Appendix: Intermediate Results and Proofs

Proofs for Section 3

Proof of Lemma 1: This result extends a similar result in [3]. We concentrate on the cases not covered there. We have to show $\mathcal{M}(DB, DB') \models p(\bar{x}): f_c \lor \exists y(q(\bar{x}', y): t_d \land q(\bar{x}', y): t_c) \lor \exists y(q(\bar{x}', y): t_d \land \exists q(\bar{x}', y): t_e)$. We have that $IC$ contains the formula $p(\bar{x}) \rightarrow \exists y(q(\bar{x}', y): t_e)$. As $DB' \models_{\Sigma} IC$ we must analyze two cases. The first one is $DB' \models_{\Sigma} \neg p(\bar{a})$. Then $IP(p(\bar{a})) = f$ or $IP(p(\bar{a})) = f_a$, so $\mathcal{M}(DB, DB') \models p(\bar{a}) : f_c$. The second case is $DB' \models_{\Sigma} q(\bar{a}', 1), \ldots, q(\bar{a}', b)$ for elements $b_1, \ldots, b_n$ in the domain ($n \geq 1$). Hence, $IP(q(\bar{a}', b_i)) = t$ or $IP(q(\bar{a}', b_i)) = t_a$, for every $1 \leq i \leq n$. Then, $\mathcal{M}(DB, DB') \models q(\bar{a}', y): f_d \land q(\bar{a}', y): t_e)$ or $\mathcal{M}(DB, DB') \models q(\bar{a}', y): f_d \land q(\bar{a}', y): t_e)$. As the analysis was done for an arbitrary value $\bar{a}$, we have that $\mathcal{M}(DB, DB') \models T(DB, IC)$. □

Proof of Lemma 2: This result extends a similar result in [3]. We concentrate on the cases not covered there. We have to show $DB_M \models_{\Sigma} p(\bar{x}) \rightarrow \exists y(q(\bar{x}', y): t_d \land q(\bar{x}', y): t_c)$. Let us suppose first $\mathcal{M} \models p(\bar{a}) : f$. Then, we either have $\mathcal{M} \models p(\bar{a}) : f$ or $\mathcal{M} \models p(\bar{a}) : f_a$. Hence, $DB_M \models_{\Sigma} p(\bar{a}) \land q(\bar{a}', y): f_d \lor q(\bar{a}', y): t_c)$. Let us suppose now $\mathcal{M} \models q(\bar{a}', y): f_d \land q(\bar{a}', y): t_c)$. Therefore, $\mathcal{M} \models q(\bar{a}', y): t_d$ for some element $b$ in the domain. Hence $DB_M \models_{\Sigma} q(\bar{a}', y)$, and from there $DB_M \models_{\Sigma} q(\bar{a}', y): t_d \land q(\bar{a}', y): t_e)$. Finally, we will assume $\mathcal{M} \models q(\bar{a}', y): t_d \land q(\bar{a}', y): t_e)$. Then, $\mathcal{M} \models q(\bar{a}', y): t_d$ for some element $b$ in the domain. Hence, $DB_M \models_{\Sigma} q(\bar{a}', y)$, and from there $DB_M \models_{\Sigma} q(\bar{a}', y): t_d \land q(\bar{a}', y): t_e)$. As this is valid for any value $\bar{a}$, we have that $DB_M \models_{\Sigma} p(\bar{x}) \rightarrow \exists y(q(\bar{x}', y))$. □

Proof of Proposition 1: By Lemma 1, we conclude that $\mathcal{M}(DB, DB') \models T(DB, IC)$. Let us suppose that $\mathcal{M}(DB, DB')$ is not $\Delta_{\Sigma}$-minimal in the class of models of $T(DB, IC)$. Then, there exists $\mathcal{M} \models T(DB, IC)$, such that $\mathcal{M} \models_{\Delta} \mathcal{M}(DB, DB')$. By using this it is possible to prove that $\Delta(DB, DB_M) \subseteq \Delta(DB, DB')$.

1. Let us suppose that $p(\bar{a}) \in \Delta(DB, DB_M)$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_M$, or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_M$. In the first case we can conclude that $p(\bar{a}) : t_d \notin T(DB, IC)$ and $\mathcal{M} \models p(\bar{a}) : f \lor p(\bar{a}) : f_a$. If we suppose that $\mathcal{M} \models p(\bar{a}) : f$, then $\mathcal{M} \models p(\bar{a}) : f_a$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}) : f_a$. But $\mathcal{M} \models_{\Delta} \mathcal{M}(DB, DB')$, and therefore $\mathcal{M}(DB, DB') \models p(\bar{a}) : f_a$. Then, we conclude that $p(\bar{a}) \notin DB'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(DB, DB')$. In the second case we can conclude that $p(\bar{a}) : f_a \notin T(DB, IC)$ and $\mathcal{M} \models p(\bar{a}) : t \lor p(\bar{a}) : t_a$. If we suppose that $\mathcal{M} \models p(\bar{a}) : t$, then $\mathcal{M} \models p(\bar{a}) : t_a$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}) : t_a$. But $\mathcal{M} \models_{\Delta} \mathcal{M}(DB, DB')$, and therefore $\mathcal{M}(DB, DB') \models p(\bar{a}) : t_a$. Then, we conclude that $p(\bar{a}) \in DB'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(DB, DB')$. Thus, we can conclude that $\Delta(DB, DB_M) \subseteq \Delta(DB, DB')$.

2. Since $\mathcal{M}(DB, DB') \models_{\Delta} \mathcal{M}$, there exists $p(\bar{a})$ such that $\mathcal{M}(DB, DB') \models p(\bar{a}) : f \land p(\bar{a}) : f_a$ and $\mathcal{M} \models p(\bar{a}) : t \lor p(\bar{a}) : f$. By using the first fact it is
possible to conclude that \( p(\bar{a}) \in \Delta(DB, DB') \). If we suppose that \( p(\bar{a}) \in DB \), then \( p(\bar{a}) : t_d \in \mathcal{T}(DB, IC) \), and therefore by considering the second fact it is possible to deduce that \( \mathcal{M} \) must satisfy \( p(\bar{a}) : t \). Thus, we can conclude that in this case \( p(\bar{a}) \in DB_{\mathcal{M}} \), and therefore \( p(\bar{a}) \not\in \Delta(DB, DB_{\mathcal{M}}) \). By the other hand, if we suppose that \( p(\bar{a}) \not\in DB \), then \( p(\bar{a}) : t_d \in \mathcal{T}(DB, IC) \), and therefore by considering the second fact it is possible to deduce that \( \mathcal{M} \) must satisfy \( p(\bar{a}) : t \). Thus, we can conclude that in this case \( p(\bar{a}) \not\in DB_{\mathcal{M}} \), and therefore \( p(\bar{a}) \not\in \Delta(DB, DB_{\mathcal{M}}) \). Finally, we conclude that \( \Delta(DB, DB') \not\subseteq \Delta(DB, DB_{\mathcal{M}}) \).

