Coverage and Connectivity in Networks with Directional Sensors (Extended Abstract)

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Abstract. We consider the problem of providing full coverage of a planar region with sensors. Likewise, we consider the connectivity of the sensor network of directional antennas formed by sensors in this region. Suppose that n sensors with coverage angle (also known as beam width) $\alpha(n)$, and reachability radius $r(n)$ are thrown randomly and independently with the uniform distribution in the interior of the unit square. Let $p(n)$ be the probability that a given sensor is active. We prove that if $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2 \alpha(n) \sin(\alpha(n)/4)}\right)$ then the probability the sensors provide full coverage of the unit square is at least $1 - n^{-O(1)}$ Likewise, we consider the connectivity of the resulting sensor network. We show that if $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2 \alpha(n) \sin^2(\alpha(n)/4)}\right)$ then the probability that a connected subnetwork of sensors provides full coverage is at least $1 - n^{-O(1)}$.

1 Introduction

Sensors are low-power communication and sensing devices that can be embedded in the physical world and spread throughout our environment (see Kahn et al [7], Sohrabi et al [14], Estrin et al [5], Hollar [6]). Large scale sensor networks are formed by sensors that can be automatically configured after being dropped over a given region. Such networks are deployed in order to handle various services over remote and/or sensitive regions, including monitoring, structural stability of buildings, quality of air, etc, and form the basis of numerous military and civilian applications. It is expected that the cost of such devices will drop significantly in the near future (see the study in [12], Agre et al [1], and Warneke et al [15]). Sensor nodes enable autonomy, self-configurability, and self-awareness. Sensor networks are built from such sensor nodes that create spontaneously an adhoc network. They can assemble themselves automatically, adapt dynamically to failures, manage movement, react to changes in network requirements, etc.

In this paper we are concerned with sensor networks formed in an adhoc manner from unreliable sensors dropped, e.g., from an airplane, on a given planar region, e.g., a unit square region. A sensor is a device that can sense events within a certain reachability radius if they are within a given coverage angle. The purpose of such a sensor network is to report events, e.g., to a certain base station located within the unit square region. To ensure that all events within this region are reported, it is important that coverage of the entire region is guaranteed as well as that the resulting sensor network is connected. There have been several studies on routing and location identification in adhoc networks in which sensors use only constant memory and local information. Several such interesting studies include Bose et al [2], Braginsky et al [3], Doherty et al [4], Kranakis et al [8], Kuhn et al [9], Meguerdichian et al [10], Ye et al [16]).

The first study on the coverage and connectivity problem we consider in this paper appeared in Shakkottai et al [13] for a restricted model in which the sensors are assumed to occupy the vertices of a square grid the size of a unit square, the reachability radius of the sensors is r, and the angular coverage of the sensors is 360 degrees. We follow closely the model of Shakkottai et al [13]. To accomodate faults we assume there is a probability p that a given sensor is active. However our sensor model is more general. First, unlike Shakkottai et al [13] where sensors may occupy only the vertices of a square unit grid, our sensors may occupy any position inside the unit square randomly and independently with uniform distribution. In addition to the position of the sensors, their orientation is also random and may occupy any angular position in the range from 0 to 360 degrees. Second, our sensors have limited angular visibility. Thus our sensors can be thought of as flood-lights with a given coverage angle of size α that can see every point in the part of the region subtended by their visibility angle, and reachability radius r. Equivalently, we can think of the sensors as antennas that can communicate provided they are within range and angle of each other.

Consider n sensors in the interior of a unit square. In order to study the limits of the coverage and connectivity problem we find it convenient to parametrize the characteristics of the sensors as a function of n. More specifically, let $\alpha := \alpha(n)$ be the coverage angle, and $r := r(n)$ the reachability radius of the sensor, respectively. Let $p := p(n)$ be the probability that a given sensor is active, in which case $1 - p(n)$ is the probability that the sensor is inactive.

We are interested in the problem of using sensors in order to cover every point of the given region, which for simplicity is assumed to be a unit square in the plane over which sensors are dropped, say from an airplane. More specifically we can formulate the following problem.

Problem 1. n sensors (i.e., directional antennas) are dropped randomly and independently with the uniform distribution (i.e., in position and rotation) over the interior of a unit square. How do we achieve full coverage of the region delimited by the unit square as well as connectivity of the resulting network? What is the probability that n sensors cover the interior of the unit square?

