Consistent Query Answering in Databases

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Tutorial held at the Italian Conference on Databases (SEBD 04), S. Margherita di Pula, June 2004.
The Context

There are situations when we want/need to live with inconsistent information in a database

With information that contradicts given integrity constraints

- The DBMS does not fully support data maintenance or integrity checking/enforcing
- The consistency of the database will be restored by executing further transactions
- Delayed updates of a datawarehouse
- Integration of heterogeneous databases without a central/global maintenance mechanism
Inconsistency wrt “soft” integrity constraints we hope to see satisfied, but do not prevent transactions from execution

User constraints than cannot be checked

Legacy data on which we want to impose semantic constraints

It may be impossible/undesirable to repair the database (to restore consistency)

No permission

Inconsistent information can be useful

Restoring consistency can be a complex and non deterministic process
The Problem

Not all data participate in the violation of the ICs

The inconsistent database can still give us “correct” or consistent answers to queries!

We want to:

- Give a precise definition of consistent answer to a query in an inconsistent database
- Find mechanisms for obtaining such consistent information from the inconsistent database
- Study the computational complexity of the problem
Part A: Basic Notions and Overview
Example

A database instance $r$

<table>
<thead>
<tr>
<th>Employee</th>
<th>Name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J.Page</td>
<td>5,000</td>
</tr>
<tr>
<td></td>
<td>J.Page</td>
<td>8,000</td>
</tr>
<tr>
<td></td>
<td>V.Smith</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td>M.Stowe</td>
<td>7,000</td>
</tr>
</tbody>
</table>

FD: Name → Salary

$r$ violates $FD$, by the tuples with $J.Page$ in Name

There are two possible ways to repair the database in a minimal way if only deletions/insertions of whole tuples are allowed
\begin{tabular}{|c|c|c|}
\hline
Employee & \multicolumn{2}{c|}{\textbf{Salary}} \\
\hline
Name & $r_1$ & \\
\hline
J. Page & 5,000 \\
V. Smith & 3,000 \\
M. Stowe & 7,000 \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline
Employee & \multicolumn{2}{c|}{\textbf{Salary}} \\
\hline
Name & $r_2$ & \\
\hline
J. Page & 8,000 \\
V. Smith & 3,000 \\
M. Stowe & 7,000 \\
\hline
\end{tabular}

\((M. Stowe, 7,000)\) persists in all repairs, and it does not participate in the violation of the FD

\((J. Page, 8,000)\) does not persist in all repairs, and it does participate in the violation of FD
Fixed: DB schema and (infinite) domain; a set of first order integrity constraints $IC$

**Definition:** (Arenas, Bertossi, Chomicki; PODS 99)

A repair of a database instance $r$ is a database instance $r'$
- over the same schema and domain
- satisfies $IC'$
- differs from $r$ by a minimal set of changes (insertions or deletions of tuples) wrt set inclusion
Given a query $Q(x)$ to $r$, we want as answers all and only those tuples obtained from $r$ that are “consistent” wrt $IC$ (even when $r$ globally violates $IC'$)

**Definition:** (Arenas, Bertossi, Chomicki; PODS 99)

A tuple $\bar{t}$ is a consistent answer to query $Q(x)$ in $r$ iff $\bar{t}$ is an answer to query $Q(x)$ in every repair $r'$ of $r$:

$$r \models KQ[\bar{t}] \iff r' \models Q[\bar{t}]$$

for every repair $r'$ of $r$

A model theoretic definition ...
**Example**

Inconsistent DB instance \( r \) wrt \( FD: Name \rightarrow Salary \)

<table>
<thead>
<tr>
<th>Employee</th>
<th>Name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>J. Page</td>
<td>5,000</td>
<td></td>
</tr>
<tr>
<td>J. Page</td>
<td>8,000</td>
<td></td>
</tr>
<tr>
<td>V. Smith</td>
<td>3,000</td>
<td></td>
</tr>
<tr>
<td>M. Stowe</td>
<td>7,000</td>
<td></td>
</tr>
</tbody>
</table>

Repairs \( r_1, \) resp. \( r_2 \)

<table>
<thead>
<tr>
<th>Employee</th>
<th>Name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>J. Page</td>
<td>5,000</td>
<td></td>
</tr>
<tr>
<td>V. Smith</td>
<td>3,000</td>
<td></td>
</tr>
<tr>
<td>M. Stowe</td>
<td>7,000</td>
<td></td>
</tr>
</tbody>
</table>

\( r \models K \ Employee (M. Stowe, 7,000) \)
\[ r \models K (Employee(J.Page, 5, 000) \lor Employee(J.Page, 8, 000)) \]

\[ r \models K \exists X Employee(J.Page, X) \]

We can see this is not the same as getting rid of the data that participates in the violation of the IC

Some information is preserved ...
Computing Consistent Answers

So far: a semantic notion of consistent answer from an inconsistent database

We want to compute consistent answers

But not by computing all possible repairs and checking answers in common to all of them

Retrieving consistent answers via computation of all database repairs is not possible/sensible/feasible
**Example:** A database instance that is inconsistent wrt

\[ FD: \ X \rightarrow Y \]

<table>
<thead>
<tr>
<th>( r )</th>
<th>( X )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

has \( 2^n \) possible repairs!
Attacking the Problem (Overview)

We have considered different alternatives for computing consistent answers

1. (Arenas, Bertossi, Chomicki; PODS 99) and (Celle, Bertossi; DOOD 00)

A computational mechanism to check and compute consistent answers

Does not compute the repairs

Queries only the available inconsistent database instance

Transforms the query and poses the new query (as usual)
New Query (enh’d SQL):

SELECT ...
FROM ...
WHERE ...
CONSIS WITH ICs

Query (SQL):

SELECT ...
FROM ...
WHERE ...
.....
2. Represent in a compact way the collection of all database repairs and get information from the representation

2.1. (Arenas, Bertossi, Kifer; DOOD 00)

Repairs are some minimal models of a theory written in annotated predicate logic

2.2. (Arenas, Bertossi, Chomicki; TPLP 03) and (Barcelo, Bertossi; PADL 03)

Repairs are stable models of a logic program

Repairs specified by means of a logic program

To obtain consistent answers, run the program
2.3. (Arenas, Bertossi, Chomicki; ICDT 01)

Repairs are maximal independent sets in a graph whose nodes are the DB tuples.

