Computing with Directional Antennae in WSNs

By
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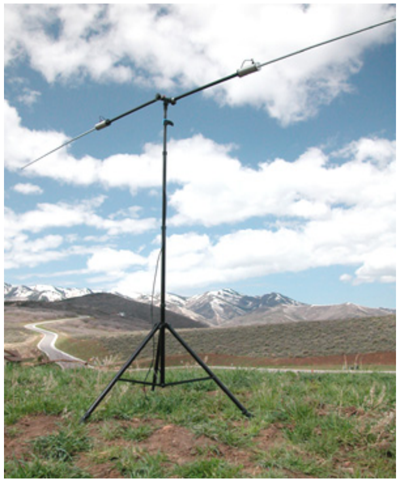
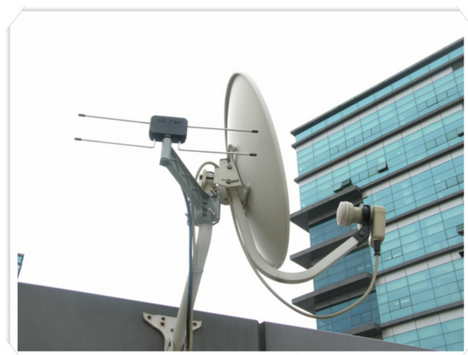
Outline of Tutorial

- Motivation
- Antennae Basics
- Network Connectivity
  - One Antenna
  - Multiple Antennae
- Neighbor Discovery
- Wormhole Attacks
- Coverage and Routing

Motivation

Antennae Everywhere...





...Beginning

- Two antennae meet on a roof, fall in love, and get married. The service wasn't all that great, but the reception was wonderful!

Beginning...

- Two antennae meet on a roof, fall in love, and get married. The service wasn't all that great, but the reception was wonderful!
- They were on the same wavelength but what did they gain? Instead of rice, Marconi was thrown, right? And in nine months or so there will be a little half wave dipole as long as the impedance was near perfect...

Comparison of Omnidirectional & Directional Antennae

	Omnidirectional	Directional
Energy	More	Less
Throughput	More	Less
Collisions	More	Less
Interference	More	Less
Connectivity	Stable	Intermittent
Discovery	Easy	Difficult
Coverage	Stable	Intermittent
Routing $SF^{(*)}$	Less	More
Security	Less	More

(*) $SF =$ Stretch Factor

Why Directional Antennae

- Transmitting in particular directions results in a higher degree of spatial reuse of the shared medium.
- Directional transmission uses energy more efficiently.
- The transmission range of directional antennas is usually larger than that of omnidirectional antennas, which can reduce hops for routing and make originally unconnected devices connected.
- Directional antennas can increase spatial reuse and reduce packet collisions and negative effects such as deafness.
- Routing protocols using directional antennas can outperform omnidirectional routing protocols.

Simple Estimate: Energy Consumption of an Antenna

- An ominidirectional antenna with range r consumes energy proportional to $\pi \cdot r^2$.
- A directional antennae with angular spread α and range R consumes energy proportional to $\frac{\alpha}{2} \cdot R^2$.
- Given energy E
  - an ominidirectional antenna can reach distance $\sqrt{E/\pi}$, and
  - a directional antenna can reach distance $\sqrt{2E/\alpha}$
- Hence the smaller the angular spread the further you can reach.

Energy Consumption of a System of Antennae

- For a network of n omnidirectional sensors having range r_i, for $i = 1, 2, \ldots, n$ respectively, the total energy consumed will be

$$\sum_{i=1}^{n} \pi \cdot r_i^2 .$$

- For a network of n directional sensors having angular spread α_i and range R_i, for $i = 1, 2, \ldots, n$ respectively, the total energy consumed will be

$$\sum_{i=1}^{n} \frac{\alpha_i}{2} \cdot R_i^2 .$$

- Given that by shortening the angular spread you can increase the range of a directional sensor the savings can be significant.

A Deeper Question

- There is a deeper question here that is worth studying:

  Give orientation algorithms that attain optimal energy/interference tradeoffs for a given set of sensors.

- Observe that in the resulting sensor network, the coverage areas of (directional) antennae overlap.

  To what extent can we analyze objectively the network performance?