We know that \( DB' \) is a database instance, and therefore \( \Delta(DB, DB') \) must be a finite set. Thus, we can conclude that \( \Delta(DB, DB_{\mathcal{M}}) \) is a finite set, and therefore \( DB_{\mathcal{M}} \) is a database instance. With the help of Lemma 2, we deduce that \( DB_{\mathcal{M}} \models IC \). But this is a contradiction, since \( DB' \) is a repair of \( DB \) with respect to \( IC \) and \( \Delta(DB, DB_{\mathcal{M}}) \not\subseteq \Delta(DB, DB_{\mathcal{M}}) \).

\[ \Box \]

**Proof of Proposition 2:** By Lemma 2, we conclude that \( DB_{\mathcal{M}} \models IC \). Now, we need to prove that \( DB_{\mathcal{M}} \) is minimal. Let us suppose this is not true. Then, there is a database instance \( DB^* \) such that \( DB^* \models IC \) and \( \Delta(DB, DB^*) \not\subseteq \Delta(DB, DB_{\mathcal{M}}) \).

1. From Lemma 1, we conclude that \( \mathcal{M}(DB, DB^*) \models \mathcal{T}(DB, IC) \).

2. Now, we are going to prove that \( \mathcal{M}(DB, DB^*) \not\leq \mathcal{M} \).

   If \( \mathcal{M}(DB, DB^*) \models p(\bar{a}) : t_a \), then we can conclude that \( p(\bar{a}) \not\in DB \) and \( p(\bar{a}) \not\in DB^* \), and therefore \( p(\bar{a}) \not\in \Delta(DB, DB^*) \). But \( \Delta(DB, DB^*) \not\subseteq \Delta(DB, DB_{\mathcal{M}}) \), and therefore \( p(\bar{a}) \not\in DB_{\mathcal{M}} \). Thus, we can conclude that \( \mathcal{M} \models p(\bar{a}) : t \vee p(\bar{a}) : t_a \). If we suppose that \( \mathcal{M} \models p(\bar{a}) : t \), then \( \mathcal{M} \not\models p(\bar{a}) : t_a \), but we know that \( \mathcal{M} \models \mathcal{T}(DB, IC) \) and \( p(\bar{a}) : t_d \in \mathcal{T}(DB, IC) \), since \( p(\bar{a}) \not\in DB \), a contradiction. Therefore, \( \mathcal{M} \not\models p(\bar{a}) : t_a \).

   If \( \mathcal{M}(DB, DB^*) \models p(\bar{a}) : t_a \), then we can conclude that \( p(\bar{a}) \in DB \) and \( p(\bar{a}) \not\in DB^* \), and therefore \( p(\bar{a}) \not\in \Delta(DB, DB^*) \). But \( \Delta(DB, DB^*) \not\subseteq \Delta(DB, DB_{\mathcal{M}}) \), and therefore \( p(\bar{a}) \not\in DB_{\mathcal{M}} \). Thus, we can conclude that \( \mathcal{M} \models p(\bar{a}) : t \vee p(\bar{a}) : t_a \). If we suppose that \( \mathcal{M} \models p(\bar{a}) : t \), then \( \mathcal{M} \not\models p(\bar{a}) : t_a \), but we know that \( \mathcal{M} \models \mathcal{T}(DB, IC) \) and \( p(\bar{a}) : t_d \in \mathcal{T}(DB, IC) \), since \( p(\bar{a}) \in DB \), a contradiction. Therefore, \( \mathcal{M} \models p(\bar{a}) : t_a \). Thus, we can deduce that \( \mathcal{M}(DB, DB^*) \not\leq \mathcal{M} \).

Finally, we know that there exists \( p(\bar{a}) \) such that it is not in \( \Delta(DB, DB_{\mathcal{M}}) \) and it is in \( \Delta(DB, DB_{\mathcal{M}}) \). Thus, \( p(\bar{a}) \in DB \) and \( p(\bar{a}) \in DB^* \), and therefore \( \mathcal{M}(DB, DB^*) \models p(\bar{a}) : t \), or \( p(\bar{a}) \not\in DB \) and \( p(\bar{a}) \not\in DB^* \), and therefore \( \mathcal{M}(DB, DB^*) \not\models p(\bar{a}) : t \). Then, we have that \( \mathcal{M}(DB, DB^*) \not\models p(\bar{a}) : t_a \) and \( \mathcal{M}(DB, DB^*) \not\models p(\bar{a}) : t_a \). Additionally, since \( p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}}) \), we can conclude that \( p(\bar{a}) \not\in DB \) and \( p(\bar{a}) \not\in DB_{\mathcal{M}} \), or \( p(\bar{a}) \not\in DB \) and \( p(\bar{a}) \in DB_{\mathcal{M}} \). In the first case, we can conclude that \( \mathcal{M} \models p(\bar{a}) : t_a \). In the second case, we can conclude that \( \mathcal{M} \models p(\bar{a}) : t_a \). Therefore, we can conclude that \( \mathcal{M} \not\models p(\bar{a}) : t_a \).}

Finally, we deduce that \( \mathcal{M} \) is not minimal in the class of the models of \( \mathcal{T}(DB, IC) \), with respect to \( \Delta \), a contradiction. \[ \Box \]
Proofs for Section 4

**Lemma 3.** For a minimal model $M$ of $T(DB, IC)$ and APC formula $\varphi(\vec{z})$, $M \models_{APC} (\neg \varphi)^m(\vec{t})$ iff $M \models_{APC} \neg \varphi^m(\vec{t})$.

**Proof.** By induction on $\varphi$.

Initial step: $\varphi(\vec{t}) = p(\vec{t})$. Trivial, by the fact that every model of $T(DB, IC)$ annotates atoms either with $t$, $f$, $t_a$ or $f_a$.

Inductive step:

- $\varphi(\vec{t}) = \neg \alpha(\vec{t})$. $M \models \neg \alpha^m(\vec{t})$ iff $M \models \alpha^m(\vec{t})$ iff $M \not\models \neg \alpha^m(\vec{t})$ iff $M \not\models (\neg \alpha)^m(\vec{t})$ (by induction hypothesis) iff $M \models (\neg \alpha)^m(\vec{t})$.

