Adding a Time Line to the Situation Calculus

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1 Introduction and Motivation.

When reasoning about actions and time we often need to incorporate factual knowledge about fluents. For example, we might know that tomorrow morning it will be sunny. Or, we may be told that the car we parked outside is no longer there. A related type of knowledge that we might acquire is about event or action *occurrences*[1]. For example, we might know that a new law will take effect tomorrow adding a new tax on foreign wine, so we might want to stock up.

Knowledge of this kind can be expressed very easily in linear temporal logics (e.g. [1, 3]). For example, we can write sentences of the form $holdsAt(F, T)$ to express that fluent F is true at time T. Or, we can write $occurs(A, T)$ to state that the action or event A has occurred (or will occur) at time T. On the other hand, in the "classical" formalism of the situation calculus [8] it is impossible to express this sort of knowledge. In this paper, we argue that this difficulty is more apparent than real and we show how to enrich the situation calculus with these features.

To realize these extensions, we endow the branching structure of time, implicit in the situation calculus formalism, with a time line. For this purpose, we identify the *temporal line* of a linear temporal logic with a single path in the tree of situations. This single path will completely describe the world in a way that parallels the time line in a linear temporal logic. We present a formalization of the situation calculus enriched with a predicate *actual* that identifies a path of situations starting in S_0 (the initial situation). The path so identified describes the world's true evolution.

2 The Language and the Axiomatization.

We have sorts $\mathcal{A}, \mathcal{S}, \mathcal{F}, \mathcal{D}$ for action types, situations, propositional fluents, and other domain objects respectively. Variables are denoted by lower case letters (with or without subscripts), and constants are denoted by upper case letters (with or without subscripts). Unless otherwise stated, letters a, s, f (A, S, F) are used for variables (constants) of sorts $\mathcal{A}, \mathcal{S}, \mathcal{F}$ respectively. We will have a

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[1] In this paper, we do not distinguish between actions and events.

single 1-place predicate variable φ over situations. Other variables and constants will be assumed to be of sort $\mathcal{D}$. Also, when free variables are omitted from formulas they are assumed to be universally quantified.

Strictly speaking, our language is second order. However, the only second order sentence we use is the induction axiom (1). Therefore, we restrict ourselves to a standard sorted first order sublanguage (with equality) to express all other axioms of our theories.

We include the special constant S_0 of sort $\mathcal{S}$, the function $do : \mathcal{A} \times \mathcal{S} \to \mathcal{S}$, the relation $< \subseteq \mathcal{S} \times \mathcal{S}$, along with other functions and relations introduced later.[2]

Basic Axioms:

$$(\forall \varphi).[\varphi(S_0) \wedge (\forall s, a).(\varphi(s) \supset \varphi(do(a, s)))] \supset (\forall s).\varphi(s), \quad (1)$$

$$(\forall s_1, s_2, a).s_1 < do(a, s_2) \equiv s_1 \leq s_2, \quad (2)$$

$$(\forall a_1, a_2, s_1, s_2).do(a_1, s_1) = do(a_2, s_2) \supset a_1 = a_2, \quad (3)$$

$$(\forall s_1, s_2).s_1 < s_2 \supset \neg\, s_2 < s_1. \quad (4)$$

Axiom (1) is an induction axiom, necessary to prove properties true in all situations. See Reiter [10] for examples of the need for, and use of, induction in the situation calculus.

Now, we need to incorporate basic axioms that introduce elements necessary to reason about the effects of actions, persistence, etc. We introduce two new primitive predicates: $precond \subseteq \mathcal{F} \times \mathcal{A}$, which is meant to say that a property is a precondition for the successful execution of an action; and the predicate $ab \subseteq \mathcal{F} \times \mathcal{A} \times \mathcal{S}$, which is a widely used *abnormality* predicate [7, 4, 5]. The intended meaning of an atom $ab(f, a, s)$ is that the fluent f is abnormal (i.e. it does not persist) with respect to action a in the situation s. Finally, we define the predicate $possible \subseteq \mathcal{A} \times \mathcal{S}$ to mean that it is possible to do an action in a situation:

$$possible(a, s) \equiv (\forall f).precond(f, a) \supset holds(f, s). \quad (5)$$

Also, we use the "commonsense law of inertia" [4]:

$$\neg ab(f, a, s) \supset [holds(f, do(a, s)) \equiv holds(f, s)]. \quad (6)$$

Proposition 2.1 *From axioms (1-6) it follows:*

$$s_1 < s_2 \wedge holds(f, s_1) \wedge \neg holds(f, s_2) \supset (\exists s, a).(s_1 < do(a, s) \leq s_2) \wedge ab(f, a, s).$$

To introduce change, *effect axioms* have to be specified; these have the following form:

$$possible(A, s) \wedge \phi_{A,F}(s) \supset holds(F, do(A, s)). \quad (7)$$

Here, $\phi_{A,F}(s)$ is a formula providing qualifications for the effect of the action A on fluent F.