Arcs are drawn between two tuples participating in the violation of a FD.
Related Work

We have used a particular notion of database repair:

- Basically no restriction on them
  (only minimality based on inclusion of sets of tuples)
- No assumption about the DB
- (Rest of this tutorial keeps referring to this notion)

However

There may be different assumptions about the DB
(in the presence of inclusion dependencies):
DB is possibly incorrect but complete:
repairs by deletion only
(Chomicki, Marcinkowski; Inf. and Comp., to appear)

DB is possibly incorrect and incomplete:
Fix FDs by deletion, referential ICs by insertion
(Cali, Lembo, Rosati; PODS 03)

Referential ICs are repaired using null values that do not propagate through other ICs
(Barcelo, Bertossi, Bravo; LNCS 2582)
Different notions of minimal repairs:

- **Minimal cardinality** set of changes
  (Arenas, Bertossi, Chomicki; TPLP 03)

- Minimal cardinality set of updates, i.e. changes of attribute values
  (as opposed to whole tuples)
  (Wijsen; ICDT 03)
  (Franconi, Laureti, Leone, Perri, Scarcello; LPAR 01)
Part B: A First Approach to CQA
Query Transformation

First-Order queries and constraints

Approach: Transform the query and keep the database instance!

Qualify the query with appropriate information derived from the interaction between the query and the ICs

- To locally satisfy the ICs

- To discriminate between tuples in the answer set

- Inspired by “Semantic Query Optimization” techniques
Consistent answers to \( Q(\bar{x}) \) in \( r \)??

Rewrite query: \( Q(\bar{x}) \mapsto Q'(\bar{x}) \)

\( Q'(\bar{x}) \) a new first order query

Retrieve from \( r \) the (ordinary) answers to \( Q'(\bar{x}) \)
**Example**

$IC: \forall x(P(x) \rightarrow Q(x)) \quad r = \{P(a), P(b), Q(b), Q(c)\}$

1. Query to $r$: $Q(x)$?

   If $Q(x)$ holds in $r$, then $P(x) \rightarrow Q(x)$ holds in $r$

   Elements in $Q$ do not participate in a violation of $IC$

2. Query: $P(x)$?

   If $P(x)$ holds in $r$, then $Q(x)$ must hold in $r$ in order to satisfy $P(x) \rightarrow Q(x)$
An answer $x$ to "$P(x)$?" is consistent if $x$ is also in table $Q$

Transform query 2. into: $P(x) \land Q(x)$?

Pose this query instead

$Q(x)$ is a residue of $P(x)$ wrt $\forall x(P(x) \rightarrow Q(x))$

Residue can be obtained by resolution between the query literal and the IC

Posing new query to $r$ we get only answer $\{b\}$

For query $Q(x)$? there is no residue, i.e. every answer to query $Q(x)$? is also a consistent answer, i.e. we get $\{b, c\}$
3. Query $\neg Q(x)$? (not safe, just for illustration)

Residue wrt $\forall x (P(x) \rightarrow Q(x))$ is $\neg P(x)$

New query: $\neg Q(x) \land \neg P(x)$

Answers to this new query (in the active domain): $\emptyset$

No consistent answers ...
Example

FD: \( \forall XYZ (\neg Employee(X, Y) \lor \neg Employee(X, Z) \lor Y = Z) \)

Query: \( Employee(X, Y) \)?

Consistent answers: \( (V. Smith, 3,000), (M. Stowe, 7,000) \)
(but not \( (J. Page, 5,000), (J. Page, 8,000) \))

Can be obtained by means of the transformed query

\[
T(Q(X, Y)) := Employee(X, Y) \land \\
\forall Z (\neg Employee(X, Z) \lor Y = Z)
\]

... those tuples \( (X, Y) \) in the relation for which \( X \) does not have and associated \( Z \) different from \( Y \) ...
SELECT Name, Salary
FROM Employee
CONSISTENT WITH
FD(Name;Salary)

SELECT Name, Salary
FROM Employee E
WHERE Not exists (SELECT E.Salary
FROM E
WHERE E.Name = Name
AND E.Salary <> Salary)

Again, the residue $\forall Z (\neg Employee(X, Z) \lor Y = Z)$ can be automatically obtained by applying resolution to the query and the $FD$

In general, $T$ is an iterative operator
Example

Relations: \(Supply(x, y, z): \text{“}x\text{ supplies item }z\text{ to }y\text{”}\)

\(Class(z, w): \text{“item }z\text{ belongs to class }w\text{”}\)

\(IC:\)  \(C\) is the only supplier of items of class \(K\)

\[\forall x, y, z (Supply(x, y, z) \land Class(z, K) \rightarrow x = C)\]

An instance \(r\) that violates \(IC\)

\[
\begin{array}{ccc}
\text{Supply} & \text{Class} \\
\hline
C & D_1 & I_1 \\
D & D_2 & I_2 \\
\hline
\end{array}
\]

\[
\begin{array}{cc}
I_1 & K \\
I_2 & K \\
\end{array}
\]
Query for items of class $K$: $\text{Class}(z, K)$?

