The Ontological Multidimensional Data Model in Datalog$^{\pm}$

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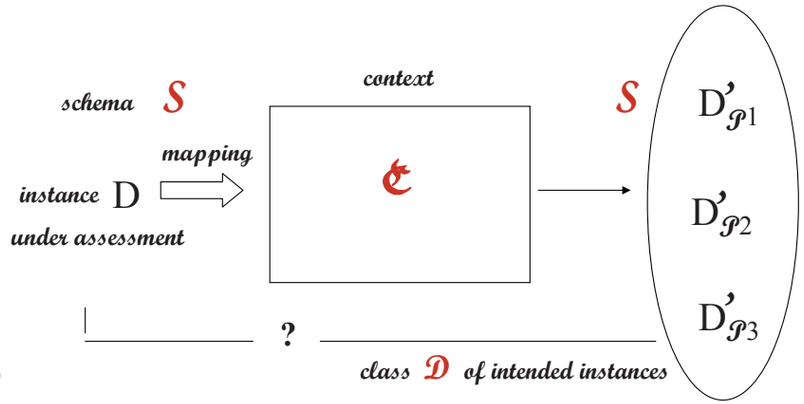
Our Initial Motivation: Contexts

- Given a data source, we may want to:
  - Analyze, understand, make sense of the data, etc.
  - Assess the data quality
- All this is a formal setting in which the data is embedded
- Contexts were introduced in previous work for data quality assessment and quality-data extraction (LB et al., VLDB'10 BIRTE WS)

Specified as a separate relational database or a (virtual) data integration system

- D can be mapped into the context
- Quality criteria imposed at contextual level

- Through the context, alternative clean versions of D can be specified, computed, compared (with each other and D), queried, etc.



  Depending on the mapping and context's ingredients

- Data dimensions were not introduced, but they are crucial for many data analysis and management problems

- The Ontological Multidimensional Data Model (OMD model) provides formal contexts for the above tasks, with explicit dimensions

Example: Doctor requires temperatures taken with oral thermometer, expecting data to correspond to this requirement

TempNoon@Ward

Patient	Value	Time	Date	Ward
Tom Waits	38.5	11:45	Sep/5	1
Tom Waits	38.2	12:10	Sep/5	1
Tom Waits	38.1	11:50	Sep/6	1
Tom Waits	38.0	12:15	Sep/6	1
Tom Waits	37.9	12:15	Sep/7	1
Lou Reed	37.9	12:10	Sep/5	2
Lou Reed	37.6	12:05	Sep/6	2
Lou Reed	37.6	12:05	Sep/7	2

Table has no elements for this assessment

An external context can provide them

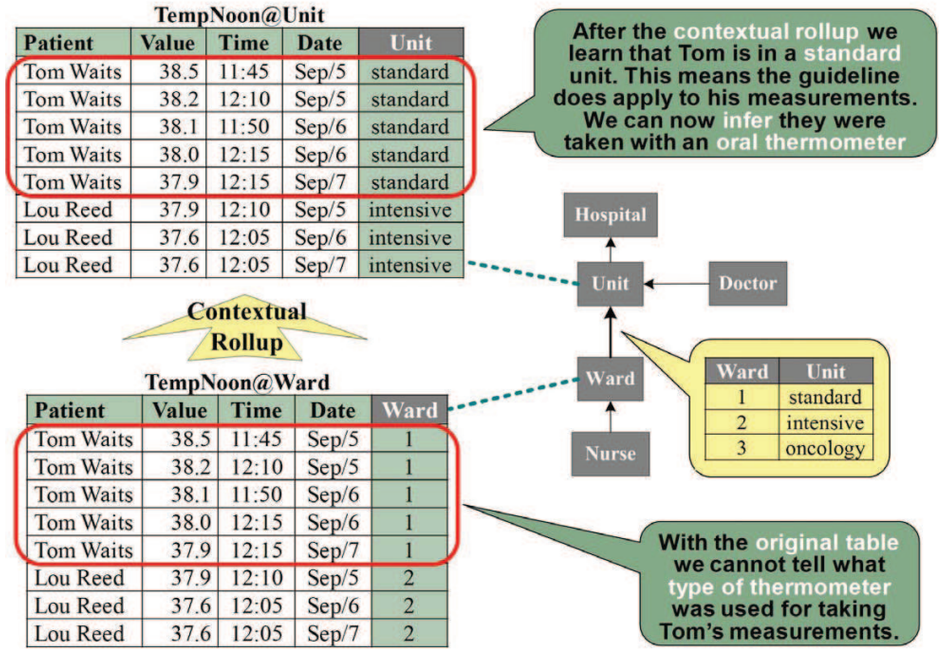
We may be missing "dimensions" above, something intrinsically "contextual"

The context could be a (multi-)dimensional database, or a dimensional ontology

A MD data model/instance

A hospital guideline

As a rule or a constraint



"Take patients' temperatures in standard care units with oral thermometers"

Can be taken advantage of through/after upward navigation in the hierarchical, dimensional hospital structure

- A MD context would enable contextual, dimensional navigation, roll-up & drill-down

  To access and generate missing data at certain levels (as in example above)

- Idea: Embed Hurtado-Mendelzon (HM) MD data model in contexts

- Go beyond: Enrich it with additional, dimension-related data, rules and constraints

  An ontological, multidimensional context!