Antennae Basics

Outline

- Essentials
- Antennae Examples
- Radiation Patterns
- Idealized Models
  - 2D
  - 3D

Ultimate Goal:
Understanding antennae basics will help us build the right models and answer the right questions!

Essentials

What is an antenna?

- An antenna is a converter!
  - **Transmission:** converts radio-frequency electric current to electromagnetic waves, radiated into space.
  - **Reception:** collects electromagnetic energy from space and converts it to electric energy.
- In two-way communications, the same antenna can be used for transmission and reception

Essental Characteristic: Wavelength

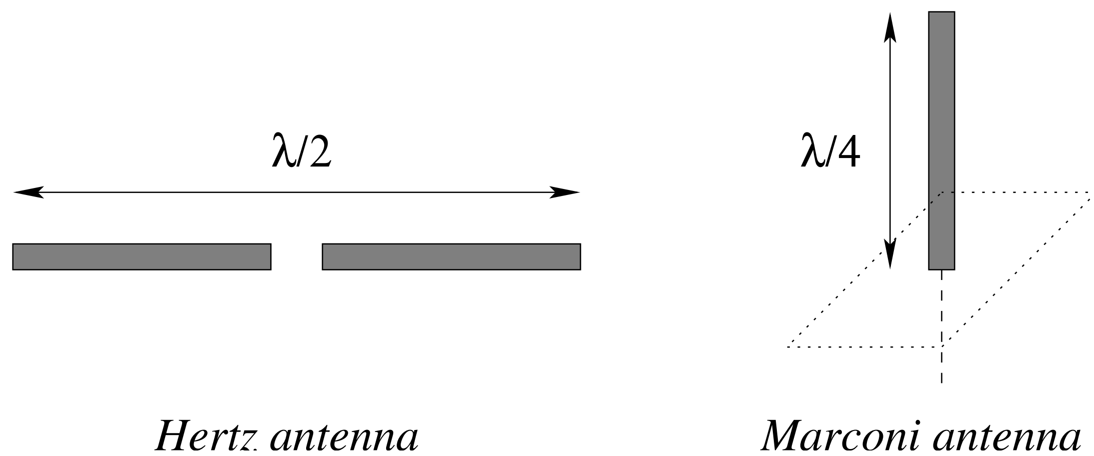
- **Wavelength:** is the distance, in free space, traveled during one complete cycle of a wave
- **Wave velocity:** Speed of light
- Therefore wavelength is given by

$$\lambda_{meters} = \frac{300 \times 10^6 \text{ meters/sec}}{\text{frequency } f \text{ in Hertz}}$$

- **Example:** You have a tooth filling that is 5 mm (= 0.005 m) long acting as a radio antenna (therefore it is equal in length to one-half the wavelength). What frequency do you receive?

Examples

- **Dipoles** are the simplest type of antenna



The **Hertz** (or half-wave) dipole consists of two straight collinear conductors of equal length separated by a small feeding gap. Length of antenna is half of the signal that can be transmitted most efficiently.

- The **Marconi** (or quarter-wave) is the type used for portable radios.

Types of Antennae

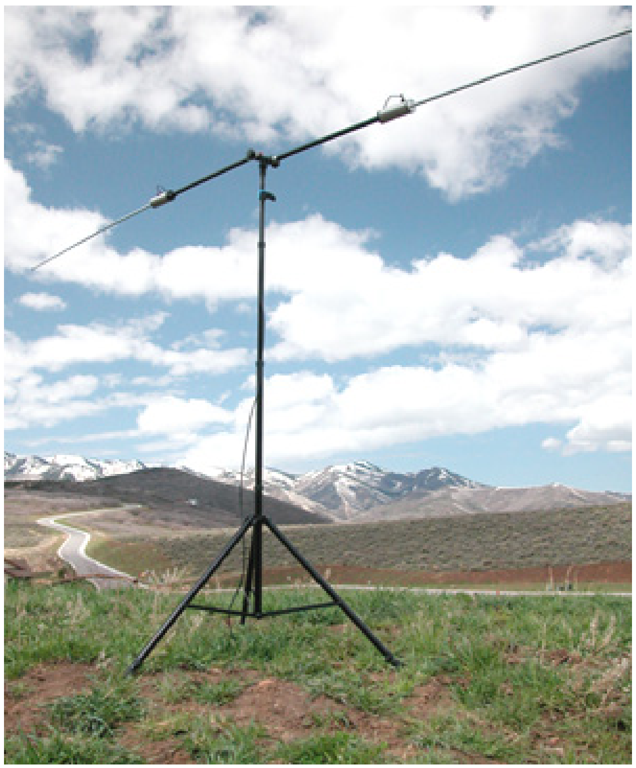
- **Isotropic**
  - Idealized, point in space
  - Radiates power equally in all directions
  - True isotropic radiation does not exist in practice!
- **Dipole**
  - Half-wave dipole (Hertz antenna)
- **Omnidirectional**
  - 2D isotropic Vertical, 1/4–wave monopole, Marconi, Groundplane
- **Directional**
  - Yagi
  - Parabolic Reflective