Answer: $I_1, I_2$

However, $IC$ has not been considered, and $I_2$ is not a consistent answer.

Instead, we query with

$$T_\omega(\text{Class}(z, K)) \equiv \text{Class}(z, K) \land \forall(x, y)(\text{Supply}(x, y, z) \rightarrow x = C)$$

Only consistent answer: $I_1$
**Example**

\[ IC : \{ R(x) \lor \neg P(x) \lor \neg Q(x), \ P(x) \lor \neg Q(x) \} \]

**Query:** \( Q(x) \)

\[ T_1(Q(x)) : = Q(x) \land (R(x) \lor \neg P(x)) \land P(x) \]

Apply \( T \) again, now to the appended residues

\[ T_2(Q(x)) : = Q(x) \land (T(R(x)) \lor T(\neg P(x))) \land T(P(x)) \]

\[ T_2(\varphi(x)) = Q(x) \land (R(x) \lor (\neg P(x) \land \neg Q(x))) \land P(x) \land (R(x) \lor \neg Q(x)) \]

And again:
$$T_3(Q(x)) := Q(x) \land (R(x) \lor (\neg P(x) \land T(\neg Q(x)))) \land$$
$$P(x) \land (T(R(x)) \lor T(\neg Q(x)))$$

Since $T(\neg Q(x)) = \neg Q(x)$ and $T(R(x)) = R(x)$, we obtain

$$T_3(Q(x)) = T_2(Q(x))$$

A finite fixed point! Does it always exist?

In general, an infinitary query: $T_\omega(\varphi(x)) := \bigcup_{n<\omega}\{T_n(\varphi(x))\}$

In the example, $T_\omega(Q(x)) = \{T_1(Q(x)), T_2(Q(x))\}$

Always finite?
Some Results

There are sufficient conditions on queries and ICs for soundness and completeness of operator $T$ (ABC; PODS 99)

- **Soundness**: every tuple computed via $T$ is consistent in the semantic sense
  
  $$ r \models T_\omega(\varphi)[t] \implies r \models K\varphi[t] $$

- **Completeness**: every semantically consistent tuple can be obtained via $T$
  
  $$ r \models K\varphi[t] \implies r \models T_\omega(\varphi)[t] $$

Natural and useful syntactical classes satisfy the conditions

But incomplete for full FO queries and ICs
There are necessary and sufficient conditions for syntactic termination

- In the iteration process to determine $T_\omega(Q)$ nothing syntactically new is obtained beyond some finite step

There are sufficient conditions for semantic termination

- From some finite step on, only logically equivalent formulas are obtained

In these favorable cases, a FO SQL query can be translated into a new FO SQL query that is posed as usual to the database
Implementation

Semantic termination is difficult to detect and implement

A new algorithm, QUECA, inspired by $T$ was introduced (Celle, Bertossi; DOOD 00)

- It syntactically terminates for a wider class of ICs
- Based on a careful syntactical analysis and memorization of residues and subsumptions between them
- Implemented on XSB
  About XSB: (Sagonas, Swift, Warren; SIGMOD 94)
Implementation in XSB makes it possible:

- Trying direct unification between residues
- Using tabling to avoid redundant computation of residues
- Interaction with DBMSs; in our case, IBM DB2

Methodology works for universal binary constraints, i.e. containing at most two database literals plus built-ins, e.g.

- FDs: \( P(u, x, y) \land P(v, x, z) \rightarrow y = z \)
- Full inclusion dependencies: \( P(\bar{x}) \rightarrow Q(\bar{x}) \)
- Range constraints: \( P(x, y) \rightarrow y < 100 \)
New Query:
SELECT ...
FROM ...
WHERE ...

Query (SQL):
SELECT ...
FROM ...
WHERE ...
.....
**Some Limitations**

First order query rewriting based approach has limitations (most of them apply to the one based on operator $T$ and to any other; see later ...)

- $T$ is defined and works for some special classes of queries and integrity constraints

- ICs are universal, which excludes referential ICs; and queries are quantifier-free conjunctions of literals

- $T$ does not work for disjunctive or existential queries, e.g. $\exists Y \ Employee(J\ Page, Y)$?
FO query reformulation has been slightly extended using other methods

- **Hypergraph representation** of the DB (the vertices) and its semantic conflicts (the hyperedges)

- Graph based algorithms on original query can be translated into SQL queries (Chomicki, Marcinkowski, Staworko; software demos at EDBT 04)

From the **logical point of view**:

- We have not logically **specified** the database repairs

- We have a **model-theoretic definition** plus an incomplete computational mechanism
From such a specification $Spec$ we might:

- Reason from $Spec$
- Consistently answer queries: $Spec \models Q(\bar{x})$
- Derive algorithms for consistent query answering

Consistent query answering is non-monotonic; then a non-monotonic semantics for $Spec$ is expected.
Part C: Specifying Database Repairs
**Specification in Annotated Logic**

We want to specify database repairs, by means of a consistent theory.

The database instance $r$ (seen as a set of ground atomic formulas) and the set of integrity constraints $IC$ are mutually inconsistent.

Use a different logic, that allows generating a consistent theory!