We want to ensure that the network of directional sensors is connected, i.e., there is a communication path between any pair of sensors in the network. Note that since the placement and orientation of the sensors is random there is a reasonable probability that some sensors will be unable to communicate with any other sensors. Rather than achieve full connectivity we require that a connected subnetwork exists that achieves full coverage. In particular, we can formulate the following problem.

Problem 2. n sensors (i.e., directional antennas) are dropped randomly and independently with the uniform distribution (i.e., in position and rotation) over the interior of a unit square. What relation between p, α, n must exist in order to ensure the existence of a connected subnetwork of sensors that provides full coverage?

Our approach is sufficiently general to solve the coverage problem for a unit square region when the sensors can be placed only on its perimeter. In this case we assume that the reachability radius of the sensors is equal to 1. More specifically we consider the following problem.

Problem 3. n sensors (i.e., directional antennas) of *reachability radius* 1 are dropped randomly and independently with the uniform distribution (i.e., in position and rotation) over the perimeter of a unit square. How do we achieve full coverage of the region delimited by the unit square,? What is the probability that n sensors cover the interior of the unit square?

1.1 Results of the paper

In this paper we prove the following results. We show that if $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2\alpha(n)\sin(\alpha(n)/4)}\right)$ then the probability that the sensors provide full coverage of the unit square is at least $1 - n^{-O(1)}$. Conversely, if the sensors provide full coverage of the unit square with probability at least $1 - 1/n$, then $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2)}{nr(n)^2\alpha(n)}\right)$. Likewise, we consider the probability that the resulting sensor network is connected. We show that if $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2\alpha(n)\sin^2(\alpha(n)/4)}\right)$. then the probability that a connected subnetwork of sensors provides full coverage is at least $1 - n^{-O(1)}$.

2 The Network of Sensors

Consider two sensors at A, B. If the coverage angle is 2π and A is reachable from B then also B is reachable from A. Clearly, this makes the sensor network an undirected graph. The situation is different when the coverage angle α of the sensors is less than 2π. Two sensors at A and B may well be within reachability range of each other but either A or B or both may not be within the coverage angle of the other sensor. Clearly, this makes such a sensor network a directed graph.

2.1 Covering a circle with a sensor

Consider a sensor at A. We would like that a circle of radius R fits inside the coverage range of A (see Figure 1) and is visible from A with an angle of size $\alpha/4$. Let d be the distance of the sensor

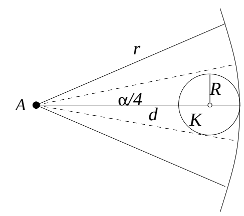

Fig. 1. A circle of radius R within the coverage range of the sensor at A.

from the center K of this circle. Since the reachability radius of the sensor is r, the circle at K is entirely visible from the sensor at A if $d + R \leq r$. Since $d = R/\sin(\alpha/4)$ the above inequality is equivalent to $\frac{R}{\sin(\alpha/4)} + R \leq r$. It is now easy to see that the following result is true.

Lemma 1. *A circle of radius R may lie within the coverage range of sensor if and only if*

$$R \leq \frac{\sin(\alpha/4)}{1 + \sin(\alpha/4)} \cdot r. \tag{1}$$

Moreover, the probability that a given sensor at distance d from a given circle of radius R such that $d + R \leq r$ is active and covers the circle, is $\Theta(p\alpha)$. ■

2.2 Chernoff bounds

Since the sensors are dropped from an airplane randomly and independently with uniform distribution we want to ensure that a given subregion of the unit square contains enough sensors with high probability. This can be done using Chernoff bounds.

Consider a given circle C of radius r that lies inside the unit square. We are interested in estimating the expected number of sensors that drop inside this circle. The random variables we are concerned are sums of independent Bernoulli trials. Let $X_1, X_2, \ldots, X_n$ be independent Bernoulli trials such that $X_i = 1$ if the i-th sensor falls inside the circle C and is 0, otherwise. Since the sensors are dropped in the unit square randomly and independently with the uniform distribution we observe that $\Pr[X_i = 1] = \pi r^2$. If $X = X_1 + X_2 + \cdots + X_n$ is the sum of these n random variables then $\mu := E[X] = n\pi r^2$, and using Chernoff bounds (see Motwani et al [11]) we derive that for any $\delta > 0$, $\Pr[X > (1+\delta)\mu] < \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^\mu$, and for any $0 < \delta \leq 1$, $\Pr[X < (1-\delta)\mu] < e^{-\mu\delta^2/2}$. It follows that with high probability the expected number of sensors that drop within any given circle C is proportional to the area of the circle.