- $\varphi(\vec{t}) = \alpha(\vec{f}_1) \lor \beta(\vec{f}_2) = (\alpha \lor \beta)(\vec{t})$, where $\vec{f}_1$ is the restriction of $\vec{t}$ to $\alpha$ (the same for $\vec{f}_2$ and $\beta$). Now, $M \models (\neg (\alpha \lor \beta))^m(\vec{t})$ iff $M \models (\neg \alpha)^m(\vec{f}_1)$ and $M \models (\neg \beta)^m(\vec{f}_2)$ iff $M \models (\neg (\alpha \lor \beta))^m(\vec{t})$ (by induction hypothesis) iff $M \models (\neg (\alpha \lor \beta))^m(\vec{t})$. □

**Proof of Proposition 3:** We will prove it by induction on $\varphi$.

Initial step: $\varphi(\vec{z}) = p(\vec{z})$. $DB \models \varphi(\vec{t})$ iff for every repair $DB'$ of $DB$, $DB' \models_{\Sigma} p(\vec{t})$ iff for every minimal model $M$ of $T(DB, IC)$, $M \models p(\vec{t}): t \lor p(\vec{t}): t_a$ iff $T(DB, IC) \models_{\Sigma} p(\vec{t}):- t \lor p(\vec{t}): t_a$.

Inductive step:

- $\varphi(\vec{z}) = \neg \alpha(\vec{z})$. $DB \models \neg \alpha(\vec{t})$ iff for every repair $DB'$ of $DB$ we have that $DB' \not\models_{\Sigma} \alpha(\vec{t})$ iff for every minimal model $M$ of $T(DB, IC)$, $M \models \alpha^m(\vec{t})$ (by induction hypothesis) iff for every minimal model $M$ of $T(DB, IC)$, $M \models \alpha^m(\vec{t})$ (by Lemma 3).

- $\varphi(\vec{z}) = \alpha(\vec{x}_1) \lor \beta(\vec{x}_2) = (\alpha \lor \beta)(\vec{t})$. $DB \models (\alpha \lor \beta)(\vec{t})$ iff for every repair $DB'$ of $DB$ it is true that $DB' \models_{\Sigma} \alpha(\vec{f}_1)$ or $DB' \models_{\Sigma} \beta(\vec{f}_2)$, where $\vec{f}_1$ is the restriction of substitution $\vec{t}$ to the variables $\vec{x}_i$, iff for every minimal model $M$ of $T(DB, IC)$, $M \models \alpha^m(\vec{f}_1)$ or $M \models \beta^m(\vec{f}_2)$ (by induction hypothesis) iff $T(DB, IC) \models_{\Sigma} \alpha^m \lor \beta^m(\vec{t})$ if $T(DB, IC) \models_{\Sigma} (\alpha \lor \beta)^m(\vec{t})$. □

Proofs for Section 5

**Lemma 4.** If $M$ is a coherent stable model of $\Pi^*(DB, IC)$, i.e. a coherent minimal model of $(\Pi^*(DB, IC))^M$, then exactly one of the following cases holds:

- $p(\vec{a}, t_a), p(\vec{a}, t^*)$ belong to $M$, and no other $p(\vec{a}, v)$, for $v$ an annotation value, belongs to $M$.

- $p(\vec{a}, t_a), p(\vec{a}, t^*), p(\vec{a}, f_a), p(\vec{a}, f^*)$ and $p(\vec{a}, f^{**})$ belong to $M$, and no other $p(\vec{a}, v)$, for $v$ an annotation value, belongs to $M$.

- $p(\vec{a}, t_a), p(\vec{a}, t^*), p(\vec{a}, f^*)$ and $p(\vec{a}, t^{**})$ belong to $M$, and no other $p(\vec{a}, v)$, for $v$ an annotation value, belongs to $M$.

- $p(\vec{a}, f^*)$ and $p(\vec{a}, f^{**})$ belongs to $M$, and no other $p(\vec{a}, v)$, for $v$ an annotation value, belongs to $M$.
Proof: For an atom \( p(\vec{a}) \) we have two possibilities:

1. \( p(\vec{a}, \vec{t}_d) \in \mathcal{M} \). Then, \( p(\vec{a}, \vec{t}^*) \in \mathcal{M} \). Two cases are possible now: \( p(\vec{a}, \vec{f}_a) \in \mathcal{M} \) or \( p(\vec{a}, \vec{f}_a) \not\in \mathcal{M} \). For the former we also have \( p(\vec{a}, \vec{f}^{**}) \), \( p(\vec{a}, \vec{f}^*) \in \mathcal{M} \) and \( p(\vec{a}, \vec{f}_a) \not\in \mathcal{M} \) (because \( \mathcal{M} \) is coherent). For the latter, \( p(\vec{a}, \vec{f}^*) \not\in \mathcal{M} \) (since \( \mathcal{M} \) is minimal), \( p(\vec{a}, \vec{f}_a) \not\in \mathcal{M} \) (because \( p(\vec{a}, \vec{f}^*) \not\in \mathcal{M} \) and \( \mathcal{M} \) is minimal) and \( p(\vec{a}, \vec{t}^{**}) \in \mathcal{M} \). This covers the first two items in the lemma.

2. \( p(\vec{a}, \vec{t}_d) \not\in \mathcal{M} \). Then, \( p(\vec{a}, \vec{f}^*) \in \mathcal{M} \). Two cases are possible now: \( p(\vec{a}, \vec{t}_d) \in \mathcal{M} \) or \( p(\vec{a}, \vec{t}_d) \not\in \mathcal{M} \). For the former we also have \( p(\vec{a}, \vec{t}^{**}) \), \( p(\vec{a}, \vec{t}^*) \in \mathcal{M} \) and \( p(\vec{a}, \vec{t}_d) \not\in \mathcal{M} \) (because \( \mathcal{M} \) is coherent).

From two database instances we can define a structure.

Definition 11. For two database instances \( DB_1 \) and \( DB_2 \) over the same schema and domain, \( \mathcal{M}^*(DB_1, DB_2) \) is the Herbrand structure \((D, I_P, I_B)\), where \( D \) is the domain of the database\(^\text{b} \) and \( I_P, I_B \) are the interpretations for the database predicates (extended with annotation arguments) and the built-ins, respectively. \( I_P \) is defined as follows:

- If \( p(\vec{a}) \in DB_1 \) and \( p(\vec{a}) \in DB_2 \), then \( p(\vec{a}, \vec{t}_d), p(\vec{a}, \vec{t}^*) \) and \( p(\vec{a}, \vec{t}^{**}) \) \in \( I_P \).
- If \( p(\vec{a}) \in DB_1 \) and \( p(\vec{a}) \not\in DB_2 \), then \( p(\vec{a}, \vec{t}_d), p(\vec{a}, \vec{t}^*), p(\vec{a}, \vec{f}_a) \) and \( p(\vec{a}, \vec{f}^*) \) \in \( I_P \).
- If \( p(\vec{a}) \not\in DB_1 \) and \( p(\vec{a}) \in DB_2 \), then \( p(\vec{a}, \vec{f}^*) \) and \( p(\vec{a}, \vec{f}^{**}) \) \in \( I_P \).
- If \( p(\vec{a}) \not\in DB_1 \) and \( p(\vec{a}) \not\in DB_2 \), then \( p(\vec{a}, \vec{t}^*), p(\vec{a}, \vec{f}_a), p(\vec{a}, \vec{t}^{**}) \) \in \( I_P \).