Use annotated predicate calculus (APC) (Kifer, Lozinskii; J. Aut. Reas. 92)

Inconsistent classical theories can be translated into consistent annotated theories.
Usual annotations: true (t), false (f), contradictory (⊤), unknown (⊥)

Atoms in an APC theory are annotated with truth values, at the object level, e.g.

$\text{Employee}(V\cdot Smith, 3000):t, \text{Employee}(V\cdot Smith, X):f$

Embed both $r$ and $IC$ into a single consistent APC theory (Arenas, Bertossi, Kifer; DOOD 00)

- ICs are hard, not to be given up
- Data is flexible, subject to repairs

Choose an appropriate truth values lattice $\mathcal{L}at$: 
- Database values: $t_d, f_d$
- Constraint values: $t_c, f_c$
- Advisory values: $t_a, f_a$  They advise to solve conflicts between $d$-values and $c$-values in favor of $c$-values
Intuitively, ground atoms $A$ for which $A:\text{t}_a$ or $A:\text{f}_a$ become true are to be inserted into, resp. deleted from $r$

Generate an APC theory $Spec$ embedding $r$ and $IC$ into APC:

- Translate the constraint:
  $$\neg \text{Employee}(X, Y) \vee \neg \text{Employee}(X, Z) \vee Y = Z$$
  into
  $$\text{Employee}(X, Y):\text{f}_c \vee \text{Employee}(X, Z):\text{f}_c \vee Y = Z:\text{t}$$

- Translate database facts, e.g. $\text{Employee}(J.\text{Page}, 5, 000)$ into $\text{Employee}(J.\text{Page}, 5, 000):\text{t}_d$

- Plus axioms for unique names assumption, closed world assumption, ...
Navigation in the lattice plus an adequate definition of APC formula satisfaction help solve the conflicts between database facts and constraint facts

- For every \( s \in \text{Lat} \), \( \bot \leq s \leq \top \)
- \( \text{lub}(t, f) = \top \), \( \text{lub}(t_c, f_d) = t_a \), etc.
- Use Herbrand structures, i.e. sets of ground annotated atoms
- Formula satisfaction: \( I \) a structure, \( s \in \text{Lat} \), \( A \) a classical atomic formula

\[
I \models A:s \quad \text{iff} \quad \text{there exists } s' \in \text{Lat} \text{ such that } A:s' \in I \text{ and } s \leq s'
\]
It can be proved that the database repairs correspond to the models of $Spec$ that make true a minimal set of atoms annotated with $t_a$, $f_a$

Change a minimal set of database atoms!!!

From the specification $Spec$ algorithmic and complexity results for consistent query answering can be obtained

Most importantly, this approach motivated a more general and practical approach to specification of database repairs based on logic programs
Specifying Repairs with Logic Programs

The collection of all database repairs can be represented in a compact form

Use disjunctive logic programs with stable model semantics (Barcelo, Bertossi; PADL 03)

Repairs correspond to distinguished models of the program, namely to its stable models

Example: Full inclusion dependency  \( IC: \forall \bar{x}(P(\bar{x}) \rightarrow Q(\bar{x})) \)

Inconsistent instance  \( r = \{P(\bar{c}), P(\bar{d}), Q(\bar{d}), Q(\bar{e})\} \)
The programs use *annotation constants* in an extra attribute in the database relations.

<table>
<thead>
<tr>
<th>Annotation</th>
<th>Atom</th>
<th>The tuple $P(\bar{a})$ is ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_d$</td>
<td>$P(\bar{a}, t_d)$</td>
<td>a fact of the database</td>
</tr>
<tr>
<td>$f_d$</td>
<td>$P(\bar{a}, f_d)$</td>
<td>a fact not in the database</td>
</tr>
<tr>
<td>$t_a$</td>
<td>$P(\bar{a}, t_a)$</td>
<td>advised to be made true</td>
</tr>
<tr>
<td>$f_a$</td>
<td>$P(\bar{a}, f_a)$</td>
<td>advised to be made false</td>
</tr>
<tr>
<td>$t^*$</td>
<td>$P(\bar{a}, t^*)$</td>
<td>true or becomes true</td>
</tr>
<tr>
<td>$f^*$</td>
<td>$P(\bar{a}, f^*)$</td>
<td>false or becomes false</td>
</tr>
<tr>
<td>$t^{**}$</td>
<td>$P(\bar{a}, t^{**})$</td>
<td>true in the repair</td>
</tr>
<tr>
<td>$f^{**}$</td>
<td>$P(\bar{a}, f^{**})$</td>
<td>false in the repair</td>
</tr>
</tbody>
</table>
Repair program $\Pi(r, IC)$:

1. The original data: $P(\bar{c}, t_d) \leftarrow$
   
   $P(\bar{d}, t_d) \leftarrow$
   
   $Q(\bar{d}, t_d) \leftarrow$
   
   $Q(\bar{e}, t_d) \leftarrow$

2. Whatever was true (false) or becomes true (false), gets annotated with $t^\ast$ ($f^\ast$):
   
   $P(\bar{x}, t^\ast) \leftarrow P(\bar{x}, t_d)$
   
   $P(\bar{x}, t^\ast) \leftarrow P(\bar{x}, t_a)$
   
   $P(\bar{x}, f^\ast) \leftarrow \text{not } P(\bar{x}, t_d)$
   
   $P(\bar{x}, f^\ast) \leftarrow P(\bar{x}, f_a)$

   ... the same for $Q$ ...
3. There may be interacting ICs (not here), and the repair process may take several steps, changes could trigger other changes