3 Achieving Coverage

We are interested in specifying conditions on the three main parameters $\alpha := \alpha(n), r := r(n), p := p(n)$ so that the active sensors in the unit square provide full coverage. Using Lemma 1 we are interested in finding sensors that with high probability cover a given set of circles of radius $R = \frac{\sin(\alpha/4)}{1+\sin(\alpha/4)} \cdot \Theta(r)$.

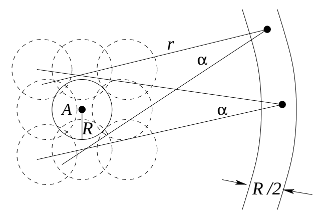

Fig. 2. Covering a given circle at A from a sensor within the strip, with high probability.

As depicted in Figure 2 any sensor within a strip of thickness $R/2$ at distance $\Omega(r)$ can potentially cover a circle of radius R. The probaility that a given sensor within this strip will cover fully such a circle is at least $\Omega(p\alpha)$. The expected number of sensors within the strip at distance $\Omega(r)$ from A is at least $\Theta(nrR)$. It follows that up to a constant

$$\begin{aligned}
&\Pr[\text{An active sensor in the strip covers the circle at } A] \\
&= 1 - \Pr[\text{No active sensor in the strip covers the circle at } A] \\
&\geq 1 - \prod_{i=1}^{\Theta(nrR)} (1 - \Omega(p\alpha)) \geq 1 - e^{-\Theta(np\alpha rR)}.
\end{aligned}$$

It is now easy to see that in order to ensure that $1 - e^{-\Theta(np\alpha rR)} \geq 1 - n^{-O(1)}$ it is enough to assume that $p \geq \Omega\left(\frac{\log n}{n\alpha rR}\right)$.

To obtain coverage of the whole unit square we decompose the given region into circular overlapping subregions $C_1, C_2, \ldots, C_N$ each of radius R, where $N = \frac{1}{R^2} = \frac{1}{\Theta(r^2 \sin^2(\alpha/4)/(1+\sin(\alpha/4))^2)} \approx \Theta\left(\frac{1}{r^2 \sin^2(\alpha/4)}\right)$. (see Figure 3). This overlap is necessary in order to guarantee that if each such

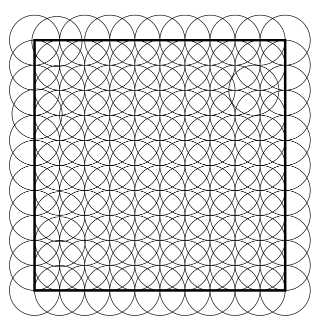

Fig. 3. Decomposition of the unit square region (square with thick perimeter) with smaller overlapping circles of equal radius.

circle is fully covered by a sensor. The whole unit square will then be also covered by the sensor network. Each circular subregion can be covered by a sensor with high probability. Let E_i be the event that C_i can be covered by a sensor. It follows that

$$\begin{aligned}&\Pr[\text{Sensors provide full coverage}] \geq \Pr[\text{Each } C_i \text{ is covered by a sensor}]\\ &= \Pr\left[\bigcap_{i=1}^{N} E_i\right] = 1 - \Pr\left[\bigcup_{i=1}^{N} \overline{E_i}\right] \geq 1 - \sum_{i=1}^{N} \Pr[\overline{E_i}] \geq 1 - \sum_{i=1}^{N} e^{-\Theta(np\alpha rR)}\\ &\geq 1 - Ne^{-\Theta(np\alpha rR)} \geq 1 - \frac{1}{\Theta(r^2 \sin^2(\alpha/4))} e^{-\Theta(np\alpha rR)}.\end{aligned}$$

It is now easy to see that in order to ensure that $1 - \frac{1}{\Theta(r^2\sin^2(\alpha/4))} e^{-\Theta(np\alpha rR)} \geq 1 - n^{-O(1)}$ it is enough to assume that $p \geq \Omega\left(\frac{\log(n/r^2 \sin^2(\alpha/4))}{nr^2 \alpha \sin(\alpha/4)}\right)$. We have proved the following theorem.

Theorem 1. *Suppose that n sensors with coverage angle $\alpha(n)$ and reachability radius $r(n)$ are thrown randomly and independently in the interior of a unit square. If the probability $p(n)$ that a given sensor is active satisfies $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2 \alpha(n) \sin(\alpha(n)/4)}\right)$ then*

$$\Pr[\textit{Sensors provide full coverage of the unit square}] \geq 1 - n^{-O(1)}.$$

■

As a special case we also obtain the result of Shakkottai et al [13] for $\alpha = 2\pi$. If $p(n) \geq \Omega\left(\frac{\log n}{nr(n)^2}\right)$ then Pr[Sensors provide full coverage of the unit square] $\geq 1 - n^{-O(1)}$.