The interpretation \( I_B \) is defined as expected: if \( q \) is a built-in, then \( q(\vec{a}) \in I_B \) iff \( q(\vec{a}) \) is true in classical logic, and \( q(\vec{a}) \not\in I_B \) iff \( q(\vec{a}) \) is false.

Notice that the database associated to \( \mathcal{M}^*(DB_1, DB_2) \) corresponds exactly to \( DB_2 \), i.e. \( DB_{\mathcal{M}}(DB_1, DB_2) = DB_2 \).

Lemma 5. If \( DB' \models_\Sigma IC \), then there is a coherent model \( \mathcal{M} \) of the program \( (IP^*(DB, IC))^{\mathcal{M}} \) such that \( DB_{\mathcal{M}} = DB' \). Furthermore, the model \( \mathcal{M} \) corresponds to \( \mathcal{M}^*(DB, DB') \).

Proof: As \( DB_{\mathcal{M}}(DB, DB') = DB' \), we only need to show that \( \mathcal{M}^*(DB, DB') \) is a model of \( (IP^*(DB, IC))^{\mathcal{M}'}(DB, DB') \). Since \( DB' \models_\Sigma \bigvee_{i=1}^n \neg p_i(\vec{a}_i) \lor \bigvee_{j=1}^m q_j(\vec{b}_j) \lor \varphi \), we have three possibilities to analyze with respect to the satisfaction of this clause. The first possibility is \( DB' \models_\Sigma \neg p_i(\vec{a}_i) \). Then, two cases arise

\(^{b}\) Strictly speaking, the domain \( D \) now also contains the annotations values.
- \( p_i(\bar{a}) \in DB \). Then, \( p_i(\bar{a}, \mathbf{f}^*) \), \( p_i(\bar{a}, \mathbf{t}_d) \), \( p_i(\bar{a}, \mathbf{f}_a) \), \( p_i(\bar{a}, \mathbf{t}^*) \) and \( p_i(\bar{a}, \mathbf{f}^{**}) \) belong to \( M^*(DB, DB') \), and the program \((II^*(DB, IC))^{\mathcal{M}'}(DB, DB')\) contains the following clauses: \( p_i(\bar{a}, \mathbf{t}_d) \leftarrow p_i(\bar{a}, \mathbf{t}^*) \leftarrow p_i(\bar{a}, \mathbf{t}_d) \), \( p_i(\bar{a}, \mathbf{t}^*) \leftarrow p_i(\bar{a}, \mathbf{t}_a) \), \( p_i(\bar{a}, \mathbf{f}^*) \leftarrow p_i(\bar{a}, \mathbf{f}_a) \), \( p_i(\bar{a}, \mathbf{t}^{**}) \leftarrow p_i(\bar{a}, \mathbf{t}_a) \) and \( p_i(\bar{a}, \mathbf{f}^{**}) \leftarrow p_i(\bar{a}, \mathbf{f}_a) \). Then, all these formulas are satisfied by \( M^*(DB, DB') \). The program also contains the clause \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{f}_a) \vee \bigwedge_{j=1}^m \bigvee_{i=1}^n q_j(\bar{a}, \mathbf{t}^*) \wedge \bigwedge_{j=1}^m \bigvee_{i=1}^n q_j(\bar{a}, \mathbf{f}^*) \wedge \varphi \), which is satisfied since \( p_i(\bar{a}, \mathbf{f}_a) \) belongs to \( M^*(DB, DB') \).

- \( p_i(\bar{a}) \notin DB \). Then, \( p_i(\bar{a}, \mathbf{f}^*) \) and \( p_i(\bar{a}, \mathbf{f}^{**}) \in M^*(DB, DB') \), and \( p_i(\bar{a}, \mathbf{f}^*) \), \( p_i(\bar{a}, \mathbf{t}^*) \leftarrow p_i(\bar{a}, \mathbf{t}_a) \), \( p_i(\bar{a}, \mathbf{f}^*) \leftarrow p_i(\bar{a}, \mathbf{f}_a) \), \( p_i(\bar{a}, \mathbf{f}^{**}) \leftarrow p_i(\bar{a}, \mathbf{t}^{**}) \leftarrow p_i(\bar{a}, \mathbf{t}_a) \) are the program \((II^*(DB, IC))^{\mathcal{M}'}(DB, DB')\). All these are satisfied by the model considered. Also the clause \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{f}_a) \vee \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{t}_a) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^*) \wedge \bigwedge_{i=1}^n q_j(\bar{a}, \mathbf{f}^*) \wedge \varphi \) is present, and is trivially satisfied since \( p_i(\bar{a}, \mathbf{t}^*) \notin M^*(DB, DB') \).

The second possibility is \( DB' \models \mathcal{F} q_j(\bar{a}) \). The following cases arise:

- \( q_j(\bar{a}) \in DB \). Then, \( M^*(DB, DB') \) contains \( q_j(\bar{a}, \mathbf{t}_d) \), \( q_j(\bar{a}, \mathbf{t}^*) \) and \( q_j(\bar{a}, \mathbf{t}^{**}) \), and \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{t}_d) \leftarrow q_j(\bar{a}, \mathbf{t}^*) \leftarrow q_j(\bar{a}, \mathbf{t}_d) \), \( q_j(\bar{a}, \mathbf{t}^*) \leftarrow q_j(\bar{a}, \mathbf{t}_a) \), \( q_j(\bar{a}, \mathbf{f}^*) \leftarrow q_j(\bar{a}, \mathbf{f}_a) \), \( q_j(\bar{a}, \mathbf{t}^{**}) \leftarrow q_j(\bar{a}, \mathbf{t}_a) \) and \( q_j(\bar{a}, \mathbf{f}^{**}) \leftarrow q_j(\bar{a}, \mathbf{f}_a) \). The structure \( M^*(DB, DB') \) satisfies all these clauses. The clause \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{f}_a) \vee \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{t}_a) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^*) \wedge \bigwedge_{i=1}^n q_j(\bar{a}, \mathbf{f}^*) \wedge \varphi \) is also in the program, and is trivially satisfied since it holds that \( q_j(\bar{a}, \mathbf{f}^*) \) does not belong to \( M^*(DB, DB') \).

- \( q_j(\bar{a}) \notin DB \). Then, \( q_j(\bar{a}, \mathbf{f}^*) \), \( q_j(\bar{a}, \mathbf{t}_a) \), \( q_j(\bar{a}, \mathbf{t}^*) \) and \( q_j(\bar{a}, \mathbf{t}^{**}) \) are in the structure \( M^*(DB, DB') \), and the following formulas are in the program \((II^*(DB, IC))^{\mathcal{M}'}(DB, DB')\): \( q_j(\bar{a}, \mathbf{t}^*) \leftarrow q_j(\bar{a}, \mathbf{t}_d) \), \( q_j(\bar{a}, \mathbf{f}^*) \leftarrow q_j(\bar{a}, \mathbf{f}_a) \), \( q_j(\bar{a}, \mathbf{t}^{**}) \leftarrow q_j(\bar{a}, \mathbf{t}_a) \) and \( q_j(\bar{a}, \mathbf{f}^{**}) \leftarrow q_j(\bar{a}, \mathbf{f}_a) \). These are satisfied by \( M^*(DB, DB') \). Also the clause \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{f}_a) \vee \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{t}_a) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^*) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^*) \wedge \varphi \) is in the program, and is satisfied since \( q_j(\bar{a}, \mathbf{t}_a) \) belongs to \( M^*(DB, DB') \).