We need annotation constants for the local changes \((t_a, f_a)\), but also annotations \((t^\ast, f^\ast)\) to provide feedback to the rules that produce local repair steps

\[
P(\bar{x}, f_a) \lor Q(\bar{x}, t_a) \leftarrow P(\bar{x}, t^\ast), Q(\bar{x}, f^\ast)
\]

One rule per IC; that says how to repair the IC in case of a violation

Passing to annotations \(t^\ast\) and \(f^\ast\) allows to keep repairing the DB wrt to all the ICs until the process stabilizes
4. Repairs must be coherent: use denial constraints at the program level to prune undesirable models

\[ \leftarrow P(\bar{x}, t_a), P(\bar{x}, f_a) \]
\[ \leftarrow Q(\bar{x}, t_a), Q(\bar{x}, f_a) \]

5. Annotations constants \( t^{**} \) and \( f^{**} \) are used to read off the literals that are inside (outside) a repair

\[ P(\bar{x}, t^{**}) \leftarrow P(\bar{x}, t_a) \]
\[ P(\bar{x}, t^{**}) \leftarrow P(\bar{x}, t_d), \text{ not } P(\bar{x}, f_a) \]
\[ P(\bar{x}, f^{**}) \leftarrow P(\bar{x}, f_a) \]
\[ P(\bar{x}, f^{**}) \leftarrow \text{ not } P(\bar{x}, t_d), \text{ not } P(\bar{x}, t_a). \ldots \text{ etc.} \]
The program has two stable models (and two repairs):

\[
\{ P(\bar{c}, t_d), \ldots, P(\bar{c}, t^*), Q(\bar{c}, f^*), Q(\bar{c}, t_a), P(\bar{c}, t^{**}), Q(\bar{c}, t^*),
Q(\bar{c}, t^{**}), \ldots \} \equiv \{ P(\bar{c}), Q(\bar{c}), P(\bar{d}), Q(\bar{d}), Q(\bar{e}) \}
\]

... insert \(Q(\bar{c})!!\)

\[
\{ P(\bar{c}, t_d), \ldots, P(\bar{c}, t^*), P(\bar{c}, f^*), Q(\bar{c}, f^*), P(\bar{c}, f^{**}), Q(\bar{c}, f^{**}),
P(\bar{c}, f_a), \ldots \} \equiv \{ P(\bar{d}), Q(\bar{d}), Q(\bar{e}) \}
\]

... delete \(P(\bar{c})!!\)
To obtain consistent answers to a FO SQL query:

1. Transform or provide the query as a logic program (this is standard methodology)

2. Run the query program together with the specification program
   ... under the skeptical or cautious stable model semantics that sanctions as true of a program what is true of all its stable models
**Example:** (continued)

Consistent answers to query \( P(\bar{x}) \land \neg Q(\bar{x}) \)?

Run repair program \( \Pi(r, IC) \) together with query program

\[
\text{Ans}(\bar{x}) \leftarrow P(\bar{x}, t^{**}), Q(\bar{x}, f^{**})
\]

The two previous stable models become extended with ground \( \text{Ans} \) atoms

None of them in the intersection of the two models

In consequence, under the skeptical SMS, \( \text{Ans} = \emptyset \), i.e. no consistent answers, as expected ...
We have successfully experimented with the *DLV* system for computing the stable models semantics (N. Leone et al.; ACM Tr. Comp. Logic)

Related methodologies:
(Arenas, Bertossi, Chomicki; TPLP 03)
(Greco, Greco, Zumpano; IEEE TKDE 03)
Consistent Answers

DBMS

DLV

ICs

Query (Logic) Program:
Ans (x) :- ....
.... :- ....
.... :- ....

Specification of Repairs:
.... :- ....
.... :- ....
.... :- ....

Query (SQL):
SELECT ...
FROM ...
WHERE ...
.....
Remarks:

- This methodology is quite general

- Existential ICs, like referential ICs, can be handled, with different repair policies, e.g. introduction of null values, cascaded deletions, ...

- The same repair program can be used for all the queries, the same applies to the computed stable models

  The query at hand adds a final layer on top (a split program)
- The program can be optimized in several ways
  In particular the materialization of the *Closed World Assumption* (in 2.) can be avoided
  (Barcelo, Bertossi, Bravo; LNCS 2582)

- There some **challenges** though:
  - Existing implementations of stable models semantics are based on grounding the rules
  - In database applications, this may lead to huge ground programs
  - Implementations are geared to computing (some) stable model(s) and answering ground queries
• For database applications, posing and answering open queries is more natural

• Computing all the stable models completely is undesirable

• Query evaluation based on skeptical stable model semantics should be guided by the query and its relevant information in the database

Relevant research:
(Eiter, Fink, G. Greco, Lembo; ICLP 03)
Part D: Aggregation Queries
We have presented first order queries only

What about aggregation queries?

- They are natural and usual in DBs, and part of SQL

- They are crucial in scenarios where inconsistencies are likely to occur, e.g. data integration, in particular, datawarehousing

We will see that aggregation is challenging for consistent answers
A restricted scenario:

- Functional dependencies

- Standard set of SQL-2 scalar aggregation operators: MIN, MAX, COUNT(*), COUNT(A), SUM, and AVG

- Atomic queries applying just one of these operators
Redefining Consistent Answers

Example: A database instance and the \( FD: Name \rightarrow Amount \)

<table>
<thead>
<tr>
<th>Salary</th>
<th>Name</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( V.\text{Smith} )</td>
<td>5000</td>
</tr>
<tr>
<td></td>
<td>( V.\text{Smith} )</td>
<td>8000</td>
</tr>
<tr>
<td></td>
<td>( P.\text{Jones} )</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>( M.\text{Stone} )</td>
<td>7000</td>
</tr>
</tbody>
</table>

The repairs:

<table>
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<th>Name</th>
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</table>

Query: MIN(Amount)?
We should get 3000 as a consistent answer: $\text{MIN}(\text{Amount})$ returns 3000 in every repair.