Ontological Contexts with Dimensions

New ingredients in MD contexts: (RuleML'15)

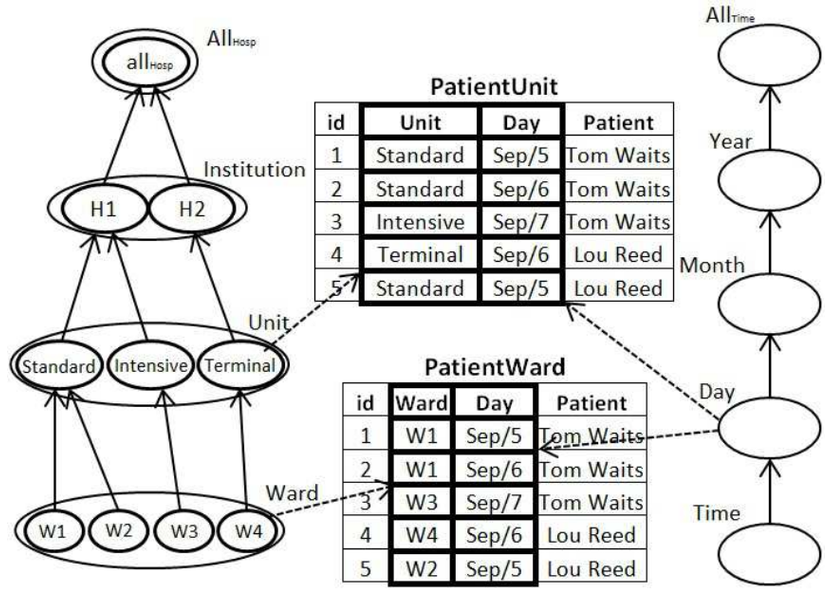
- A (relational reconstruction of) the HM model
- Categorical relations: Generalize fact tables

  Not necessarily numerical values, linked to different levels of dimensions, possibly incomplete
- Dimensional rules: generate data where missing, enable navigation
- Dimensional constraints: on (combinations of) categorical relations, involving values from dimension categories

Example:

- Categories *Ward* and *Unit* in *Hospital* dimension
- $UnitWard(unit, ward)$: parent/child relation
- *PatientWard*: categorical relation

  Ward and *Day* categorical attributes take values from categories



PatientUnit

id	Unit	Day	Patient
1	Standard	Sep/5	Tom Waits
2	Standard	Sep/6	Tom Waits
3	Intensive	Sep/7	Tom Waits
4	Terminal	Sep/6	Lou Reed
5	Standard	Sep/5	Lou Reed

PatientWard

id	Ward	Day	Patient
1	W1	Sep/5	Tom Waits
2	W1	Sep/6	Tom Waits
3	W3	Sep/7	Tom Waits
4	W4	Sep/6	Lou Reed
5	W2	Sep/5	Lou Reed

- Categorical relations are subject to dimensional constraints
- Need rules for dimensional navigation

What language to express all this? Datalog$^{\pm}$ (Gottlob et al., ∞)

$\mathbf{Datalog^{\pm}}$ MD Ontologies

Dimensional Constraints:

- A referential constraint restricting units in $PatientUnit$ to elements in the $Unit$ category, as a negative constraint

$$\bot \leftarrow PatientUnit(\underline{u, d}; p), \neg Unit(u)$$

- *"All thermometers used in a unit are of the same type"* :

$$t = t' \leftarrow Thermometer(\underline{w, t}; n), Thermometer(\underline{w', t'}; n'),$$
$$UnitWard(u, w), UnitWard(u, w')$$

An EGD

$Thermometer(ward, thermometertype; nurse)$ is categorical relation, t, t' for categorical attributes

- *"No patient in intensive care unit on August /2005"*:

$$\bot \leftarrow PatientWard(\underline{w, d}; p), UnitWard(\texttt{Intensive}, w),$$
$$MonthDay(\texttt{August/2005}, d)$$

An NC

Dimensional Rules:

- Data in $PatientWard$ generate data about patients for higher-level categorical relation $PatientUnit$

$$PatientUnit(\underline{u, d}; p) \leftarrow PatientWard(\underline{w, d}; p),\ UnitWard(u, w)$$

To navigate from $PatientWard.Ward$ up to $PatientUnit.Unit$ via $UnitWard$

A TGD

Once at the level of *Unit*, take advantage of guideline (a rule):

"Temperatures of patients in a standard care unit are taken with oral thermometers"

Data at $Unit$ level that can be used there and at $Ward$ level

- Data in categorical relation *WorkingSchedules* generate data in categorical relation *Shifts*

WorkingSchedules

Unit	Day	Nurse	Type
Intensive	Sep/5	Cathy	cert.
Standard	Sep/5	Helen	cert.
Standard	Sep/6	Helen	cert.
Standard	Sep/9	Mark	non-c.