In order to achieve the desired effects, we need to introduce some mechanism to enforce the notion of minimal abnormality. Since the frame and ramification problems are not the primary focus of this paper, we will assume that either we have a complete monotonic specification of the theory of action (e.g. using Reiter's *successor state axioms* [9] to complete the set of effect axioms (7)); or some non-monotonic mechanism to *complete* the theory (e.g. Lin and Shoham [5]).

2.1 An Actual Path of Situations.

So far, we have introduced axioms that allow for the specification of what is true and

[2]We use the abbreviation $s_1 \leq s_2$ to mean $s_1 < s_2 \vee s_1 = s_2$. Also, in order to avoid excessive clutter, we use the notation $a < b < c$ to mean $a < b \wedge b < c$, and similarly when the expression combines $<$ and $\leq$.

what truths change along different paths that start in the initial situation S_0. Each one of these paths is interpreted as a different way in which the world could evolve. The world evolves through a specific path iff the actions along the path are executed in the order prescribed by that path. A limitation of the situation calculus view, which we tackle here, is that we have no way of describing how the world actually evolves. To this end, we incorporate a predicate *actual* for situations. The intended meaning is that a situation is *actual* if it lies on the path that describes the world's real evolution. The axioms for *actual* are:

$$actual(S_0), \tag{8}$$

$$(\forall\, a, s).actual(do(a, s)) \supset actual(s) \wedge possible(a, s), \tag{9}$$

$$(\forall\, a_1, a_2, s).actual(do(a_1, s)) \wedge actual(do(a_2, s)) \supset a_1 = a_2. \tag{10}$$

Axioms (8-9) express that the initial situation is always actual, and if a situation is actual, then its immediate predecessor must also be. Axiom (10) says that an actual situation has at most one actual successor situation. An important characteristic of actual situations is that they all lie on the same path (i.e., they constitute the time line), as the following shows:

Proposition 2.2

$$(\forall\, s_1, s_2).s_2 \leq s_1 \supset [actual(s_1) \supset actual(s_2)]$$

$$actual(s_1) \wedge actual(s_2) \supset s_1 < s_2 \vee s_2 < s_1 \vee s_1 = s_2.$$

We also introduce a notion of time, which will allow us to establish a direct relationship between the situation calculus and linear temporal logics. Intuitively, each situation has a starting time and an ending time. During the time span of a situation no fluents change truth values[3]. We incorporate the sort $\mathcal{T}$, interpreted as a continuous time line, into our language and introduce the functions *start* and *end* from $\mathcal{S}$ to $\mathcal{T}$. Events/actions are considered instantaneous and occur at the ending time of situations. This is captured by the following axioms[4]:

$$(\forall\, s, a).end(s) = start(do(a, s)),$$
$$(\forall\, s).start(s) < end(s),$$
$$start(S_0) = 0.$$

An immediate consequence of these axioms is the following:

$$(\forall\, s, s').s < s' \supset end(s) \leq start(s').$$

Occurrences are introduced as a relation between *event types* and situations. For example, $occurs(pickup(x), s)$ says that a pickup event occurred in situation s. Occurrences are defined in terms of the actual path as follows:

$$occurs(a, s) \equiv actual(do(a, s)).$$

In some situations, it might be convenient to establish a relationship between events/actions that occur and the time at which they occur (rather than the situation). For this purpose, we introduce a predicate $occurs_{\mathcal{T}} \subseteq \mathcal{A} \times \mathcal{T}$, defined as:

$$occurs_{\mathcal{T}}(a, t) \equiv (\exists\, s).occurs(a, s) \wedge end(s) = t.$$

We define a relation $holds_{\mathcal{T}}$ between fluents and time points and a relation *during* between time points and situations:

$$holds_{\mathcal{T}}(f, t) \equiv (\exists\, s).actual(s) \wedge during(t, s) \wedge holds(f, s),$$

[3] This restricts the properties of the world that can be represented as fluents, e.g. the position of a moving ball may not be conveniently represented by a fluent. This topic is left for future research.