**Query:** $\text{MAX}(\text{Amount})$?

The maximum, 8000, comes from a tuple that participates in the violation of $FD$.

$\text{MAX}(\text{Amount})$ returns a different value in each repair: 7000 or 8000.

There is no consistent answer as previously defined.

Modify the definition of consistent answer:
**Definition:** A consistent answer to an aggregate query \( Q \) in the database instance \( r \) is a numerical interval that contains all the answers to \( Q \) obtained from the repairs of \( r \).

An optimal consistent answer to \( Q \) is the smallest interval \([a, b]\) such that ...

In the example:

- \([6000, 9000]\) is a consistent answer to the query \( \text{MAX(Amount)} \)

- \([7000, 8000]\) is an optimal consistent answer to \( \text{MAX(Amount)} \)

(Arenas, Bertossi, Chomicki; ICDT 01)
Problems: Find and determine

- Algorithms for computing the optimal bounds:

\[ \begin{array}{c|c|c}
  a & & b \\
  \hline
  \end{array} \]

- \( a \): the max-min answer; and
- \( b \): the min-max answer

By querying \( r \) only!

- Computational complexities

We need the right tools to attack these problems ...
Graph Representation of Repairs

For both purposes it was crucial to appeal to a graph representation of repairs

Given a set of FDs $FD$ and an instance $r$, the conflict graph $CG_{FD}(r)$ is an undirected graph:

- **Vertices** are the tuples $\bar{t}$ in $r$

- **Edges** are of the form $(\bar{t}_1, \bar{t}_2)$ for which there is a dependency in $FD$ that is simultaneously violated by $\bar{t}_1, \bar{t}_2$
Example: Schema \( R(A, B) \)  

FDs: \( A \rightarrow B \) and \( B \rightarrow A \)

Instance \( r = \{(a_1, b_1), (a_1, b_2), (a_2, b_2), (a_2, b_1)\} \)

Each repair of \( r \) corresponds to a maximal independent set in \( CG_{FD}(r) \)

... or to a maximal clique in the complement graph
Some Complexity Results

- MAX(A) can be different in every repair

  Maximum of the MAX(A)’s is MAX(A) in \( r \)

  Then computing the min max-answer to MAX(A) from \( r \) is direct

- Computing directly from \( r \) the minimum of the MAX(A)’s, i.e. the maximal min-answer to MAX(A), is not that immediate

But still, computing the maximal min-answer to MAX(A) for one FD \( F \) is in PTIME (in data complexity)
- For more than one FD, the problem of deciding whether the maximal min-answer to $\text{MAX}(A) \leq k$ is NP-complete (reduction from SAT)

- Even for one FD, the problem of deciding if the maximal min-answer to $\text{COUNT}(A) \leq k$ is NP-complete (reduction from HITTING SET)
In general:

<table>
<thead>
<tr>
<th></th>
<th>maximal min-answer</th>
<th>minimal max-answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>FD</td>
<td>= 1$</td>
</tr>
<tr>
<td>$</td>
<td>FD</td>
<td>\geq 2$</td>
</tr>
<tr>
<td>MIN(A)</td>
<td>PTIME</td>
<td>NP-complete</td>
</tr>
<tr>
<td>MAX(A)</td>
<td>PTIME</td>
<td>PTIME</td>
</tr>
<tr>
<td>COUNT(*)</td>
<td>PTIME</td>
<td>NP-complete</td>
</tr>
<tr>
<td>COUNT(A)</td>
<td>NP-complete</td>
<td>NP-complete</td>
</tr>
<tr>
<td>SUM(A)</td>
<td>PTIME</td>
<td>NP-complete</td>
</tr>
<tr>
<td>AVG(A)</td>
<td>PTIME</td>
<td>NP-complete</td>
</tr>
</tbody>
</table>

(Arenas, Bertossi, Chomicki, He, Raghavan, Spinrad; Th. Comp. Sci. 03)
We have identified normalization conditions, e.g. BCNF, (and other conditions) on the DB under which more efficient algorithms can be designed.

However, improvements are not impressive.

CQA for aggregate queries is an intrinsically complex problem.

It seems necessary to approximate optimal consistent answers to aggregate queries, but “maximal independent set” seems to have bad approximation properties ...
Part E: Complexity of CQA
When the first order query rewriting approach works (correct and terminating), consistent answers to FO queries can be obtained in \textit{PTIME} (data complexity).

Graph theoretic techniques for CQA for aggregate queries were extended (hypergraphs now) to:

- Extend the \textit{PTIME} computation to other classes of FO queries, e.g. with very restricted forms of projection (existential quantifiers), but denial constraints.

- Study the complexity of CQA for FO queries for wider classes of integrity constraints, e.g. including referential ICs (but only deletions for repair).

(Chomicki, Marcinkowski; Inf. Comp., to appear)
Some Complexity Results

In those cases where CQA can be done in PTIME, the problem of repair checking can be solved in PTIME.

Repair checking is also PTIME for arbitrary FDs and acyclic inclusion dependencies (deletions only).