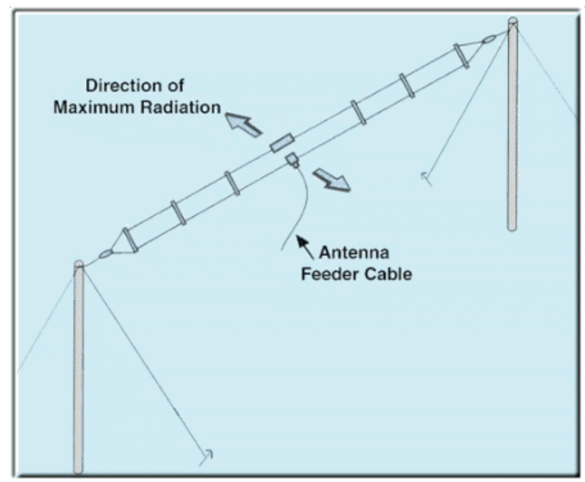
Antennae Examples

Dipole (1/2)

- Emission is maximal in the plane perpendicular to the dipole and zero in the direction of wires which is the direction of the current.

Dipole (2/2)

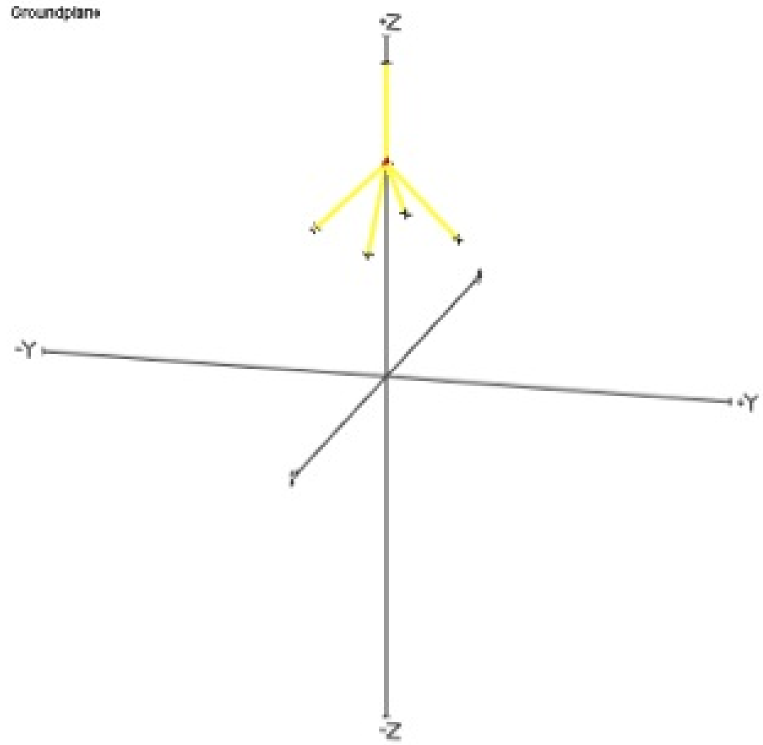




Groundplane (1/2)

- Main element of a ground-plane antenna is almost always oriented vertically.
- This results in transmission of, and optimum response to, vertically polarized wireless signals.
- When the base of the antenna is placed at least 1/4 wavelength above the ground or other conducting surface, the radials behave as a near-perfect ground system for an electromagnetic field, and the antenna is highly efficient.