The third possibility is \( DB' \models \mathcal{F} \varphi \). Then, \( \varphi \) is true. The clause \( \bigvee_{j=1}^m p_i(\bar{a}, \mathbf{f}_a) \vee \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{t}_a) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^*) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^*) \wedge \varphi \) is in \((II^*(DB, IC))^{\mathcal{M}'}(DB, DB')\), and is satisfied since \( M^*(DB, DB') \models \varphi \).

As the analysis was done for an arbitrary value \( \bar{a} \), it holds that the Herbrand structure \( M^*(DB, DB') \) is a model of \((II^*(DB, IC))^{\mathcal{M}'}(DB, DB')\). Moreover, it is also coherent, since \( M^*(DB, DB') \) was defined in such a way that does not contain both \( p(\bar{a}, \mathbf{t}_a) \) and \( p(\bar{a}, \mathbf{f}_a) \). \( \square \)

The next lemma shows that if \( M \) is a coherent and minimal model of the program \((II^*(DB, IC))^{\mathcal{M}'} \), and represents a finite database instance, then the instance satisfies the constraints.

**Lemma 6.** If \( M \) is a coherent stable model of the program \((II^*(DB, IC))^{\mathcal{M}'} \) and \( DB_M \) is finite, then \( DB_M \models \mathcal{F} IC \).
Proof: We want to show $DB_M |\models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x}_i) \lor \bigvee_{j=1}^{m} q_j(\bar{y}_j) \lor \varphi$, for every constraint in $IC$. Since $M$ is a model of $(II^*(DB, IC))^\mathcal{M}$, we have that $M |\models \bigvee_{i=1}^{n} p_i(\bar{x}_i, \bar{a}) \lor \bigvee_{j=1}^{m} q_j(\bar{y}_j, \bar{t}_j) \cup \bigwedge_{i=1}^{n} p_i(\bar{x}_i, \bar{t}^*) \land \bigwedge_{j=1}^{m} q_j(\bar{y}_j, \bar{f}^*) \land \varphi$. Then, at least one of the following cases is satisfied:

- $M |\models p_i(\bar{a}, \bar{a}_j)$. Then, $M |\models p_i(\bar{a}, \bar{f}^*)$ and $p(\bar{a}) \notin DB_M$ (by lemma 4).
  Hence, $DB_M |\models_{\Sigma} \neg p_i(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_M |\models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x}_i) \lor \bigvee_{j=1}^{m} q_j(\bar{y}_j) \lor \varphi$ holds.
- $M |\models q_j(\bar{a}, \bar{a}_j)$. It is symmetrical to the previous one.
- It is not true that $M |\models \varphi$. Hence, $\varphi$ is true, and $DB_M |\models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x}_i) \lor \bigvee_{j=1}^{m} q_j(\bar{y}_j) \lor \varphi$ holds.

Lemma 7. Consider two database instances $DB$ and $DB'$ over the same schema and domain. If $M$ is a coherent and minimal model of $(II^*(DB, IC))^\mathcal{M}$, such that $M \subseteq M^*(DB, DB')$, then there exists model $M'$ such that $M'$ is a coherent and minimal model of $(II^*(DB, IC))^\mathcal{M}$ and $\Delta(DB, DB_M') \subseteq \Delta(DB, DB')$.

Proof: Since $M$ is a coherent and minimal model of $(II^*(DB, IC))^\mathcal{M}$, we have that $p(\bar{a}, \bar{a}_j) \in M$ if $p(\bar{a}) \in DB$. By the way we defined $M^*(DB, DB')$ and given $M \subseteq M^*(DB, DB')$, the only two ways that both models can differ is that, for some $p(\bar{a}) \in DB$, $\{p(\bar{a}, \bar{a}_j), p(\bar{a}, \bar{f}^*), p(\bar{a}, \bar{f}**)\} \subseteq M^*(DB, DB')$ and none of these atoms belong to $M$, or for some $p(\bar{a}) \notin DB$, $\{p(\bar{a}, \bar{a}_j), p(\bar{a}, \bar{t}^*), p(\bar{a}, \bar{t}**), p(\bar{a}, \bar{t}***), p(\bar{a}, \bar{f}**), p(\bar{a}, \bar{f}**)\} \subseteq M^*(DB, DB')$ and none of these atoms belong to $M$. Now, some of the atoms in $M$ may have not received an interpretation in terms of $\bar{t}^*$ and $\bar{f}^*$, i.e., $M$ is not a minimal model of $(II^*(DB, IC))^\mathcal{M}$. Anyway, if we use the interpretation rules over $M$, we will finish with a model $M'$ that is a minimal model of $(II^*(DB, IC))^\mathcal{M}$.

It is clear that $M'$ is a coherent and minimal model of $(II^*(DB, IC))^\mathcal{M}$. It just rests to prove that $\Delta(DB, DB_M') \subseteq \Delta(DB, DB')$. First, we will prove $\Delta(DB, DB_M') \subseteq \Delta(DB, DB')$. Let us suppose $p(\bar{a}) \in \Delta(DB, DB_M')$. Then, either $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_M'$ or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_M'$. In
the first case, \( p(\bar{a},t_4), p(\bar{a},t^*) \) and \( p(\bar{a},f^*) \) are in \( \mathcal{M}' \). These atoms are also in \( \mathcal{M} \) and, by our assumption, they are also in \( \mathcal{M}^*(DB, DB') \). Hence, \( p(\bar{a}) \in \Delta(DB, DB') \). In the second case, \( p(\bar{a},f^*) \), \( p(\bar{a},t_4) \) and \( p(\bar{a},t^*) \) are in \( \mathcal{M}' \). These atoms are also in \( \mathcal{M} \) and, by our assumption, these are also in \( \mathcal{M}^*(DB, DB') \). Hence, \( p(\bar{a}) \in \Delta(DB, DB') \).