Shifts

Ward	Day	Nurse	Shift
W4	Sep/5	Cathy	night
W1	Sep/6	Helen	morning
W4	Sep/5	Susan	evening

$$\exists z\ Shifts(\underline{w, d}; n, z) \leftarrow WorkingSchedules(\underline{u, d}; n, t), UnitWard(u, w)$$

Captures guideline: *"If a nurse works in a unit on a specific day, she has shifts in every ward of that unit on the same day"*

Existential variable z for missing values for the non-categorical *shift* attribute

Rule for downward- navigation and value invention, with join via categorical attribute between categorical and parent-child predicate

Properties of MD Ontologies

- With reasonable and natural conditions, Datalog$^{\pm}$ MD ontologies become weakly-sticky Datalog$^{\pm}$ programs [Cali et al., AIJ'12]

  Important that join variables in TGDs are for categorical attributes (with values among finitely many category members)

- The chase (that enforces TGDs) may not terminate

  Weak-Stickiness guarantees tractability of conjunctive QA: only a "small", initial portion of the chase has to be queried

  Boolean conjunctive QA is tractable for weakly-sticky (WS) Datalog$^{\pm}$ ontologies

- Separability condition on the (good) interaction between TGDs and EGDs becomes application dependent

  If EGDs have categorical head variables (as in page 9), separability holds

  Separability guarantees decidability of conjunctive QA, etc.

Next goals were:

(a) Develop a practical QA algorithm for WS Datalog$^{\pm}$

(b) Optimization of ontologies (as programs)

Query Answering on WS MD-Ontologies

- There was a non-deterministic PTIME algorithm for WS Datalog$^{\pm}$ (Cali et al., AIJ'12)
- Our goal was to develop a practical chase-based QA algorithm
- Apply magic-sets techniques to optimize QA

  There is such a technique (MS) available for (a class of) "existential programs" ($\exists$-Datalog) (Alviano et al., Datalog 2.0'12)
- WS Datalog$^{\pm}$ is not closed under MS
- We extended WS Datalog$^{\pm}$ to a still tractable class of *Joint-Weakly-Sticky* programs, which is closed under magic sets

  We proposed a QA algorithm (Milani & Bertossi, RR'16, AMW'15)

Discussion

- Datalog$^{\pm}$ is an expressive and computationally nice family of existential programs (with constraints)
- We have used Datalog$^{\pm}$ to create multidimensional ontologies

  They can be seen as logic-based, relational extensions of MD DBs
- The OMD data model considerably extends the HM MD data model

  OMD includes general TGDs, EGDs and NCs

  A (relational reconstruction of the) HM model, data and queries are seamlessly integrated into a uniform logico-relational framework

- The usual constraints considered in the HM model are specific for the dimensional structure of data

  Most prominently, to guarantee summarizability (i.e. correct aggregation, no double-counting)

  In the HM model we find constraints enforcing strictness and homogeneity

  Strictness: Every category element rolls up to a single element in a parent category

  In OMD can be expressed by EGDs

  Homogeneity: Category elements have parent elements in parent categories

  In OMD can be expressed by TGDs

- OMD supports general, possibly incomplete categorical relations

  Not only complete fact tables linked to bottom categories

- Our MD ontologies belong to well-behaved classes of Datalog$^{\pm}$
- We proposed chase-based QA algorithms for (extensions of) WS Datalog$^{\pm}$
- We are working on the implementation of the QA algorithm
- We applied magic-sets techniques
- MD ontologies were motivated by data quality concerns

  They are interesting by themselves

  QA can be used to extract quality data from dirty data (RuleML'15)

- Open problems in our setting:
  - Sometimes we have to deal with closed predicates, e.g. categories
  - Inconsistency tolerance

    What if constraints are not satisfied?

Next some more specific technical issues ...

Downward Navigation and Categorical Attributes

- TGDs as in page 11 can be used for "deterministic" downward navigation: only values for non-categorical attributes are created, with determinism wrt. the categories involved

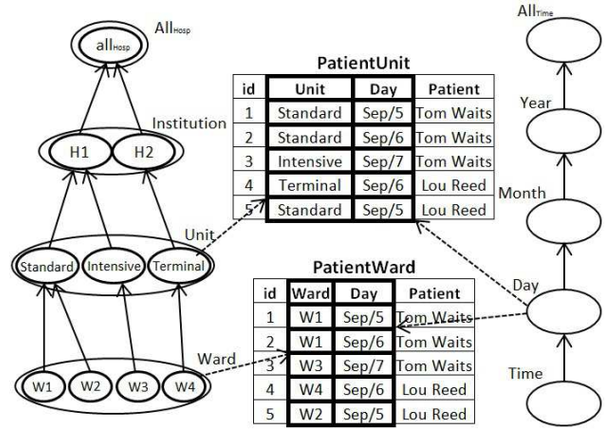
- In some applications there may be incomplete data about categorical attributes