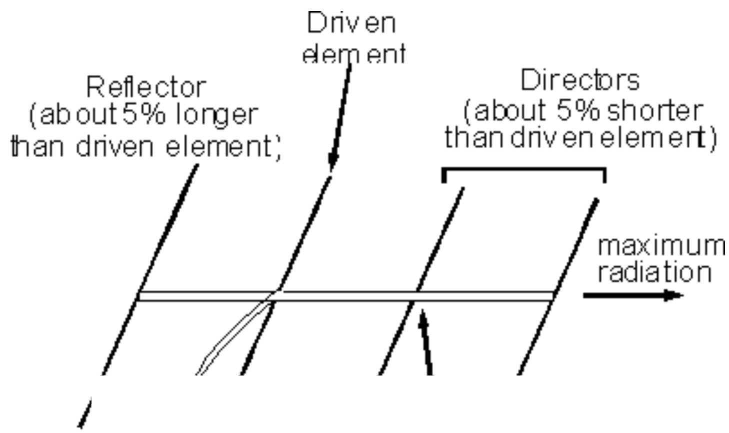
Groundplane (2/2)

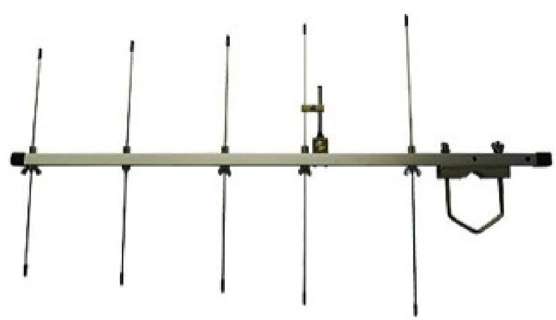


Yagi (1/2)

- Is a directional antenna consisting of a driven element (typically a dipole or folded dipole) and additional elements (usually a so-called reflector and one or more directors).
- It is directional along the axis perpendicular to the dipole in the plane of the elements, from the reflector toward the driven element and the director(s).

Yagi (2/2)

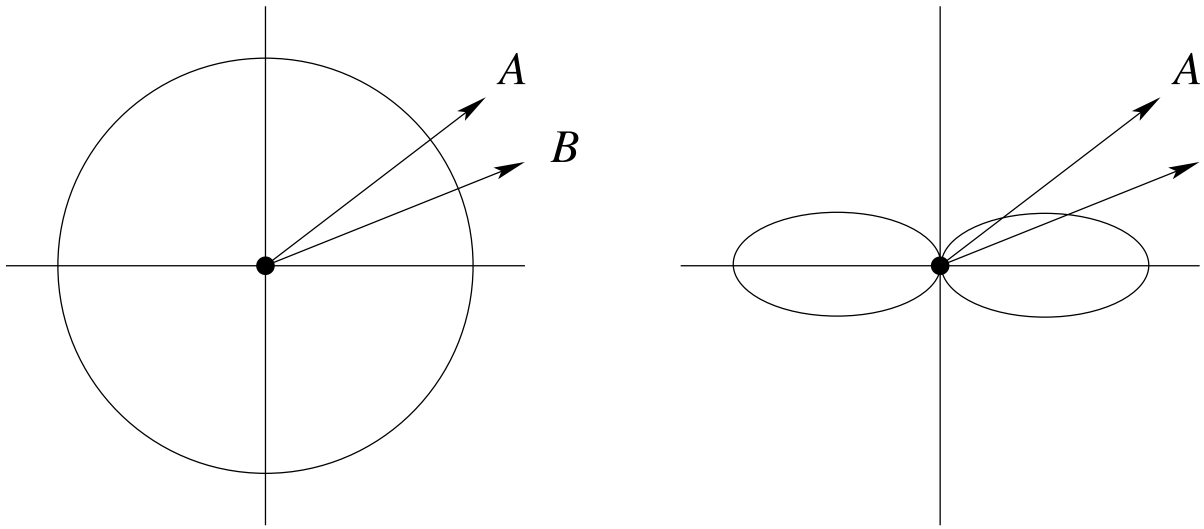

Driven
element
Reflector
(about 5% longer
than driven element)
Directors
(about 5% shorter
than driven element)
maximum
radiation