[4] In what follows, we use $<$ and $\leq$ as relations between situations (as defined earlier) and as the standard ordering relation between the elements of $\mathcal{T}$.

$$during(t, s) \equiv start(s) < t \leq end(s) \vee \\ start(s) < t \wedge \neg(\exists\, s').actual(s') \wedge s' > s.$$

Intra-state persistence is derivable:

Proposition 2.3

$$(\forall\, f, s, t, t').during(t, s) \wedge during(t', s) \supset \\ holds_{\mathcal{T}}(f, t) \equiv holds_{\mathcal{T}}(f, t').$$

We can also introduce the notion of an event occurring between situations as follows:

$$occursBet(e, s_1, s_2) \equiv \\ (\exists\, s).s_1 < s < s_2 \wedge occurs(e, s).$$

Proposition 2.4

$$actual(s_1) \wedge s_1 < s_2 \wedge \neg(\exists\, e).occursBet(e, s_1, s_2) \\ \supset (\exists\, a).s_2 = do(a, s_1).$$

3 Some Applications.

Explaining Observations. Assume that our car is parked in the morning:

$$holds(Parked, S_m) \wedge actual(S_m).$$

However, we observe some time later that the car is no longer parked:

$$actual(S_e) \wedge \neg holds(Parked, S_e) \wedge S_m < S_e.$$

Furthermore, assume that our theory includes the following successor state axiom (Reiter [9]):

$$possible(a, s) \supset \\ (holds(Parked, do(a, s)) \equiv \\ (holds(Parked, s) \wedge a \neq Steal \wedge \\ a \neq TowAway) \vee a = Park).$$

Then the following is a consequence of our axioms:

$$(\exists\, s).(S_m < s < S_e) \wedge [occurs(Steal, s) \vee \\ occurs(TowAway, s))].$$

Behavioural Constraints. It is possible to express behavioural constraints for agents. For example: "Never cross the street against a red light:"

$$holds(Red, s) \supset \neg occurs(Cross, s).$$

"If you drink, don't drive:"

$$\neg occurs(Drive, do(Drink, s)).$$

We also have a natural notion of prevention. To prevent F from ever becoming false:

$$actual(s) \supset holds(F, s).$$

External Events. We can formalize event occurrences not under an agent's control, but which she can use in planning, for example, that every 15 minutes between 0800 and 2400, a bus will arrive.

$$(\forall n).0 \leq n \leq 64 \supset \\ occurs_{\mathcal{T}}(BusArrives, 8 + n/4).$$

Hypothetical Reasoning. By preserving the branching state property of the situation calculus, we can express and answer a variety of hypothetical queries, although counterfactuals cannot be expressed. For example "At time T_p in the past, when you put A on B, could A have been put on C instead?" can be simply expressed as:

$$during(T_p, s) \wedge actual(s) \supset \\ possible(put(A, C), s).$$

"If I had performed $put(A, C)$, would F have been true?"

$$holds(F, do(put(A, C), S_p)) \wedge \\ possible(put(A, C), S_p).$$

None of these features is possible in linear temporal logics. We need the branching structure of the situation calculus, coupled with a linear time line in that branching structure.

Causality. We can express causality between events. For example, that shooting a loaded gun causes a noise to occur:

$$holds(Loaded, s) \wedge occurs(Shoot, s) \supset \\ occurs(Noise, do(Shoot, s)).$$

4 Conclusions and Future Research.

4.1 Why Is this Interesting?

- The standard alternatives to the situation calculus in the literature (e.g. Allen [1], Kowalski and Sergot [3]) are linear time theories which provide for the representation of event occurrences and fluent truths at time t. We have shown how, with a minimal change to its ontology, the situation calculus can realize the essential features of these alternatives, without sacrificing its capacity for hypothetical reasoning provided by its branching state structure.

- We can do hypothetical reasoning in a first order setting; there is no need to go modal.

- Although we did not discuss it in this paper, there is a close correspondence between our enriched situation calculus and the *calculus of events* of Kowalski and Sergot. One advantage of this is the clean semantics provided by our axiomatization, in contrast to the event calculus's reliance on the *negation as failure* feature of logic programming, whose semantics is not well understood. Moreover, our axioms lead to a logic programming implementation with comparable computational properties to those of the event calculus.

4.2 Things to Do

- **Enriched event ontologies.** The axiomatization of this paper assumes nonconcurrent, discrete, durationless events. These limitations need to be removed, perhaps by appealing to ideas of Lin and Shoham [6] and Gelfond, Lifschitz and Rabinov [2].

- **Foundations for reasoning about physical processes.** With the ability to represent continuity, we will be able to describe processes like: "If I provide a continuous supply of heat to a pot of water, the water will eventually boil." A language of this sort is necessary for formally specifying systems that reason about physical processes.

- **Planning with External Events.** We are currently studying how to formulate planning tasks in the framework proposed here.

- **Time lines and complex events.** The approach of this paper supposes that all events are primitive. Levesque, Lin and Reiter[5] have augmented the "classical" situation calculus with the ability to define complex events from primitive ones using quantification, sequencing, conditionals, nondeterministic choice and iteration. Augmenting this theory with a time line would provide all the benefits we have described in this paper, but generalized to this much richer language.

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[5]forthcoming.

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