  Existential quantifications over categorical variables may be needed

Categorical relation *DischargePatients*, linked to *Institution*, with data about patients leaving the hospital

DischargePatients

Inst.	Day	Patient
H1	Sep/9	Tom Waits
H1	Sep/6	Lou Reed
H2	Oct/5	Elvis Costello



Query on *PatientUnit* about the dates that 'Elvis Costello' was in a unit at institution 'H2'

No answer directly from *PatientUnit* (as derived from *PatientWard*)

If each patient is in a (only one) unit, *DischargePatient* can generate data downwards for *PatientUnit*

Knowledge about lower-level unit (category value) is uncertain:

$$\exists \underline{\boldsymbol{u}}\ \mathit{InstitutionUnit}(\boldsymbol{i}, \underline{\boldsymbol{u}}), \mathit{PatientUnit}(\boldsymbol{u}, \boldsymbol{d}; p) \leftarrow \mathit{DischargePatients}(\boldsymbol{i}, \boldsymbol{d}; p)$$

- With rules of this kind, an MD ontology is still weakly-sticky

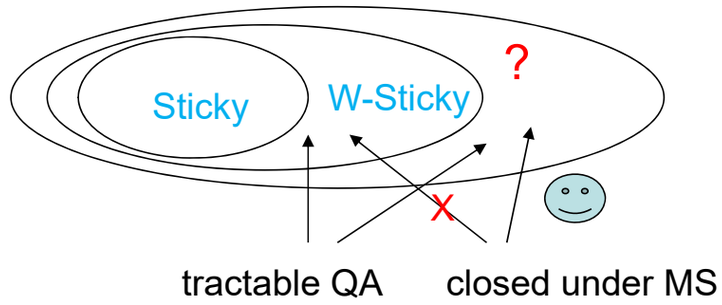
  In particular only a limited number of nulls can be generated with the chase

- EGDs with only categorical attributes in heads do not guarantee separability anymore, and becomes application dependent

Going Not-Too-Far Beyond WS Ontologies

- WS Datalog$^{\pm}$ is a syntactic class defined by a combination of:
  - The notion of finite-rank position (predicate/attribute) found in weakly-acyclic TGDs (Π_F in data exchange)
  - A variable-marking procedure developed for sticky Datalog$^{\pm}$, to keep track of value propagation via joins

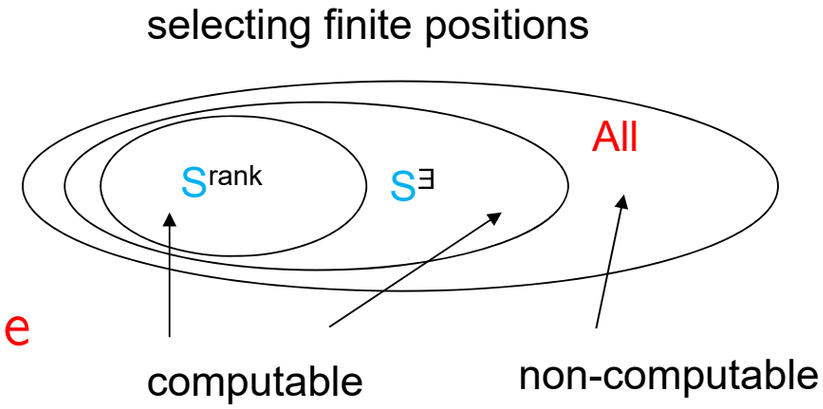
    (A better-behaved, less expressive subclass of WS Datalog$^{\pm}$)



- It captures "finite positions": finitely many nulls in them during the chase (not necessarily *all* finite positions, which is undecidable)

  A "selection function", S^{rank}, of finite positions via finite-rank positions

- We started investigating more general selections functions (AMW'15, RR'16)



- Determining a new, syntactic, computable selection function: $S^{rank} \subseteq S^{\exists}$
- $S^{\exists}$ uses:
  - *Existential-dependency graph* (Krötzsch & Rudolph, IJCAI'11)
  - Marking procedure via join variables in TGDs (neglected by S^{rank})
- We identified and characterized via $S^{\exists}$ the *Joint-Weakly-Sticky* (JWS) class

  A syntactic class with tractable QA that extends WS Datalog$^{\pm}$ and is closed under MS!