Radiation Patterns

Radiation Patterns of Antennae

- Antennae transmit radiation according to specific patterns:

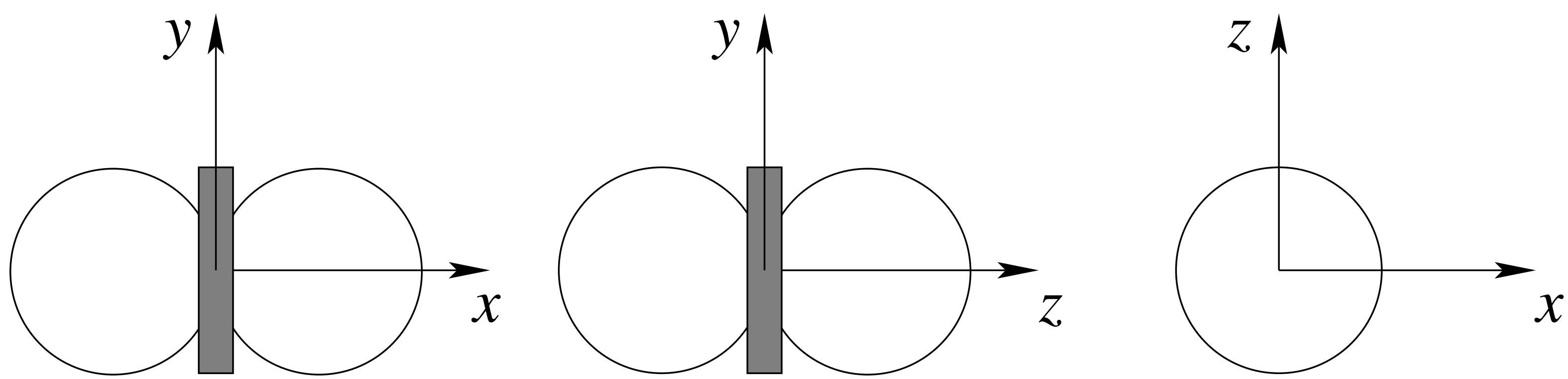


Omnidirectional *Directional*

- Omnidirectional are *isotropic* in the sense that same power (radiation) is transmitted in all directions.
- Directional antennas have preferred patterns (like an ellipse): E.g., in the picture above B receives more power than A.

Dipole Radiation Patterns

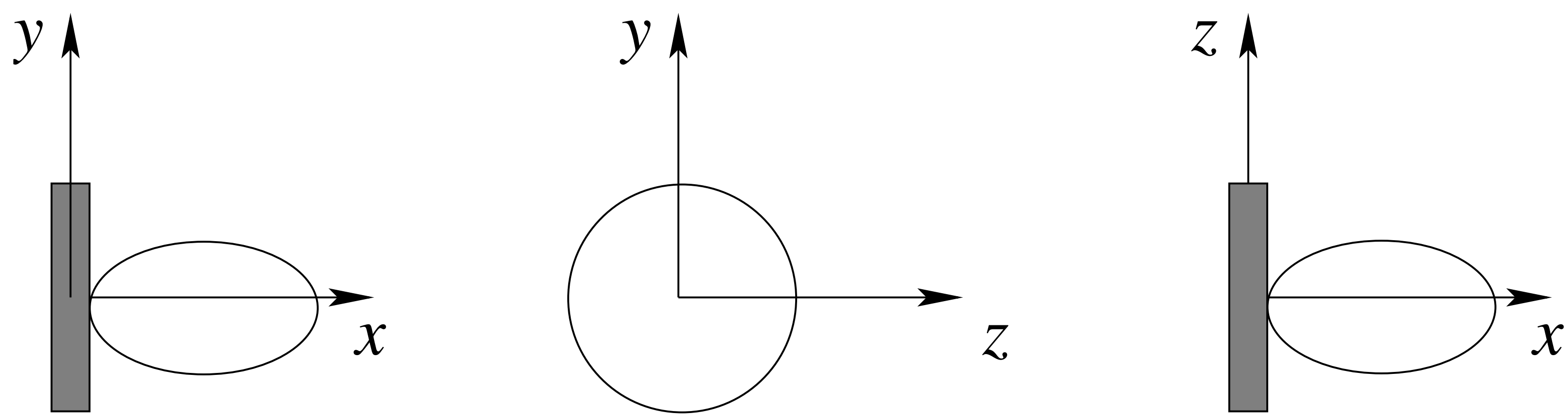
- The half-wave dipole has an omnidirectional pattern only in one planar dimension and a figure eight in the other two.