It is also interesting to look at the reverse problem: if coverage is assured with high probability asymptotically in n, what is an upper bound on the probability $p(n)$ that a sensor is active?

Partition the square into N pairwise disjoint circles each of radius r as depicted in Figure 4. If any given of these circles has no active sensor (say the circle centered at K) then K cannot be covered by any sensor lying in a neighboring circle. Consider the event E: the center of a circle is covered by an active sensor. It is clear that $p\frac{\alpha}{2} \leq \Pr[E] \leq p\alpha$. There are approximataly $N = \Theta(1/r^2)$ such circles. It follows that

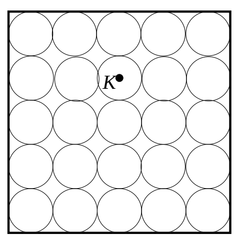

Fig. 4. Partitioning of the unit square region (square with thick perimeter) with smaller circles of equal radius.

$$\begin{aligned}&\Pr[\text{Unit square is covered}] \leq \Pr[\text{Centers of all the circles are covered}]\\ &= \prod_{i=1}^{N} \Pr[E] = (1 - \Pr[\bar{E}])^N \leq (1 - (1 - \Theta(p\alpha))^{\Theta(nr^2)})^N\\ &\leq \exp\left(-(1 - \Theta(p\alpha))^{\Theta(nr^2)} N\right) = \exp\left(-(1 - \Theta(p\alpha))^{\Theta(nr^2)} / \Theta(r^2)\right)\end{aligned}$$

It follows from the inequalities above that if Pr[Unit square is covered] $\geq 1 - 1/n$ then also $\exp\left(-(1 - \Theta(p\alpha))^{\Theta(nr^2)}/\Theta(r^2)\right) \geq 1 - 1/n$, which in turn implies that $\frac{(1-\Theta(p\alpha))^{nr^2}}{r^2} \leq \log\left(\frac{n}{n-1}\right) \leq \frac{1}{n-1}$. It follows that $p \geq \Omega\left(\frac{\log(n/r^2)}{nr^2\alpha}\right)$. We have proved the following theorem.

Theorem 2. *Suppose that n sensors with coverage angle $\alpha(n)$ and reachability radius $r(n)$ are thrown randomly and independently in the interior of a unit square. If*

$$\Pr[\textit{Sensors provide full coverage of the unit square}] \geq 1 - 1/n$$

as $n \to \infty$, then the probability $p(n)$ that a given sensor is active satisfies

$$p(n) \geq \Omega\left(\frac{\log(n/r(n)^2)}{nr(n)^2\alpha(n)}\right).$$

■

3.1 Achieving coverage only from the perimeter

Our approach in this section is sufficiently general to solve the coverage problem for a unit square region when the sensors can be placed only on its perimeter. In this case we assume that the reachability radius of the sensors is equal to 1. We mention without proof the following result whose proof is similar to the proofs of Theorems 1 and 2 above.

Theorem 3. *Suppose that n sensors with coverage angle $\alpha(n)$ and reachability radius 1 are thrown randomly and independently on the perimeter of a unit square. If the probability $p(n)$ that a given sensor is active satisfies $p(n) \geq \Omega\left(\frac{\log(n/\sin^2(\alpha(n)/4))}{n\alpha(n)\sin(\alpha(n)/4)}\right)$ then*

$$\Pr[\textit{Sensors provide full coverage of the unit square}] \geq 1 - n^{-O(1)}. \quad (2)$$

Similarly, we can prove that if Inequality 2 is valid then $p(n) \geq \Omega\left(\frac{\log n}{n\alpha(n)}\right)$. ■

4 Achieving Coverage and Connectivity

Connectivity is not always assured in a network of directional antennas. For example, it can happen that a sensor lying in the convex hull formed by the sensors is such that its angle of coverage points outside of the convex hull. Thus it cannot connect to any other sensor. We resolve this problem by showing in the sequel that there exists a connected subgraph of the graph of sensors that provides complete coverage of the unit square. We also provide an efficient routing algorithm in this setting.

