Formal Methods in Security

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Summary

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- The role of formal methods

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- Probabilistic reasoning

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- Channel capacity as a measure of anonymity

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- Games, Capacities and Previsions
- Conclusions

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- Attackers have access to every message and can synthesize messages.
- They can perform statistical analysis of intercepted messages.
- What can be done to preserve secrecy or anonymity?

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- use tools like model checkers, bisimulation checkers to verify properties of the protocol.
- The models may be probabilistic.
- Legendary success: Gavin Lowe and the Needham-Schroeder protocol.

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- Probabilistic process algebra and metrics were used by John Mitchell et al.
- Anonymity protocols analyzed by Palamidessi et al.
- Probabilistic model checking developed by Kwiatkowska et al.; the PRISM system.

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  - File sharing, refereeing (!), ...
- In some sense “dual” to secrecy.

Example Systems

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- Freenet [Clarke et. al. 2001]: anonymous information retreival

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- The probabilistic approach is essential when the protocols themselves use randomization
- However, usually both probability and nondeterminism is present.

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- Beyond suspicion: to the observer, the culprit is not more likely than any other agent to be the culprit.
- Probable innocence: the culprit has less than 50% chance of being the culprit.
- Possible innocence: the culprit has less than 100% chance of being the culprit.

Dining Cryptographers: Chaum 1988

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  - Three cryptographers share a meal
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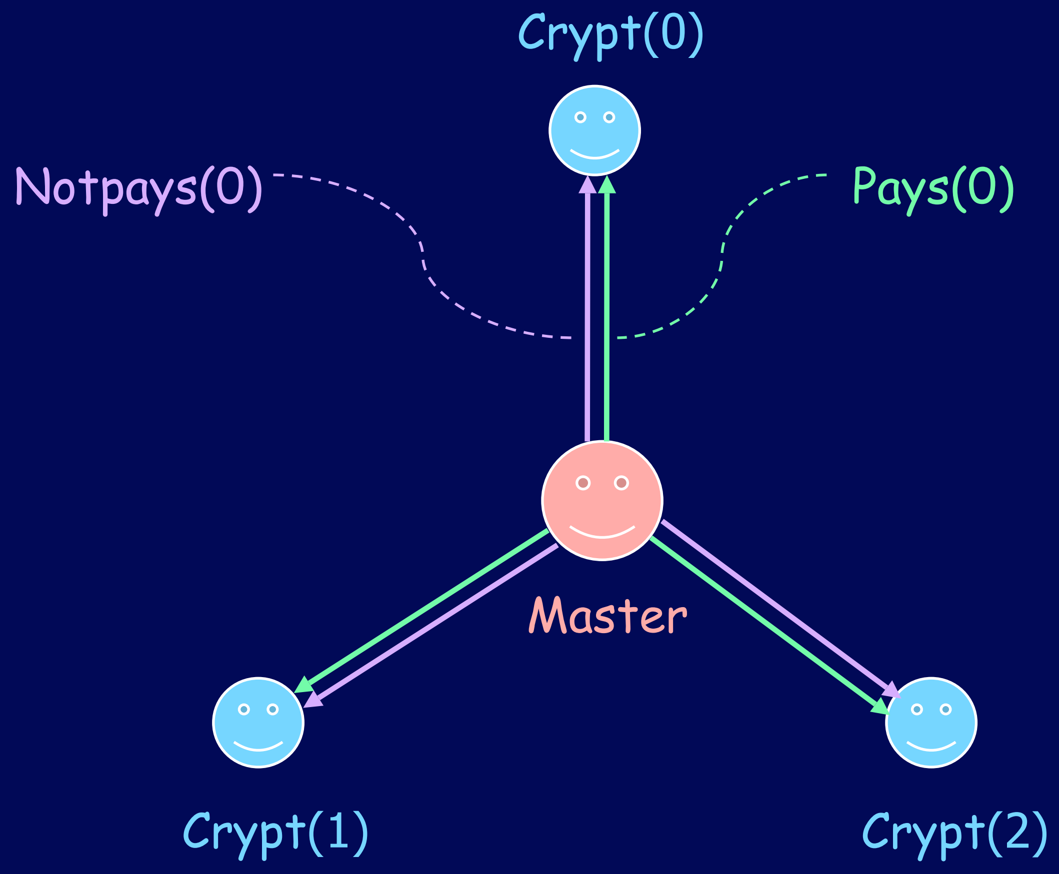
  - Three cryptographers share a meal
  - The meal is either paid by M or by one of the diners, M decides who will pay
  - M informs each one whether they will pay or not
- The goal: the cryptographers want to find out if one of them is paying without knowing who.