Joint-Weak-Stickiness

Set of TGDs Σ: $p(\hat{X}, \hat{Y}), u(\hat{Y}) \rightarrow \exists Z\, p(Y, Z)$

$u(X), p(X, \hat{Y}), p(\hat{Y}, \hat{W}) \rightarrow t(X)$

Marks body variables that either:

(a) do not appear in heads, e.g. X in the first rule, and Y in the second, or

(b) occur in heads only in positions of marked variables (maybe another rule), e.g. Y in first rule (Y occurs in $p[1]$ in the head, where marked variable Y appears in the body of second rule)

- $\mathcal{S}^{rank}(\Sigma) = \Pi_F(\Sigma) = \{u[1]\}$
- With marked variables as for WS programs
- Σ is WS if marked join variables appear in some "finite position"
- Join variable Y appears in $p[1], p[2] \notin \mathcal{S}^{rank}(\Sigma)$ Σ is not WS!
- Σ is JWS: $\mathcal{S}^{\exists}(\Sigma) = \{p[1], p[2], u[1], t[1]\}$

- We proposed a PTIME chase-based QA algorithm for JWS Datalog$^{\pm}$

  For QA a finite initial fragment of the chase is good enough

- The (generic) algorithm takes into account during the chase if a position is finite or not

  As determined by the selection function (which acts as an oracle)

  And behaves accordingly

- As such it can be applied both to WS and JWS, but some finite positions will be missed when applied to WS

QA Algorithm

- Σ :

$$p(\hat{X}, Y) \rightarrow \exists Z\, p(Y, Z)$$
$$u(\hat{X}), p(\hat{X}, Y), p(Y, \hat{W}) \rightarrow t(Y)$$

- Σ is JWS: X appears in $\mathcal{S}^{\exists}(\Sigma) = \{u[1]\}$
- Algorithm with $D = \{p(a, b), u(b)\}$ and $\mathcal{Q}$: $\exists Y\, t(Y)$

  - Initialize $I := D$, and apply first TGD, creating $p(b, \zeta_1)$
  - First TGD cannot be applied again: $p(\zeta_1, \zeta_2)$ homomorphic to $p(b, \zeta_1)$
  - No applicable rules
  - Resume with frozen ζ_1 (as a constant, relevant for homo tests)

- As many resumptions as existentials in query (one here)

$D = \{p(a, b), u(b)\}$
$\mathcal{Q}\colon\ \exists Y\ t(Y)$

$$p(\hat{X}, Y) \ \rightarrow\ \exists Z\ p(Y, Z)$$
$$u(\hat{X}), p(\hat{X}, Y), p(Y, \hat{W}) \ \rightarrow\ t(Y)$$

- Algorithm continues
- Apply first and second TGDs, creating $p(\zeta_1, \zeta_2)$ and $t(\zeta_1)$, resp.
- No applicable rules (due to homo test), no more resumptions
- The algorithm stops with instance $I = D \cup \{p(b, \zeta_1), p(\zeta_1, \zeta_2), t(\zeta_1)\}$
- $I \models \mathcal{Q}$, so answer is *true* in $\Sigma \cup D$

Algorithm stops, producing a query-dependant, initial, finite portion of the regular chase, and is good enough to answer the query

Magic-Sets Rewriting

- We consider a magic-sets rewriting method (MS) for Datalog$^{\exists}$

  [Alviano et al., Datalog 2.0'12]

  Quite general, and does not bound existential variables

  Nothing like this: $\exists w\ Assist^{f\,b}(v, w) \leftarrow\ Assist^{ff}(u, v)$

- WS not closed under MS, but JWS is

- $AL(\mathcal{S}^{ext})$ can be applied both to a JWS program and its MS rewriting

  Whereas $AL(\mathcal{S}^{rank})$ applied to a WS program's MS rewriting (possibly no longer WS) will be sound, but possibly incomplete

Example: Σ below is WS

$$\sigma_1: \quad \exists z\ Assist(z,x) \leftarrow Assist(x,y)$$
$$\sigma_2: \quad \exists w\ Assist(v,w) \leftarrow Assist(u,v)$$
$$\sigma_3: \quad Certified(x') \leftarrow Assist(x',y'), Assist(y',z'), Doctor(y')$$

Query $\mathcal{Q}$: $Certified(\texttt{Marie})$?

Adorned program Σ^a:

$$r_1: \quad \exists z\ Assist^{fb}(z,x) \leftarrow Assist^{bf}(x,y)$$
$$r_2: \quad \exists w\ Assist^{bf}(v,w) \leftarrow Assist^{fb}(u,v)$$
$$r_3: \quad Certified^{b}(x') \leftarrow Assist^{bf}(x',y'), Assist^{bf}(y',z'), Doctor(y')$$

Still WS

The MS rewriting Σ^M:

$$m_1: \quad \exists z\ Assist^{fb}(z,x) \leftarrow mg\_Assist^{fb}(x), Assist^{bf}(x,y)$$

$$m_2: \quad \exists w\ Assist^{bf}(v,w) \leftarrow mg\_Assist^{bf}(v), Assist^{fb}(u,v)$$

$$m_3: \quad Certified^{b}(x') \leftarrow mg\_Certified^{b}(x'), Assist^{bf}(x',y'), Assist^{bf}(y',z'), Doctor(y')$$

And the magic rules:

$$m_4: \quad mg\_Certified^{b}(\texttt{Marie}).$$

$$m_5: \quad mg\_Assist^{bf}(x') \leftarrow mg\_Certified^{b}(x')$$

$$m_6: \quad mg\_Assist^{bf}(y') \leftarrow mg\_Certified^{b}(x'), Assist^{bf}(x',y')$$

$$m_7: \quad mg\_Assist^{fb}(v) \leftarrow mg\_Assist^{bf}(v)$$

$$m_8: \quad mg\_Assist^{bf}(x) \leftarrow mg\_Assist^{fb}(x)$$

Σ^M is not WS!