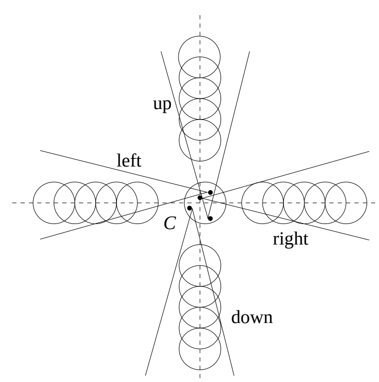

Fig. 5. For each circle we can find with high probability four sensors within the circle whose directional antennas fully cover a circle in the *up, down, left, right* directions, respectively.

Condider the configuration decomposing the unit square into circles of radius $R = \frac{\sin(\alpha/4)}{1+\sin(\alpha/4)} \cdot \Theta(r)$ and depicted in Figure 3. Recall that the coverage range of each sensor is r and consider the radius R. For each circle in this configuration we can find with high probability four active sensors within the circle whose directional antennas fully cover a circle in the *up, down, left, right* directions, respectively (see Figure 5). For example, for the two circles at distance $\leq r - 2R$ depicted in Figure 6 we can find with high probability a sensor in C to cover the circle C'.

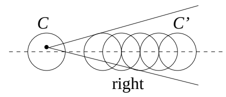

Fig. 6. Finding a sensor in C in order to cover the circle C'.

$$\begin{aligned}
&\Pr[\text{An active sensor in } C \text{ covers the circle } C'] \\
&= 1 - \Pr[\text{no active sensor in } C \text{ covers the circle } C'] \\
&\geq 1 - \prod_{i=1}^{\Theta(nR^2)} (1 - \Theta(p\alpha)) \geq 1 - e^{-\Theta(np\alpha R^2)}.
\end{aligned}$$

It is now easy to see that in order to ensure that $1 - e^{-\Theta(np\alpha r R^2)} \geq 1 - n^{-O(1)}$ it is enough to assume that $p \geq \Omega\left(\frac{\log n}{n\alpha R^2}\right) = \Omega\left(\frac{\log n}{nr^2\alpha \sin^2(\alpha/4)}\right)$.

The same idea as before can now be used to achieve full coverage of the unit square by a connected subgraph of the resulting network of sensors. The circles must be such that there is sufficient overlap in order to guarantee connectivity. To obtain coverage of the whole unit square we partition the given region into circular subregions $C_1, C_2, \ldots, C_N$ each of radius R, where $N = \frac{1}{R^2} = \frac{1}{\Theta(r^2 \sin^2(\alpha/4)/(1+\sin(\alpha/4))^2)} = \Theta\left(\frac{1}{r^2 \sin^2(\alpha/4)}\right)$. (see Figure 3). Each circular subregion can be covered by a sensor with high probability. Let E_i be the event that C_i can be covered by a sensor. It follows that

$$\begin{aligned}
&\Pr[\text{Sensors provide full coverage}] \geq \Pr[\text{Each } C_i \text{ is covered by a sensor}] \\
&= \Pr\left[\bigcap_{i=1}^{N} E_i\right] = 1 - \Pr\left[\bigcup_{i=1}^{N} \overline{E_i}\right] \geq 1 - \sum_{i=1}^{N} \Pr[\overline{E_i}] \geq 1 - \sum_{i=1}^{N} e^{-\Theta(np\alpha R^2)} \\
&\geq 1 - Ne^{-\Theta(np\alpha R^2)} \geq 1 - \frac{1}{\Theta(r^2 \sin^2(\alpha/4))} e^{-\Theta(np\alpha R^2)}.
\end{aligned}$$

It is now easy to see that in order to ensure that $1 - \frac{1}{\Theta(r^2 \sin^2(\alpha/4))} e^{-\Theta(np\alpha R^2)} \geq 1 - n^{-O(1)}$ it is enough to assume that $p \geq \Omega\left(\frac{\log(n/r^2 \sin^2(\alpha/4))}{nr^2\alpha \sin^2(\alpha/4)}\right)$.

It remains to prove that a connected subgraph of the sensor network provides full coverage of the unit square. Consider the set S of active sensors that fully cover at least one of the circles of radius R depicted in Figure 3. The previous argument shows that this set of sensors indeed provides full coverage of the unit square. It remains to prove that it is also connected. Let s, t be two arbitrary active sensors in S. It is enough to show that there is a path from s to t with high probability asymptotically in n. By definition of S there is a circle, say C, of radius R in Figure 3 which is fully covered by the sensor s. Let C' be a circle of radius again in Figure 3 in whose interior the sensor t belongs. It was shown before (see also Figure 5) that we can find a path from circle C to circle C' like in the Manhattan routing on a two dimensional mesh in order to reach every node at C' (and hence also the target node t. Summing up we have proved the following theorem.