The dining cryptographers



Solution

- We insert a coin between each pair of cryptographers and toss it
- The result of each coin toss is visible only to the adjacent cryptographers
- Each cryptographer examines the two adjacent coins and says "agree" or "disagree"
- The one who pays (if any) will say the opposite of the truth.

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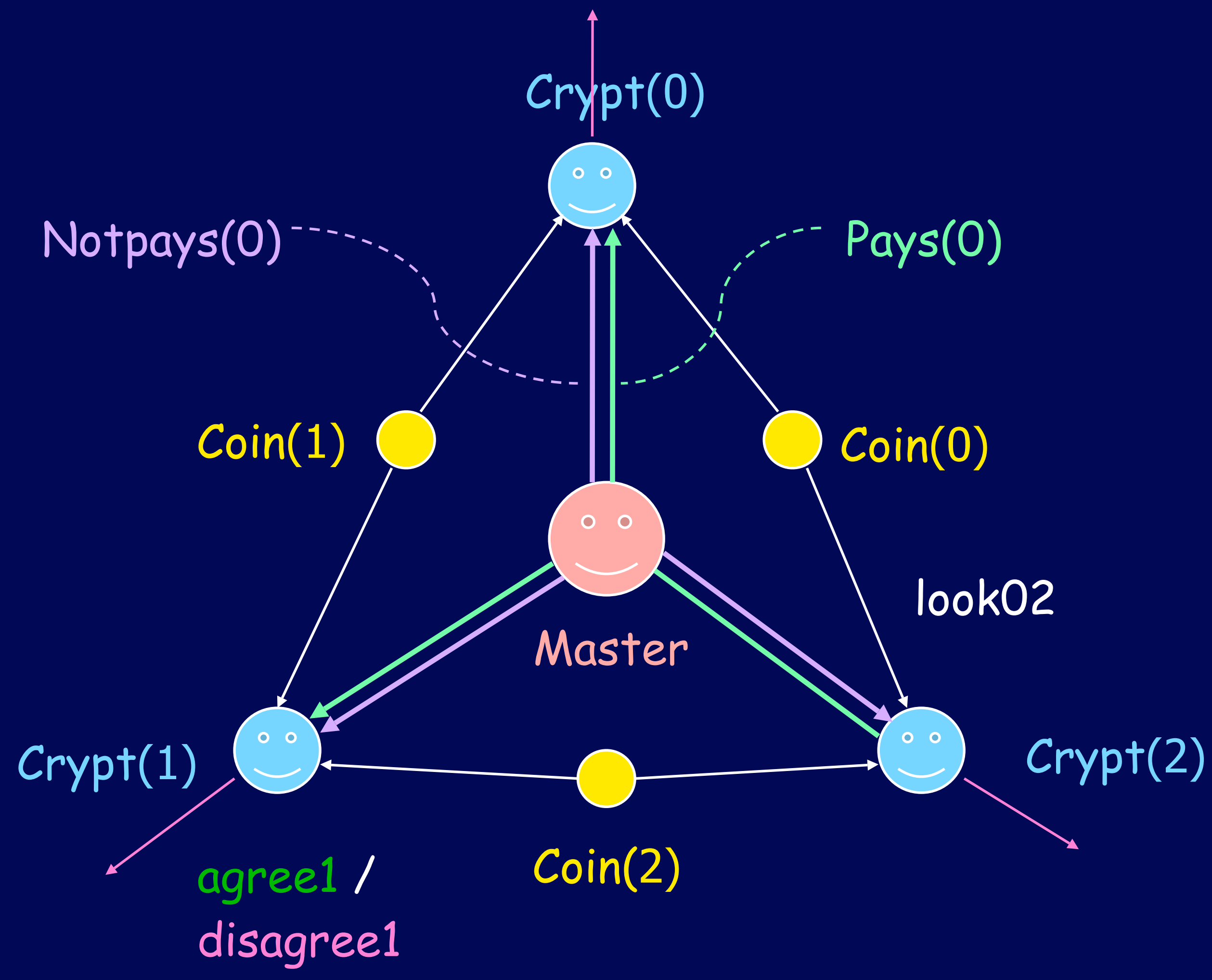