Σ is JWS since it is WS

Σ^M is also JWS

EXTRA SLIDES

Existential Dependency Graph and Join Acyclicity

Example:

Assume a set Σ of tgds (a variable only appears in one rule):

$$\sigma_1 : \quad \exists z\ Assist(x, z) \leftarrow Nurse(x, y), Doctor(x)$$
$$\sigma_2 : \quad \exists w\ Nurse(w, u) \leftarrow Assist(t, u)$$

Π_x^B and Π_x^H are the set of all positions where a variable x occurs in the body and head of a rule

I.e. $\Pi_x^B = \{Nurse[1], Doctor[1]\}$ and $\Pi_x^H = \{Assist[1]\}$

For any $\exists$-variable x, Ω_x is the set of positions in which values invented for x may appear

Ω_x can be computed as the smallest set that:

(1) $\Pi_x^H \subseteq \Omega_x$ and

(2) $\Pi_y^H \subseteq \Omega_x$ for every $\forall$-variable y with $\Pi_y^B \subseteq \Omega_x$

That is, $\Omega_z = \{Assist[2], Nurse[2]\}$ and $\Omega_w = \{Nurse[1]\}$

EDG of Σ has:

(1) $\exists$-variables as its nodes,

(2) There is an edge from x to y if the rule where y occurs contains a $\forall$-variable z in its body with $\Pi_z^B \subseteq \Omega_x$

In this example, EDG of Σ has two nodes: z and w

There is only one edge from z to w

A set of tgds Σ is joint acyclic (JA) if its EDG is acyclic

Σ is JA (because EDG is acyclic)

We now define $\exists$-infinite positions of Σ:

$\Pi^{\exists}_{\infty}(\Sigma) := \bigcup \Omega_{x_i}$, with x_is variables that appear in a cycle in the EDG

$\Pi^{\exists}_{F}(\Sigma)$ are $\exists$-finite positions (the rest of the positions)

Proposition 1: $\Pi_F(\Sigma) \subseteq \Pi^{\exists}_{F}(\Sigma)$ $(\Pi^{\exists}_{\infty}(\Sigma) \subseteq \Pi_{\infty}(\Sigma))$

In this example:

$\Pi_{\infty}(\Sigma) = \{Assist[1], Assist[2], Nurse[1], Nurse[2]\}$, while

$\Pi^{\exists}_{\infty}(\Sigma) = \emptyset$

Example of Magic Rewriting

Example: Consider a set Σ of tgds:

$$\sigma_1 : \quad \exists z\ Assist(z, x) \leftarrow Assist(\underline{x}, \underline{y})$$
$$\sigma_2 : \quad \exists w\ Assist(v, w) \leftarrow Assist(\underline{u}, \underline{v})$$
$$\sigma_3 : \quad Certified(x') \leftarrow Assist(x', \underline{y'}), Assist(\underline{y'}, \underline{z'}), Doctor(\underline{y'})$$

$\Pi_F(\Sigma) = \{Doctor[1]\}$

$\Pi_F(\Sigma) = \{Assist[1], Assist[2], \ \ Certified[1]\}$

Σ is WS!

y' is repeated and marked but appears in $Doctor[1] \in \Pi_F(\Sigma)$

Dashed lines represent special edges

Given a query $\mathcal{Q}$: $Certified(\texttt{Marie})$ the adorned program Σ^{μ} is:

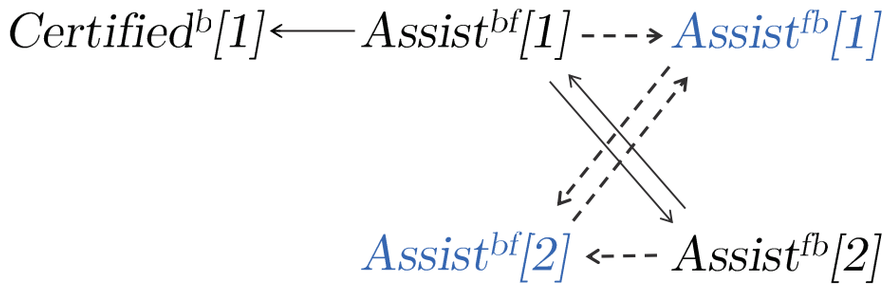
$$r_1: \quad \exists z\ Assist^{fb}(z,x) \leftarrow Assist^{bf}(\underline{x},\underline{y})$$

$$r_2: \quad \exists w\ Assist^{bf}(v,w) \leftarrow Assist^{fb}(\underline{u},\underline{v})$$

$$r_3: \quad Certified^{b}(x') \leftarrow Assist^{bf}(x',\underline{y'}), Assist^{bf}(\underline{y'},\underline{z'}), Doctor(\underline{y'})$$