However: (deletions only)

- For arbitrary FDs and inclusion dependencies, repair checking becomes coNP-complete.
- For arbitrary FDs and inclusion dependencies, CQA, i.e., deciding if a tuple is CA, becomes $\Pi_2^P$-complete.
- (Query answering from disjunctive logic programs under skeptical stable models semantics is also $\Pi_2^P$-complete!!)
More complexity theoretic results:
(Cali, Lembo, Rosati; PODS 03)

Among others:

- For arbitrary FDs and inclusion dependencies (in particular, referential ICs), CQA becomes undecidable

Issues?

- Inclusion dependencies repaired through insertions
- Cycles in the set of inclusion dependencies
- Infinite underlying domain that can be used for insertions
Remarks:

- Complexity of query evaluation from disjunctive logic programs (DLPs) coincides with the complexity of CQA.

- From this point of view the problem of CQA is not being overkilled by the use of the DLP approach.

- However, it is known that for wide classes of queries and ICs, CQA has a lower complexity, e.g. in P time.

- It becomes relevant to identify classes of ICs and queries for which the DLP can be automatically “simplified” into, e.g. a FO query.

- There is ongoing research on this ...
Part F: CQA in Virtual Data Integration Systems
Virtual Data Integration

Scenario:

- A collection of material data sources $S_1, \ldots, S_n$ with relational schemas

- A virtual database $\mathcal{G}$ with a global relational schema that integrates the data in the sources

- A collections of mappings descriptions that specify the relationship between the data “in” the virtual relations and the data in the source relations

Queries are posed at the global level
Global Query (SQL):

```
SELECT ... 
FROM ... 
WHERE ... 
.....
```
The mediator $G$ receives a global query $Q$ and generates a query plan that when executed, extracts and combines the information from the sources.

Plan generation depends on how the contents of the sources are described by the mappings.

Several approaches to virtual data integration:

- **Global-as-view approach (GAV)**: Global relations are specified as views over the source relations.
- **Local-as-view approach (LAV)**: Data sources are specified as views over the global schema.
- **Mixed approaches** ... (a survey in Lenzerini; PODS 02)
Example: (LAV) Sources relations $V_1, V_2$ described by:

$$S_1: \; V_1(Title, Year, Director) \leftarrow \text{Movie}(Title, Year, Director, Genre), \text{American}(director), Year \geq 1960, Genre = \text{comedy}.$$  

$$S_2: \; V_2(Title, Review) \leftarrow \text{Movie}(Title, Year, Director, Genre), Year \geq 1990, \text{Review}(Title, Review).$$

Global schema $G: \text{Movie}, \text{Review}, \text{American}$

In addition, there are material extensions for the source relations: $v_1, v_2$
Query to $G$: Comedies w/reviews since 1950?

$$Q(Title, Review) \leftarrow$$

$$\begin{array}{l}
\text{Movie}(Title, Year, Director, Genre), \\
\text{Review}(Title, Review), Year \geq 1950, \\
Genre = \text{comedy}.
\end{array}$$

Information is in the sources, now, views ...

A query plan: (there are methodologies for obtaining them)

$$Q'(Title, Review) \leftarrow$$

$$\begin{array}{l}
V_1(Title, Year, Director), V_2(Title, Review).
\end{array}$$

Usually one assumes that certain ICs hold at the global level; and they are used in the generation of query plans.
How can we be sure that those global ICs hold?

They are not maintained at the global level

Most likely they are not fully satisfied

A natural scenario for applying CQA: retrieve only information from the global, virtual database that is consistent with $IC$

New issues appear:

- What is a repair of the global virtual database?
- Only one global database to repair?
- How to retrieve consistent information from the global virtual DB $G$ at query time ...
Global Query (SQL):

```
SELECT ...
FROM ...
WHERE ...
CONSISTENT WITH .....;
```
We will assume:

- The LAV approach is adopted
- The source relations are declared as open:

  The global relations can be materialized in different ways, still satisfying the source descriptions; so different global instances are possible.

  A global (material) instance $D$ is legal if the view definitions applied to it compute extensions $V_1(D), V_2(D)$ such that $v_1 \subseteq V_1(D)$ and $v_2 \subseteq V_2(D)$.

  That is, each source relation contains a possibly proper subset of the data of its kind in the global system.
Example

Global system $G_1$ with sources

\[ V_1(X, Y) \leftarrow R(X, Y) \quad \text{with} \quad v_1 = \{(a, b), (c, d)\} \]
\[ V_2(X, Y) \leftarrow R(Y, X) \quad \text{with} \quad v_2 = \{(c, a), (e, d)\} \]

\[ D = \{ R(a, b), R(c, d), R(a, c), R(d, e) \} \] and its supersets are the legal instances

Global query $Q$: $R(X, Y)$?

\[ Certain_{G_1}(Q) = \{(a, b), (c, d), (a, c), (d, e)\} \]

Certain answers to a query are true in all the legal instances
What if we had a global functional dependency $R: X \rightarrow Y$?

(local FDs $V_1: X \rightarrow Y$, $V_2: X \rightarrow Y$ satisfied in the sources)

Global FD not satisfied by $D = \{(a, b), (c, d), (a, c), (d, e)\}$
(nor by its supersets)

From the certain answers to the query $Q: R(X, Y)$?, i.e. from

$$\text{Certain}_{G_1}(Q) = \{(a, b), (c, d), (a, c), (d, e)\}$$

only $(c, d), (d, e)$ should be consistent answers
Much effort made by the DB community to find algorithms for generating plans to obtain the certain answers (still some limitations)

Not much for obtaining consistent answers

Here we do both, in stages ...

