COMP 4900C: Assignment 1
Due: Tuesday, Feb. 7, 2008

1. (2 points) The rotation matrix in the x-y plane is
$$R(\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$
   (a) Verify $R(\phi_1)R(\phi_2) = R(\phi_1 + \phi_2)$ from the identities for $\cos(\phi_1 + \phi_2)$ and $\sin(\phi_1 + \phi_2)$.
   (b) What is $R(\phi)R(-\phi)$?

2. (4 points) Scale a vector $[x \quad y]^T$ in the plane can be achieved by
$$x' = sx \text{ and } y' = sy$$
   where s is a scalar.
   (a) Write out the matrix form of this transformation.
   (b) Write out the transformation matrix for homogeneous coordinates.
   (c) If the transformation also includes a translation
$$x' = sx + t_x \text{ and } y' = sy + t_y$$
   Write out the transformation matrix for homogeneous coordinates.
   (d) What is the equivalent of the above matrix for three-dimensional vectors?

3. (2 points) Find the least square solution $\bar{x}$ for $Ax = b$ if
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$
   Verify that the error vector $b - A\bar{x}$ is orthogonal to the columns of A.

4. (2 points) A pinhole camera has focal length $f = 500$, pixel sizes $s_x = s_y = 1$, and its principal point is at $(o_x, o_y) = (320,240)$. The world coordinate frame and the camera coordinate frame can be related by $X_c = RX_w + T$, where
$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$
   (a) Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.
   (b) What are the pixel coordinates of the world point
$$X_w = [150 \quad 200 \quad 400]^T \text{ ?}$$