- For example, the side view along the xy- and zy-plane are figure eight, while in the zx-plane it is uniform (or omnidirectional).

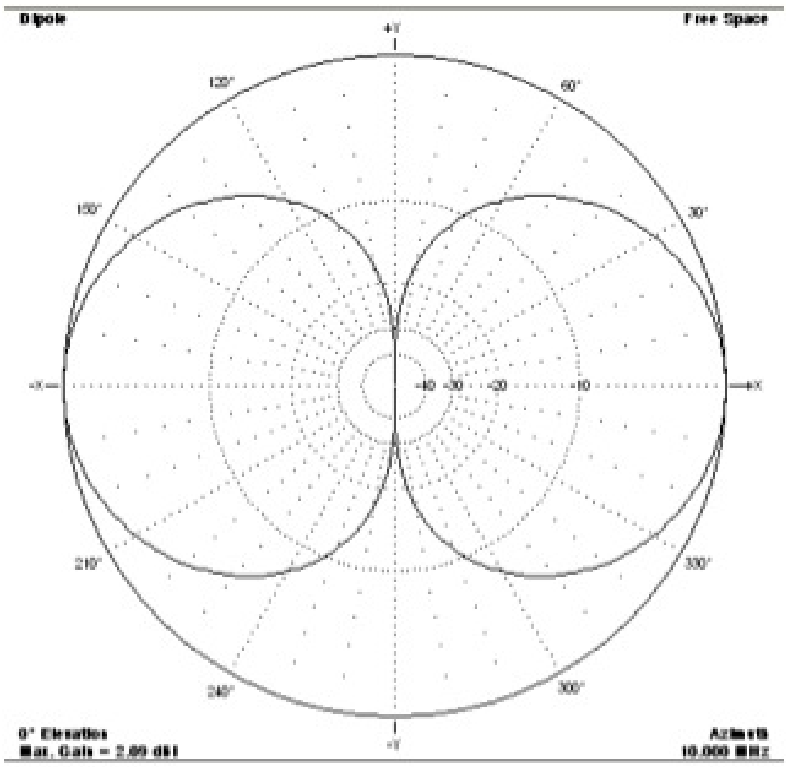
Dipole Radiation Patterns

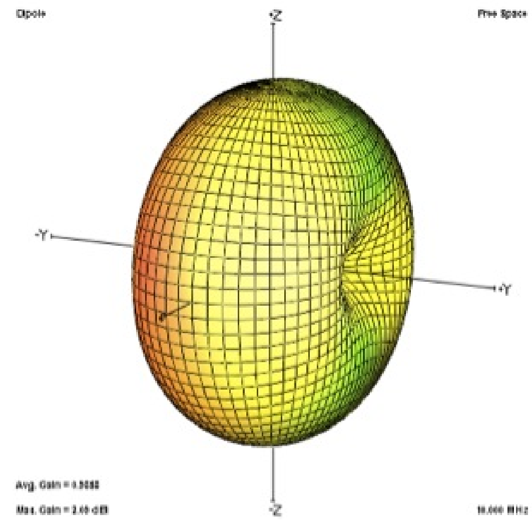
- A typical directional radiation pattern is shown below.