We will now prove \( \Delta(DB, DB_{AX}) \subseteq \Delta(DB, DB') \). We know for some fact \( p(\bar{a}) \) there is an element related to it which is in \( \mathcal{M}^*(DB, DB') \) and which is not in \( \mathcal{M} \). One possible case is \( p(\bar{a},t_4) \) and \( p(\bar{a},f^*) \) are in \( \mathcal{M}^*(DB, DB') \) and not in \( \mathcal{M} \). Then, \( p(\bar{a}) \in \Delta(DB, DB') \), but \( p(\bar{a}) \notin \Delta(DB, DB_{AX}) \). The other possible case is that \( p(\bar{a},t_4) \) and \( p(\bar{a},t^*) \) are in \( \mathcal{M}^*(DB, DB') \) and not in \( \mathcal{M} \). Then, \( p(\bar{a}) \in \Delta(DB, DB') \), but \( p(\bar{a}) \notin \Delta(DB, DB_{AX}) \).

\[ \square \]

**Proposition 5.** If \( DB' \) is a repair of \( DB \) with respect to \( IC \), then there is a coherent stable model \( \mathcal{M} \) of the program \( \Pi^*(DB, IC) \) such that \( DB_{AX} = DB' \).

Furthermore, the model \( \mathcal{M} \) corresponds to \( \mathcal{M}^*(DB, DB') \).

**Proof:** By Lemma 5 we have \( \mathcal{M}^*(DB, DB') \) is a coherent model of the program \( \Pi^*(DB, IC, \mathcal{M}^*(DB, DB')) \). We just have to show it is minimal. Let us suppose first there exists a model \( \mathcal{M} \) of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \) such that the case that \( \mathcal{M} \subseteq \mathcal{M}^*(DB, DB') \) (it is also coherent since it is contained in \( \mathcal{M}^*(DB, DB') \). Since \( \mathcal{M} \subseteq \mathcal{M}^*(DB, DB') \), the model \( \mathcal{M} \) contains the atom \( p(\bar{a},t_4) \) if \( p(\bar{a}) \in DB \). Then, we can assume without loss of generality that \( \mathcal{M} \) is minimal (if it is not minimal, we can always generate from it a minimal model \( \mathcal{M}' \) such that \( \mathcal{M}' \subseteq \mathcal{M} \), by deleting its non-supported atoms).

By Lemma 7, there exists model \( \mathcal{M}' \) such that \( \Delta(DB, DB_{AX}) \subseteq \Delta(DB, DB') \) and \( \mathcal{M}' \) is a coherent and minimal model of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \). By Lemma 6, \( DB_{AX} \models IC \). This contradicts our fact that \( DB' \) is a repair.

\[ \square \]

**Proposition 6.** If \( \mathcal{M} \) is a coherent and minimal model of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \) and \( DB_{AX} \) is finite, then \( DB_{AX} \) is a repair of \( DB \) with respect to \( IC \).

**Proof:** From Lemma 6, we have \( DB_{AX} \models IC \). We just have to show minimality. Let us suppose there is a database instance \( DB' \), such that \( DB' \models IC \) and \( \Delta(DB, DB') \subseteq \Delta(DB, DB_{AX}) \). Then, by Lemma 5, \( \mathcal{M}^*(DB, DB') \) is a coherent model of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \). We will first show it is the case that \( \mathcal{M}^*(DB, DB') \subseteq \mathcal{M} \) and that \( \mathcal{M}^*(DB, DB') \) is a model of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \). Notice that since \( \mathcal{M} \) is a minimal model of \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \), this program contains the clause \( p(\bar{a},f^*) \) for every \( p(\bar{a}) \not\in DB \). The rest of the program must look exactly like \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \). This is true because the only other clauses in \( \Pi^*(DB, IC) \) that contain negation in their bodies are the interpretation rules \( p(\bar{a},f^{**}) \leftrightarrow not p(\bar{a},t_4), not p(\bar{a},t_4) \) and \( p(\bar{a},t^{**}) \leftrightarrow p(\bar{a},t_4), not p(\bar{a},t_4) \). Since \( \Delta(DB, DB') \subseteq \Delta(DB, DB_{AX}) \), if \( \mathcal{M} \) does not satisfy \( p(\bar{a},t_4) \) then \( \mathcal{M}^*(DB, DB') \) does not satisfy it either (this is, either both programs, \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \) and \( (\Pi^*(DB, IC))_{\mathcal{M}^*(DB, DB')} \), contain the clause \( p(\bar{a},t^{**}) \leftrightarrow p(\bar{a},t_4) \) or both do not

\[ \square \]
contain it) and if $\mathcal{M}$ does not satisfy $p(\bar{a}, \mathbf{t}_\mathbf{a})$ then $\mathcal{M}^*(DB, DB')$ does not satisfy it either (this is, either both programs, $(\Pi^*(DB, IC))^{\mathcal{M}^*(DB, DB')}$ and $(\Pi^*(DB, IC))^\mathcal{M}$, contain the clause $p(\bar{a}, \mathbf{f}^*)$ or both do not contain it). By Definition 11, for an arbitrary atom $p(\bar{a})$ in a model $\mathcal{M}^*(DB, DB')$, we just have to analyze four cases:

1. Let us suppose just $p(\bar{a}, \mathbf{t}^{**})$, $p(\bar{a}, \mathbf{t}^*)$ and $p(\bar{a}, \mathbf{t}_\mathbf{a})$ belong to $\mathcal{M}^*(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB'$. Since $p(\bar{a}) \notin \Delta(DB, DB')$, we have two possibilities. The first one says $p(\bar{a}) \notin \Delta(DB, DB')$. Then, $p(\bar{a}, \mathbf{t}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^{**})$ also belong to $\mathcal{M}$ and $\mathcal{M}^*(DB, DB')$ is clearly a model of the clauses in $(\Pi^*(DB, IC))^\mathcal{M}$ concerning $p(\bar{a})$. The second one says $p(\bar{a}) \in \Delta(DB, DB')$. Again, $p(\bar{a}, \mathbf{t}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^{**})$ belong to $\mathcal{M}$ and $\mathcal{M}^*(DB, DB')$ is clearly a model of the clauses in $(\Pi^*(DB, IC))^\mathcal{M}$ concerning $p(\bar{a})$.

2. Let us suppose now, just $p(\bar{a}, \mathbf{f}^*)$ and $p(\bar{a}, \mathbf{f}^{**})$ belong to $\mathcal{M}^*(DB, DB')$. Again we have two possibilities. The first one says that $p(\bar{a}) \notin \Delta(DB, DB')$. Then, $p(\bar{a}, \mathbf{f}^*)$ and $p(\bar{a}, \mathbf{f}^{**})$ also belong to $\mathcal{M}$. The program $(\Pi^*(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^*) \leftarrow$, that is satisfied by the program $\mathcal{M}^*(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^*(DB, IC))^\mathcal{M}^*(DB, DB')$. The second one says that $p(\bar{a}) \in \Delta(DB, DB')$. Again, $p(\bar{a}, \mathbf{f}^*)$ and $p(\bar{a}, \mathbf{f}^{**})$ belong to $\mathcal{M}$. The program $(\Pi^*(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^*) \leftarrow$, that is satisfied by $\mathcal{M}^*(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because they are also present in $(\Pi^*(DB, IC))^\mathcal{M}^*(DB, DB')$.

