, and Walker 1987). Thus, we can define a numbering scheme in which each fluent receives a number. A set T_{rc} is considered stratified if there is a numbering scheme in which, for each causal rule, the numbers assigned to the fluents in the antecedent, are strictly smaller that the numbers assigned to the fluent in the consequent. It is not difficult to show that if the set T_{rc} is stratified, a translation would yield a theory in which stability is guaranteed.

8 CONCLUSION

In this article we have discussed the problem of formalizing theories of action and change for worlds in which instantaneous actions can be executed concurrently. We showed that axiomatizations in which the effects of concurrently executed actions interact, due to the presence of state constraints, admit multiple minimal models. This result holds for theories in which there is a single intended minimal model. The existence of this unwanted models leads to the conclusion that there is relevant information about the dynamics of the world which is not present in the original axiomatization.

In order to deal with this problem, we propose the use of natural actions as primitive elements in the axiomatization of physical domains. Thus, an axiomatizer should directly encode domain knowledge using axioms about natural actions, as opposed to writing state constraints, from which the axioms about natural actions could be mechanically obtained. However, the mechanism proposed allows one to have a better understanding of the applicability of the approach. We believe that an approach to modeling based on the characterization of triggered or natural actions should be more natural (sic). The advantages of this approach are:

- We can use a standard minimal model semantics. In particular, this allows to use Reiter's monotonic approach to deal with the frame problem.
- The axiomatization is modular with respect to the modeling of actions. One does not need to specify what are the effects of actions in combination with other actions that might be performed concurrently. Instead, intervening actions are triggered depending on the state of the world (independently of the manner in which the state was reached).

Also, we discuss an approach to deal with interacting effects based on the interpretation of ramification state constraints as *causal rules*. This approach depends crucially upon the syntactic form in which the state constraints are written. Thus, the results that are obtained are different depending upon how the rules are written. For instance, the constraint $f_1(s) \supset f_2(s)$ would lead to a theory of action that would not be equivalent to one in which the constraint were written as $f_2(s) \supset f_1(s)$. However, assuming that the rules are written with their intended causal interactions in mind, they can be translated to the approach based on triggered events.

The research that is presented in this article is closely related to research on ramification and causality. In particular, there are interesting relationships to the work of Lin (1995, 1998), Thielscher (1997), Baral and Gelfond (1997) and Gustafsson and Doherty (1996). For future research, we would like to explore this relationship.

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