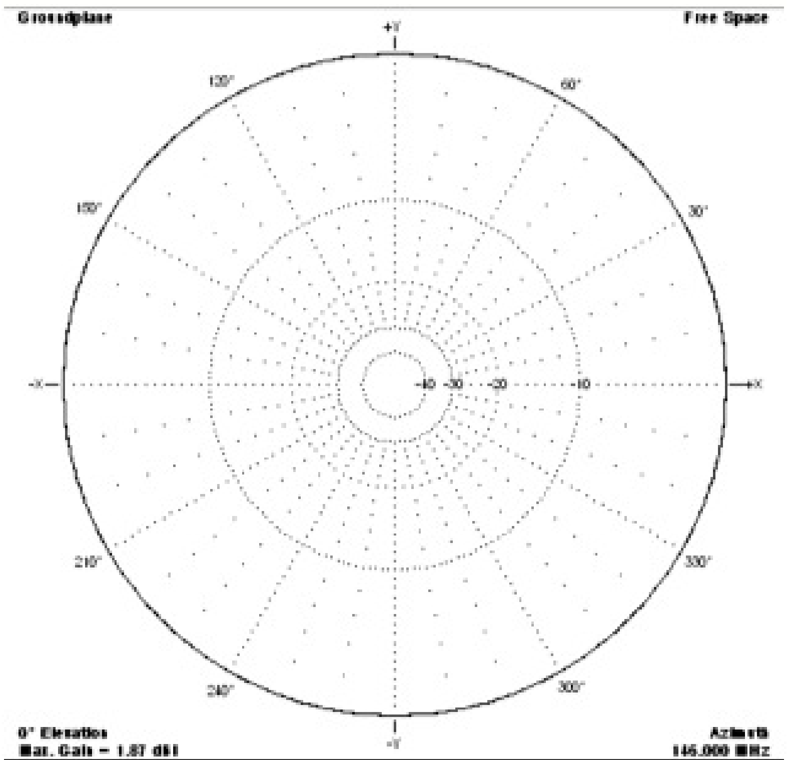
- Here the main strength of the signal is on the x-direction.