3. Let us suppose just $p(\bar{a}, \mathbf{t}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$, $p(\bar{a}, \mathbf{f}_\mathbf{a})$, $p(\bar{a}, \mathbf{f}^*)$ and $p(\bar{a}, \mathbf{f}^{**})$ belong to the model $\mathcal{M}^*(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB'$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB')$. Therefore, $p(\bar{a}, \mathbf{t}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$, $p(\bar{a}, \mathbf{f}_\mathbf{a})$, $p(\bar{a}, \mathbf{f}^*)$ and $p(\bar{a}, \mathbf{f}^{**})$ belong to $\mathcal{M}$. Moreover, $\mathcal{M}^*(DB, DB')$ is clearly a model of the clauses in $(\Pi^*(DB, IC))^\mathcal{M}$ concerning $p(\bar{a})$.

4. Finally, we will suppose just $p(\bar{a}, \mathbf{f}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$, $p(\bar{a}, \mathbf{t}^*)$ and $p(\bar{a}, \mathbf{t}^{**})$ belong to the model $\mathcal{M}^*(DB, DB')$. Then, $p(\bar{a}) \notin DB$ and $p(\bar{a}) \notin DB'$. Hence, $p(\bar{a}) \notin \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB')$. Therefore, $p(\bar{a}, \mathbf{f}^*)$, $p(\bar{a}, \mathbf{t}^*)$, $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^{**})$ belong to $\mathcal{M}$. The program $(\Pi^*(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^*) \leftarrow$, that is satisfied by $\mathcal{M}^*(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^*(DB, IC))^\mathcal{M}^*(DB, DB')$.

We will now show $\mathcal{M}^*(DB, DB') \subset \mathcal{M}$. We have assumed there is an element of $\Delta(DB, DB')$ that is not an element of $\Delta(DB, DB')$. Thus, for some element $p(\bar{a})$, either $p(\bar{a}) \in DB$, $p(\bar{a}) \notin DB$ and $p(\bar{a}) \notin DB$, or $p(\bar{a}) \notin DB$, $p(\bar{a}) \notin DB'$ and $p(\bar{a}) \in DB$. For the first one we have $\mathcal{M}^*(DB, DB')$ satisfies $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^*)$, and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^*)$, but also satisfies $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^*)$. In the second one, $\mathcal{M}^*(DB, DB')$ satisfies $p(\bar{a}, \mathbf{f}^*)$ and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{f}^*)$, but also $p(\bar{a}, \mathbf{t}_\mathbf{a})$ and $p(\bar{a}, \mathbf{t}^*)$. Then, $\mathcal{M}$ is not a minimal model; a contradiction. \qed
Proof of Theorem 1: From Propositions 5 and 6.

Proofs for Section 7

The following is an extension of Lemma 4, considering the introduction of null values.

Lemma 8. If $\mathcal{M}$ is a coherent stable model of $\Pi^*(DB, IC)$, i.e. a coherent minimal model of $(\Pi^*(DB, IC))^\mathcal{M}$, then exactly one of the following cases holds:

- $p(\tilde{a}, t_d)$, $p(\tilde{a}, t^*)$ and $p(\tilde{a}, t^{**})$ belong to $\mathcal{M}$, and no other $p(\tilde{a}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $p(\tilde{a}, t_d)$, $p(\tilde{a}, t^*)$, $p(\tilde{a}, f_a)$, $p(\tilde{a}, f^*)$ and $p(\tilde{a}, f^{**})$ belong to $\mathcal{M}$, and no other $p(\tilde{a}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $p(\tilde{a}, t_a)$, $p(\tilde{a}, f^*)$, $p(\tilde{a}, t^*)$ and $p(\tilde{a}, t^{**})$ belong to $\mathcal{M}$, and no other $p(\tilde{a}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $p(\tilde{a}, f^*)$ and $p(\tilde{a}, f^{**})$ belong to $\mathcal{M}$, and no other $p(\tilde{a}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $p(\tilde{a}, \text{null}, t_d)$ and $p(\tilde{a}, \text{null}, t^*)$ belong to $\mathcal{M}$, and no other $p(\tilde{a}, \text{null}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $p(\tilde{a}, \text{null}, t_a)$, $p(\tilde{a}, \text{null}, t^{**})$ belongs to $\mathcal{M}$, and no other $p(\tilde{a}, \text{null}, v)$, for $v$ an annotation value, belongs to $\mathcal{M}$.
- $\forall v p(\tilde{a}, \text{null}, v)$, for $v$ an annotation value.

Proof: The first four cases where already proven in Lemma 4. The two new cases are deduced directly considering the new rules involving the referential ICs and the inclusion of null values.

Definition 11 is extended to consider the atoms with null values as follows:

Definition 12. For two database instances $DB_1$ and $DB_2$ over the same schema and domain, $\mathcal{M}^*(DB_1, DB_2)$ is the Herbrand structure $\{D, I_P, I_B\}$, where $D$ is the domain of the database and $I_P, I_B$ are the interpretations for the database predicates (extended with annotation arguments) and the built-ins, respectively. $I_P$ is defined as follows:

- If $p(\tilde{a}) \in DB_1$ and $p(\tilde{a}) \in DB_2$, then $p(\tilde{a}, t_d)$, $p(\tilde{a}, t^*)$ and $p(\tilde{a}, t^{**}) \in I_P$.
- If $p(\tilde{a}) \in DB_1$ and $p(\tilde{a}) \notin DB_2$, then $p(\tilde{a}, t_d)$, $p(\tilde{a}, t^*)$, $p(\tilde{a}, f^*)$ and $p(\tilde{a}, f^{**}) \in I_P$.
- If $p(\tilde{a}) \notin DB_1$ and $p(\tilde{a}) \notin DB_2$, then $p(\tilde{a}, f^*)$ and $p(\tilde{a}, f^{**}) \in I_P$.
- If $p(\tilde{a}) \notin DB_1$ and $p(\tilde{a}) \in DB_2$, then $p(\tilde{a}, f^*)$, $p(\tilde{a}, t_a)$, $p(\tilde{a}, t^*)$ and $p(\tilde{a}, t^{**}) \in I_P$.
- If $p(\tilde{a}, \text{null}) \in DB_1$ and $p(\tilde{a}, \text{null}) \in DB_2$, then $p(\tilde{a}, \text{null}, t_d)$ and $p(\tilde{a}, \text{null}, t^{**}) \in I_P$.
- If $p(\tilde{a}, \text{null}) \notin DB_1$ and $p(\tilde{a}, \text{null}) \in DB_2$, then $p(\tilde{a}, \text{null}, t_a)$ and $p(\tilde{a}, \text{null}, t^{**}) \in I_P$.