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- The number saying "disagree" is even if and only if M is paying. (This works for arbitrary graphs.)
- If the coins are fair then an external observer and the non-paying cryptographers will not be able to deduce who is paying.
- In fact they will not even be able to increase their probabilistic estimates.

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- This is not detected by the purely nondeterministic approaches.
- In less extreme cases of bias the situation is harder to analyze but clearly some information can leak out.

Coin 12 and Coin 13 are H, Coin 23 is T

M chooses the payer uniformly at random.

	1 pays	2 pays	3 pays
1 says	d	a	a
2 says	d	a	d
3 says	d	d	a

We never see 1 saying d while 2 and 3 say a.

If we say "almost never" then the nondeterministic approach will say this is fine!

Information Theory Summarized

X, Y are random variables and x, y represent possible values.

Entropy: $H(X) = -\sum_x p(x) \log p(x)$
Uncertainty in X.

Conditional Entropy: $H(X|Y) = -\sum_y p(y)[\sum_x p(x|y) \log p(x|y)]$
Uncertainty in X when Y is known.

Mutual Information: $I(X;Y) = H(X) - H(X|Y)$
What Y reveals about X and vice versa.

Channel Capacity

A channel is just a triple

$$(\mathcal{X}, \mathcal{Y}, p(\cdot|\cdot))$$

where $\mathcal{X}$ is the set of input symbols, $\mathcal{Y}$ is the set of output symbols and $p(y|x)$ is the probability of observing y if x is input.

Given an input distribution $p(x)$ we can define random variables X and Y.

The **channel capacity** is given by

$$C = \max_{p(x)} I(X;Y).$$

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- we want the channel capacity to be as low as possible.

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- We are viewing the protocol itself as an abstract channel and thus adopting channel capacity as a quantitative measure of anonymity.

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- It corresponds precisely to strong anonymity, i.e. to the statement that A and O are independent.

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- Palamidessi's group has modelled the DC protocol in the PRISM language and shown how to compute the capacity.
- One can consider the theory of hypothesis testing and analyze attacks made using Bayesian decision rules. We have bounds on the probability of error. This has been greatly extended in a new paper which uses some ideas from convexity theory to give new bounds.

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- This causes an interaction between probability and nondeterministic choices.
- One has capacities rather than measures. Used in economics and in concurrency theory by Gupta, Jagadeesan, Desharnais and Panangaden.

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- He has a 641 page document (in French)!!
- Related work by Mislove, Keimel, Plotkin and Tix.
- The theory is ready to be used.

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- The theory of games and capacities needs to be combined with information theory.
- All kinds of beautiful mathematics: convexity theory, domain theory in addition to traditional information theory.

Existing Collaborations

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- I am designated an Équipe étranger of INRIA Futur and work closely with Catuscia Palamidessi. Her part of the collaboration is supported by INRIA and mine by McGill university and to a small extent by FQRNT.

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- Josée Desharnais and François Laviolette (U. Laval) collaborate with Jean Goubault-Larrecq. Looser ties with me, Vincent Danos and others.