$\Pi_F(\Sigma^{\mu}) = \{Certified^{b}[1], Assist^{bf}[1], Assist^{fb}[2], Doctor[1]\}$

$\Pi_{\infty}(\Sigma^{\mu}) = \{Assist^{bf}[2], Assist^{fb}[1]\}$



Σ^{μ} is still WS (y' in r_3 appears in $Doctor[1] \in \Pi_F(\Sigma^{\mu})$)

The MS rewriting Σ^M contains modified rules:

$$m_1 : \quad \exists z\ Assist^{fb}(z, x) \leftarrow mg\_Assist^{fb}(\underline{x}), Assist^{bf}(\underline{x}, \underline{y})$$

$$m_2 : \quad \exists w\ Assist^{bf}(v, w) \leftarrow mg\_Assist^{bf}(\underline{v}), Assist^{fb}(\underline{u}, \underline{v})$$

$$m_3 : \quad Certified^{b}(x') \leftarrow [mg\_Certified^{b}(x'), Assist^{bf}(x', \underline{y'}),$$
$$Assist^{bf}(\underline{y'}, \underline{z'}), Doctor(\underline{y'})]$$

And the magic rules:

$$m_4 : \quad mg\_Certified^{b}(\texttt{Marie})$$

$$m_5 : \quad mg\_Assist^{bf}(x') \leftarrow mg\_Certified^{b}(\underline{x'})$$

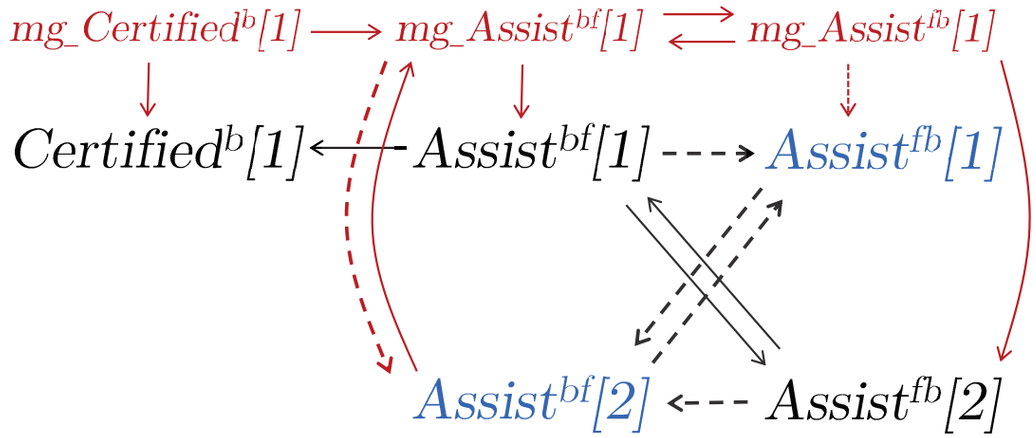
$$m_6 : \quad mg\_Assist^{bf}(y') \leftarrow mg\_Certified^{b}(\underline{x'}), Assist^{bf}(\underline{x'}, \underline{y'})$$

$$m_7 : \quad mg\_Assist^{fb}(v) \leftarrow mg\_Assist^{bf}(\underline{v})$$

$$m_8 : \quad mg\_Assist^{bf}(x) \leftarrow mg\_Assist^{fb}(\underline{x})$$

$\Pi_F(\Sigma^M) = \{mg\_Certified^b[1], Doctor[1]\}$

Σ^M is not WS! Because of repeated variables in m_1,m_2 and m_6



This proves that WS is not closed under MS rewriting

Σ is JWS since it is WS

Now consider the EDG of Σ^M:

Ω_z contains $Assist^{fb}[1]$ and Ω_w has $Assist^{bf}[2]$

Therefore $\Pi^{\exists}_{\infty}(\Sigma)$ contains $Assist^{fb}[1]$ and $Assist^{bf}[2]$

Σ^M is JWS

Example of QA

Example: A WS Σ:

$$\exists z\, Assist(z, x) \leftarrow Assist(x, y)$$
$$\exists w\, Nurse(x, w) \leftarrow Doctor(x)$$
$$Certified(z, x) \leftarrow Assist(x, y), Nurse(x, z)$$

$D = \{Doctor(\texttt{john}), Certified(\texttt{alice}), Assist(\texttt{john}, \texttt{alice})\}$

CQ $\mathcal{Q} : \exists x \exists y (Assist(x, y) \wedge Assist(y, \texttt{john}))$

We use $\mathcal{S}^{rank}$ and $\Pi_F(\Sigma) = \{Nurse[1], Nurse[2], Doctor[1]\}$