Strictly speaking, the domain $D$ now also contains the annotations values.
The interpretation $I_B$ is defined as expected: if $q$ is a built-in, then $q(\bar{a}) \in I_B$ iff $q(\bar{a})$ is true in classical logic, and $q(\bar{a}) \notin I_B$ iff $q(\bar{a})$ is false. 

Notice that, as before, the database associated to $\mathcal{M}^*(DB_1, DB_2)$ corresponds exactly to $DB_2$, i.e. $DB_{\mathcal{M}^*(DB_1, DB_2)} = DB_2$. The next lemma states that Lemma 6 still holds when considering universal and referential ICs.

**Lemma 9.** If $\mathcal{M}$ is a coherent stable model of the program $\Pi^*(DB, IC)$ and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}} \models IC$. 

**Proof:** As in Lemma 6 it was already proven that universal constraints are satisfied. As $\mathcal{M}$ satisfies: $\{aux(\bar{x}'), q(\bar{x}', y, t_0) \land \neg q(\bar{x}', y, t_0); aux(\bar{x}') \leftarrow q(\bar{x}', y, t_0); p(\bar{x}, t_0) \lor q(\bar{x}', y, null, t_0) \leftarrow p(\bar{x}, t_0) \land \neg aux(\bar{x}'), not q(\bar{x}', null, t_0)\}$ we have that it can be proved, as in Lemma 6 that the RICs of the form $p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)$ are satisfied by $\mathcal{M}$. 

The next lemma is a variation of Lemma 5 that considers universal and referential ICs and the fact that a database that is inconsistent wrt a RIC of the form $p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)$ can be repaired only deleting a tuple or inserting a tuple with the null value.

**Lemma 10.** If $DB'$ is a repair of $DB$, then there is a model $\mathcal{M}$ of $\Pi^*(DB, IC)^M$ such that $DB_{\mathcal{M}} = DB'$. 

**Proof:** This lemma is proved like Lemma 5, but instead of considering that $\mathcal{M} = \mathcal{M}^*(DB, DB')$, it considers $\mathcal{M} = \mathcal{M}^*(DB, DB') \cup \{aux(\bar{a}) \mid IC_i \in IC' and IC_i is of the form p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)\}$ and $\exists y ((q(\bar{a}', y, t_0) \in M^*(DB, DB') and q(\bar{a}', y, t_0) \notin M^*(DB, DB')) or q(\bar{a}', y, t_0) \in M^*(DB, DB'))\}. 

The next proposition shows that Proposition 5 holds also for $\Pi^*(DB, IC)$ extended for RICs.

**Proposition 7.** If $DB'$ is a repair of $DB$ with respect to $IC$, then there is a coherent stable model $\mathcal{M}$ of $\Pi^*(DB, IC)$ such that $DB_{\mathcal{M}} = DB'$. 

**Proof:** By Lemma 10 we have that $\mathcal{M} = \mathcal{M}^*(DB, DB') \cup \{aux(\bar{a}') \mid IC_i \in IC and IC_i is of the form p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)\}$ and $\exists y ((q(\bar{a}', y, t_0) \in M^*(DB, DB') and q(\bar{a}', y, t_0) \notin M^*(DB, DB')) or q(\bar{a}', y, t_0) \in M^*(DB, DB'))\) is a coherent model of the program $\Pi^*(DB, IC)^M$. Its minimality can be proved as done for $M^*(DB, DB')$ in Lemma 5.

**Proposition 8.** If $\mathcal{M}$ is a coherent and stable model of $\Pi^*(DB, IC)$, and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}}$ is a repair of $DB$ with respect to $IC$. 

**Proof:** From Lemma 9, we have $DB_{\mathcal{M}} \models IC$. We only need to prove that it is $\leq DB$-minimal. This is proven in a similar way as it was done in Proposition 6, but considering $\leq DB$ instead of minimality under set inclusion. 

**Proof of of Theorem 2:** From Propositions 7 and 8.
Proofs for Section 8

Proof of Theorem 3: \(\iff\) If the set of \(\text{ground}(IC)\) does not have a pair of bilateral literals in the same IC, we want to prove that the program \(II^*(DB, IC)\) is HCF for any DB.

We will suppose that the program \(II^*(DB, IC)\) is not HCF. Then the program \(\text{ground}(II(DB, IC))\) has a directed cycle that goes through two literals that belong to the head of the same rule from \(\text{ground}(II(DB, IC))\). The only rules with more than one literal in the head are the rules capturing the ICs, i.e. those of the form \(\bigvee_{i=1}^n p_i(\tilde{a}_i, f_a) \lor \bigvee_{j=1}^m q_j(\tilde{b}_j, t_a) \leftrightarrow \bigwedge_{i=1}^n p_i(\tilde{a}_i, t^*) \land \bigwedge_{j=1}^m q_j(\tilde{b}_j, t^*) \land \varphi\).

For the program no to be HCF there has to be a cycle involving:

- \(P_1(\tilde{a}_1, f_a)\) and \(P_2(\tilde{a}_2, f_a)\) or
- \(Q_1(\tilde{b}_1, t_a)\) and \(Q_2(\tilde{b}_2, t_a)\) or
- \(P_1(\tilde{a}_1, f_a)\) and \(Q_1(\tilde{b}_1, t_a)\)

If we analyze the first case, we can consider that only \(P_1(\tilde{a}_1, f_a)\) might be bilateral. Figure 2 shows that no directed cycle involving \(P_1(\tilde{a}_1, f_a)\) and \(P_2(\tilde{a}_2, f_a)\) is possible. The dependency graph of the other two cases is analogous, and it is not possible to have cycles involving to literals of the head of a rule. So the program cannot be HCF.

\(\Rightarrow\) If the program \(II^*(DB, IC)\) is HCF for any DB then the set of instantiated ICs do not have a pair of bilateral literals in the same IC.

Let us suppose there is a pair of bilateral literals, \(P_1(\tilde{a}_1)\) and \(Q_1(\tilde{b}_1)\), in the same IC. As \(P_1(\tilde{a}_1)\) and \(Q_1(\tilde{b}_1)\) are in the same IC, there are three different cases to study. Note that \(P\) and \(Q\) can be the same predicate.

1. \(P_1(\tilde{a}_1)\) and \(Q_1(\tilde{b}_1)\) are in the head of the IC. In this case, \(P_1(\tilde{a}_1, f_a)\) and \(Q_1(\tilde{b}_1, f_a)\) are in the head of a rule of \(II^*(DB, IC)\), and as it can be seen in Figure 3 there is a cycle that includes them, so the program is not HCF.
2. $P_1(\tilde{a}_1)$ and $Q_1(\tilde{b}_1)$ are in the body of the IC. Analogous to first case.
3. $P_1(\tilde{a}_1)$ is in the head and $Q_1(\tilde{b}_1)$ is in the body of the IC. Analogous to the first case.

So, if there is a pair of bilateral literals in the same IC, the program cannot be HCF, i.e. if the program is HCF, then it cannot have a pair of bilateral literals in the same IC. \qed