First concentrating on the **minimal legal instances** of a virtual systems, i.e. those that do not properly contain any other legal instance

- They do not contain unnecessary information; that could, unnecessarily, violate global ICs
In the example, \( D = \{ R(a, b), R(c, d), R(a, c), R(d, e) \} \) is the only minimal instance

The minimal answers to a query are those that can be obtained from every minimal legal instance:

\[
\text{Certain}_G(Q) \subseteq \text{Minimal}_G(Q)
\]

For monotone queries they coincide

Consistent answers to a global query wrt \( IC \) are those obtained from all the repairs of all the minimal legal instances wrt \( IC \)

(Bertossi, Chomicki, Cortes, Gutierrez; FQAS 02)
In the example:

- The only minimal legal instance

\[ D = \{ R(a, b), R(c, d), R(a, c), R(d, e) \} \]

violates the FD \( R : X \rightarrow Y \)

- Its repairs wrt FD are

\[ D^1 = \{ R(a, b), R(c, d), R(d, e) \} \quad \text{and} \]
\[ D^2 = \{ R(c, d), R(a, c), R(d, e) \} \]

- Consistent answers to query \( Q: R(X, Y) \)?

  Only \( \{(c, d), (d, e)\} \)
Computing consistent answers? (Bravo, Bertossi; IJCAI 03)

- Answer set programming (ASP) based specification of minimal instances of a virtual data integration system

- ASP based specification of repairs of minimal instances (we saw how to do this, e.g. programs with annotation constants)

- Global query in Datalog (or its extensions) to be answered consistently

- Run combined programs above under skeptical answer set semantics (stable model semantics)
- Methodology works for first-order queries (and Datalog extensions), and universal ICs combined with (acyclic) referential ICs

- **Important subproduct:** A methodology to **compute certain answers to monotone queries**
Mappings

Global Relations

Global ICs

Query

Answer Set Programming (ASP) specification of a set of Legal Global Instances

ASP specification of the repairs

Query Program (Datalog)

Consistent Answers to Query

DLV
Run under skeptical answer set semantics
Example

Domain: $\mathcal{D} = \{a, b, c, \ldots \}$  

Global system $\mathcal{G}_2$:

\[
\begin{align*}
V_1(X, Z) & \leftarrow P(X, Y), R(Y, Z) & v_1 = \{(a, b)\} & \text{open} \\
V_2(X, Y) & \leftarrow P(X, Y) & v_2 = \{(a, c)\} & \text{open} \\
\end{align*}
\]

Mininst($\mathcal{G}_2$) = \{\{P(a, c), P(a, z), R(z, b)\} \mid z \in \mathcal{D}\}

Specification of minimal instances: $\Pi(\mathcal{G}_2)$

\[
\begin{align*}
P(X, Z) & \leftarrow V_1(X, Y), F_1(X, Y, Z) \\
P(X, Y) & \leftarrow V_2(X, Y) \\
R(Z, Y) & \leftarrow V_1(X, Y), F_1(X, Y, Z) \\
F_1(X, Y, Z) & \leftarrow V_1(X, Y), \mathit{dom}(Z), \mathit{choice}((X, Y), (Z)) \\
\mathit{dom}(a)., & \mathit{dom}(b)., \mathit{dom}(c)., \ldots, V_1(a, b)., V_2(a, c). \\
\end{align*}
\]
Inspired by inverse rules algorithm for computing certain answers (Duschka, Genesereth, Levy; JLP 00)

\( F_1 \) is a functional predicate, whose functionality on the second argument is imposed by the choice operator

\[
\text{choice}( (\bar{X}), (Z) ) : \text{non-deterministically chooses a unique value for } Z \text{ for each combination of values for } \bar{X}
\]

(Giannotti, Pedreschi, Sacca, Zaniolo; DOOD 91)

Models of \( \Pi(\mathcal{G}_2) \) are the choice models, but the program can be transformed into one with stable models semantics
\( M_b = \{ \text{dom}(a), \ldots, V_1(a, b), V_2(a, c), P(a, c), \text{diffChoice}_1(a, b, a), \text{chosen}_1(a, b, b), \text{diffChoice}_1(a, b, c), F_1(a, b, b), R(b, b), P(a, b) \} \)

\( M_a = \{ \text{dom}(a), \ldots, V_1(a, b), V_2(a, c), P(a, c), \text{chosen}_1(a, b, a), \text{diffChoice}_1(a, b, b), \text{diffChoice}_1(a, b, c), F_1(a, b, a), R(a, b), P(a, a) \} \)

\( M_c = \{ \text{dom}(a), \ldots, V_1(a, b), V_2(a, c), P(a, c), \text{diffChoice}_1(a, b, a), \text{diffChoice}_1(a, b, b), \text{chosen}_1(a, b, c), F_1(a, b, c), R(c, b) \} \)

\[ \ldots \]

Here: 1-1 correspondence with minimal instances of \( \mathcal{G}_2 \)
In general:

- The minimal instances are all among the models of the program

- All the models of the program are (determine) legal instances

- In consequence, the program can be used to compute all the certain answers to monotone queries

- The program can be refined to compute all and only the minimal legal instances
Related Work

- More details about this approach to CQA in VDISs: (Bravo, Bertossi; J. Appl. Logic, to appear)

- Extension to open, closed and clopen sources in (Bertossi, Bravo; in forthcoming book by Springer)

- Consistency handling, repairs and different semantics for CQA under GAV
  - (Lembo, Lenzerini, Rosati; KRDB 02)
  - (Cali, Lembo, Rosati; IJCAI 03)
There are clear connections between query answering in VDISs and query answering in peer-to-peer data exchange systems.

Peers exchange data at query answering time according to certain data exchange constraints or data exchange mappings.

No central data repository; no centralized management; data resides at peers’ sites ...