Theorem 4. *Suppose that n sensors with coverage angle $\alpha(n)$ and coverage radius $r(n)$ are thrown randomly and independently in the interior of the unit circle. If $p(n) \geq \Omega\left(\frac{\log(n/r(n)^2 \sin^2(\alpha(n)/4))}{nr(n)^2\alpha(n)\sin^2(\alpha(n)/4)}\right)$ then* $\Pr[\exists$ *a connected subnetwork of sensors which provides full coverage*$] \geq 1 - n^{-O(1)}$. ■

As a special case we also obtain the result that for $\alpha = 2\pi$, if $p(n) \geq \Omega\left(\frac{\log n}{nr(n)^2}\right)$ then $\Pr[$A connected subnetwork of sensors provides full coverage$] \geq 1 - n^{-O(1)}$.

5 Conclusion

We have considered the problem of coverage and connectivity of a sensor network established over a unit square. We established conditions on the probability that a given sensor is active in order to guarantee that a subnetwork of the resulting sensor network is connected and provides full coverage of the unit square. We also note that a similar approach will work for coverage and connectivity of sensor networks in more general planar regions.

References

1. J. Agre and L. Clare, An integrated architecture for cooperative sensing networks, IEEE Computer, vol. 33, no. 5, May 2000, 106-108.
2. P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia, Routing with guaranteed delivery in ad hoc wireless networks. Wireless Networks, 7(6):609-616, 2001.
3. D. Braginsky and D. Estrin, Rumor routing algorithm for sensor networks, 2001. Available at http://lecs.cs.ucla.edu/ estrin/.
4. L. Doherty, L. E. Ghaoui, and K. S. J. Pister, Convex position esti- mation in wireless sensor networks, in Proceedings of IEEE Infocom, (Anchorage, AK), April 2001.
5. D. Estrin, R. Govindan, J. Heidemann and S. Kumar: Next Century Challenges: Scalable Coordination in Sensor Networks. In Proc. 5th ACM/IEEE International Conference on Mobile Computing Â? MOBICOM'1999.
6. S.E.A. Hollar: COTS Dust. Msc. Thesis in Engineering-Mechanical Engineering, University of California, Berkeley, USA, 2000.
7. J. M. Kahn, R. H. Katz, and K. S. J. Pister, Mobile networking for smart dust, in Proceedings of MobiCom 99, (Seattle, WA), August 1999.
8. E. Kranakis, H. Singh, and J. Urrutia, Compass Routing in Geometric Graphs, in proceedings of 11th Canadian Conference on Computational Geometry, CCCG-99, pages 51-54, Vancouver Aug. 15-18, 1999.
9. F. Kuhn, R. Wattenhofer, Y. Zhang, A. Zollinger. Geometric Ad-Hoc Routing: Of Theory and Practice, Proceedings of the 22nd ACM Symposium on the Principles of Distributed Computing (PODC), Boston, Massachusetts, USA, July 2003.
10. S. Meguerdichian, F. Koushanfar, M. Potkonjak, and M. Srivastava, Coverage problems in wireless ad-hoc sensor networks, in Proceedings of IEEE Infocom, (Anchorage, AK), 2001.
11. R. Motwani, P. Raghavan, Randomized Algorithms, Cambridge University Press, 1995.
12. National Research Council, Embedded, Everywhere: A Research Agenda for Systems of Embedded Computers, Committee on Networked Systems of Embedded Computers, for the Computer Science and Telecommunications Board, Division on Engineering and Physical Sciences, Washington, DC, 2001.
13. S. Shakkottai, R. Srikant, N. Shroff, Unreliable Sensor Grids: Coverage, Connectivity and Diameter, In proceedings of IEEE INFOCOM, 2003, held in San Francisco, March 30 to April 2, 2003.
14. K. Sohrabi, J. Gao, V. Ailawadhi, and G. Pottie, Protocols for self- organization of a wireless sensor network, IEEE Personal Communica- tions, vol. 7, pp. 16-27, October 2000.
15. B. Warneke, M. Last, B. Leibowitz, and K. Pister, SmartDust: communicating with a cubic- millimeter computer, IEEE Computer, vol. 34, no. 1, January 2001, 44-51.
16. W. Ye, J. Heidemann, and D. Estrin, An energy-efficient MAC protocol for wireless sensor networks, in Proceedings of IEEE Infocom, (New York, NY), June 2002.