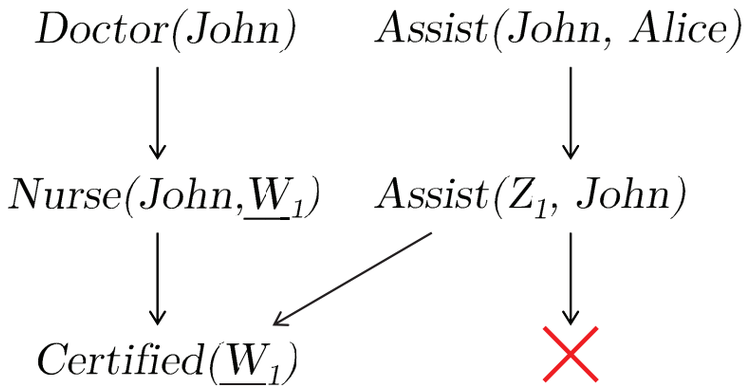
The two phases for QA:

1. *pChase* runs until termination

   However, after a pChase-step the generated nulls appearing in $\Pi_F(\Sigma)$- positions are immediately frozen

   W_1 is frozen (hence underlined)
   immediately, because it appears in $Nurse[2] \in \Pi_F(\Sigma)$

   Z_1 is not frozen, because
   $Assist[1] \in \Pi_\infty(\Sigma)$

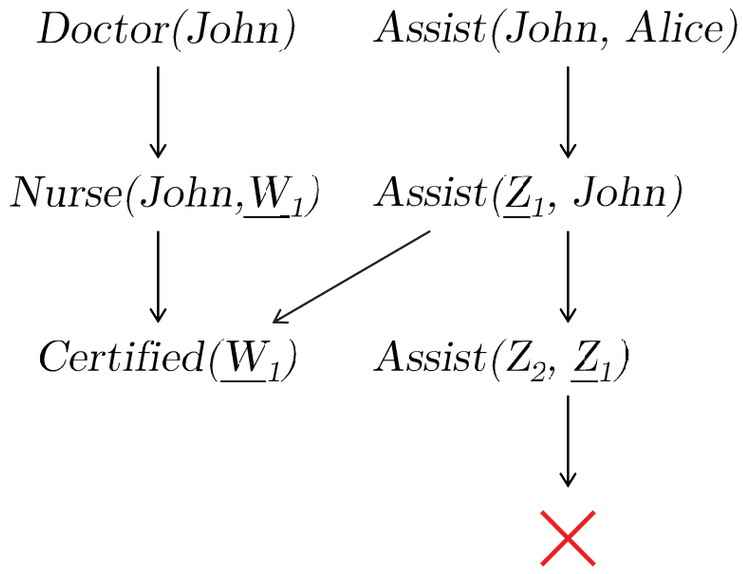


2. *pChase* iteratively resumes for a number of times that depends on the number of distinct $\exists$-variables that appear in a join in the query (deals with joins in the query)

y is the only $\exists$-variable that also appears in a join in $\mathcal{Q}$

Therefore, we freeze all nulls (e.g. Z_1), and resume the chase only once

$Assist(Z_2, Z_1)$ is entailed since Z_1 is frozen now!

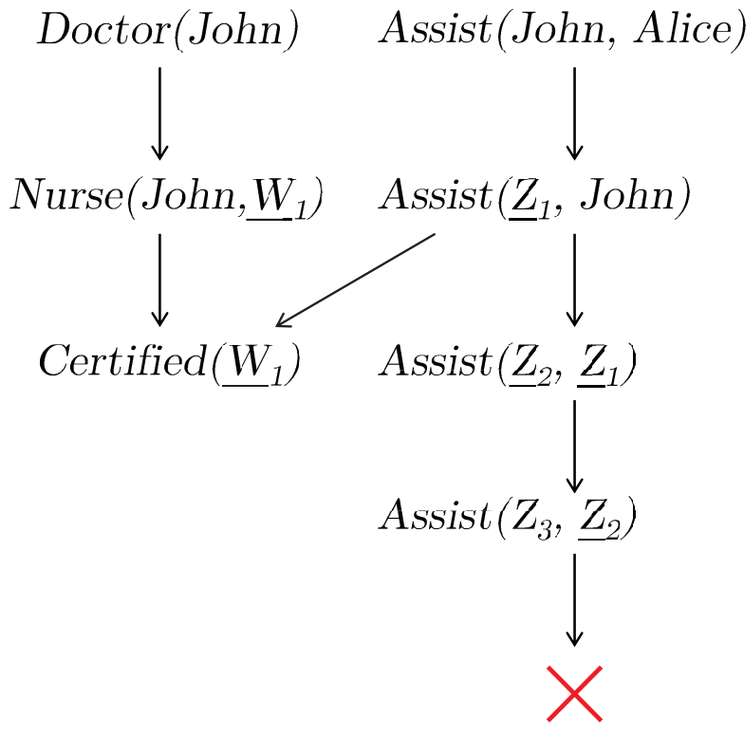


$\mathcal{Q}$ true after the chase resumption!

It was false without it!

Let us now pose the query:

$$\mathcal{Q}'\colon\ \exists x \exists y \exists z\ (Assist(x, y) \wedge Assist(y, z) \wedge Assist(z, \texttt{john}))$$



Now the algorithm runs with two chase resumptions (due to y and z),returning true!