

Number Theory and Cryptography

Alfred Menezes

University of Waterloo

Outline

- ▶ Univ Waterloo research areas:
 - Cryptography, Security and Privacy (CrySP)
 - Symmetric-key cryptography
 - Quantum cryptography
- ▶ Univ Calgary & Waterloo research areas:
 - Number-theoretic cryptography
 - Implementation

CrySP

- ▶ Cryptography, Security, and Privacy Research Group
David R. Cheriton School of Computer Science,
University of Waterloo
- ▶ Faculty: [Ian Goldberg](#), [Urs Hengartner](#), [Doug Stinson](#)
- ▶ Research topics include:
 - Distributed cryptographic protocols (key distribution, broadcast encryption, secret sharing,...)
 - Useful security and privacy technologies (Off-The-Record messaging, Tor, private information retrieval,...)
 - Security and privacy in emerging computing environments (pervasive computing, location-based services, vehicular networks, RFIDs, ...)

Symmetric-key cryptography

- ▶ Faculty: [Guang Gong](#)
- ▶ Research topics include:
 - Constructions of Boolean function (high nonlinearity, algebraic immunity, bent functions,...)
 - Design and analysis of stream ciphers
 - Sequence design for wireless CDMA communications
- ▶ Research ties with Nicolas Sendrier, Claude Carlet, Pascale Charpin, Anne Canteaut

Quantum cryptography

- ▶ Institute for Quantum Computing (IQC), U Waterloo
- ▶ Faculty: Daniel Gottesman, Debbie Leung, Norbert Lütkenhaus, Michele Mosca, John Watrous, Richard Cleve
- ▶ Research topics include:
 - Quantum cryptographic protocols (key distribution, interactive proof systems and zero-knowledge, multi-party computation,...)
 - Experimenting with quantum key distribution (free-space optical link)

Number-theoretic cryptography

- ▶ Calgary faculty: Mark Bauer, Michael Jacobson, Renate Scheidler, Hugh Williams
- ▶ Waterloo faculty: David Jao, Alfred Menezes, Edlyn Teske, Scott Vanstone
- ▶ Ottawa faculty: Isabelle Déchène
- ▶ Toronto faculty: Kumar Murty

Elliptic curve cryptography

- ▶ Discrete-log cryptography using the group of points on an elliptic curve $E : Y^2 = X^3 + aX + b$ defined over a finite field \mathbb{F}_q .
- ▶ The order of the group is $\approx q$.
- ▶ First proposed by Koblitz and Miller in 1985.
- ▶ For a well-chosen elliptic curve, the best attack on the discrete logarithm problem is Pollard's rho method, which takes $\approx q^{1/2}$ steps.
- ▶ Security for ECC scales nicely:
 - 160-bit ECC versus 1024-bit RSA
 - 256-bit ECC versus 3072-bit RSA
 - 384-bit ECC versus 7680-bit RSA
 - 512-bit ECC versus 15360-bit RSA
- ▶ ECC has been widely standardized and deployed

Hyperelliptic curve cryptography

- ▶ Discrete-log cryptography using the divisor class group of a genus- g hyperelliptic curve $C : Y^2 = X^{2g+1} + \dots$ defined over a finite field \mathbb{F}_q .
- ▶ The order of the group is $\approx q^g$.
- ▶ First proposed by Koblitz in 1989.
- ▶ Pollard's rho method for computing discrete logs has running time $\approx q^{g/2}$.
- ▶ Potential advantage over elliptic curve systems:
 - Use a smaller field \mathbb{F}_q for the same level of security.

Hyperelliptic curve discrete logs

- ▶ (1994; Adleman, DeMarrais and Huang): Subexponential-time algorithm for large genus.
- ▶ (2000; Gaudry): $O(q^2)$ for small $g \geq 5$.
- ▶ (2000; Harley): $O(q^{2-2/(g+1)})$ for small $g \geq 4$.
- ▶ (2003; Thériault): $O(q^{2-2/(g+0.5)})$ for small $g \geq 3$.
- ▶ (2005; Diem, Gaudry, Thomé, Thériault): $O(q^{2-2/g})$ for small $g \geq 3$.
- ▶ (2005; Diem) $O(q)$ for genus-3 non-hyperelliptic curves.
- ▶ (2007; Smith) $O(q)$ for 18.75% of genus-3 hyperelliptic curves.
- ▶ Still untouched: $g = 1$ and $g = 2$.

Pairing-based cryptography

- ▶ Use bilinear pairings from low-embedding degree elliptic and hyperelliptic curves to design cryptographic protocols
- ▶ Joux (2000) and Boneh-Franklin (2001)
- ▶ Lots of practical questions remain:
 - Real benefits of identity-based encryption?
 - Other applications (identity-based signatures, aggregate signatures, group signatures,...)
 - Optimal parameters and implementation?
 - Security?

Some ongoing projects

- ▶ Implementation of Weil descent attacks for the DLP in elliptic curves over \mathbb{F}_{2^m} (m composite)
- ▶ Analysis of the effectiveness of Weil descent attack for the DLP in elliptic curves over optimal extension fields (OEFs) \mathbb{F}_{p^m}
- ▶ Rigorous and tighter bounds for computing isogenies between families of random elliptic curves (and designing signature schemes)
- ▶ Security of individual Diffie-Hellman bits over families of isogenous elliptic curves
- ▶ Analysis of methods for generating pairing-friendly elliptic curves

Algebraic number fields

- ▶ Development and implementation of algorithms for computing invariants of number fields and function fields
 - Fast arithmetic in the class group, regulator computation, discrete log computation, ...
- ▶ Cryptographic protocols using real and imaginary quadratic fields
- ▶ Cubic function fields
- ▶ Experiments on a cluster of 152 Dual Intel P4 2.4/2.8 GHz processors

Efficient implementation

- ▶ Faculty: [Vassil Dimitrov](#) (Calgary), [Anwar Hasan](#) (Waterloo), [Alfred Menezes](#) (Waterloo) [Ali Miri](#) (Ottawa)
- ▶ Research topics include:
 - Finite field arithmetic
 - Scalar multiplication
 - Edwards coordinates (Bernstein/Lange) and Theta functions (Gaudry)
 - Pairings
 - Protocol arithmetic
 - Fault-tolerant hardware design
- ▶ Existing research ties with Jean-Claude Bajard, Arnauld Tisserand, Laurent Imbert, Christopher Negre

Side-channel attacks

- ▶ Faculty: [Catherine Gebotys](#)
- ▶ Experimental testbed for electromagnetic (EM) and power-analysis attacks on embedded systems