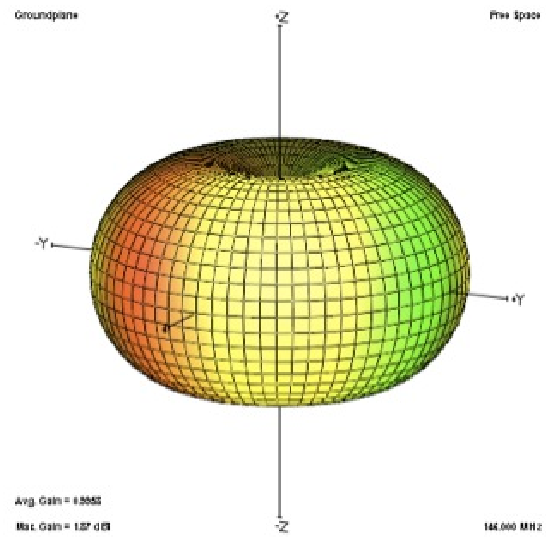
Dipole: Radiation Pattern



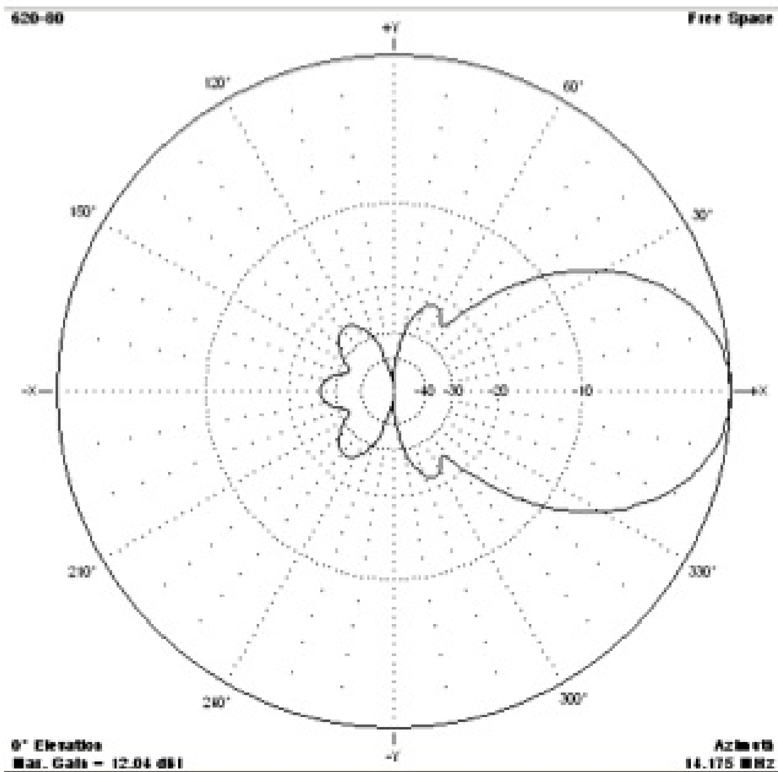


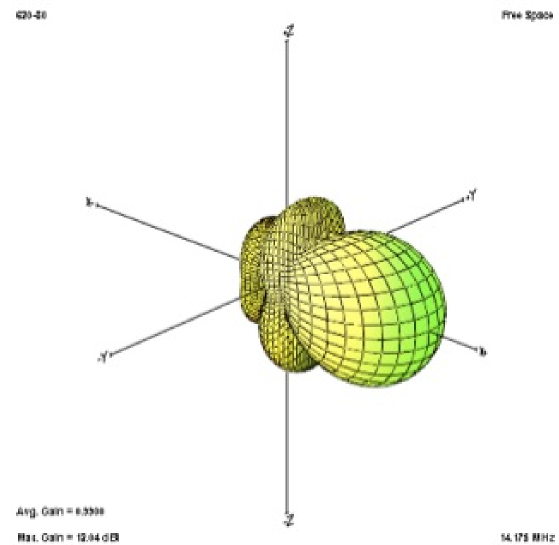
Groundplane: Radiation Pattern





Yagi: Radiation Pattern

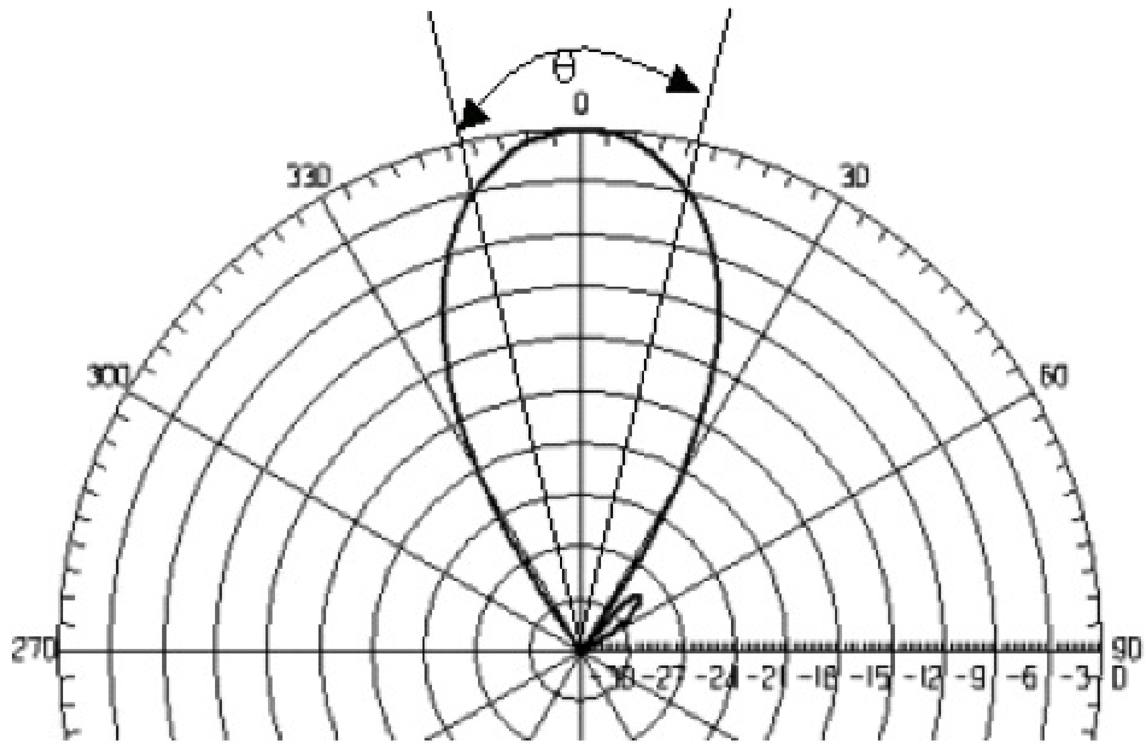




Beamwidth of Antennae

- The **beamwidth** is a measure of the directivity of the antenna.
- It is the angle within which the power radiated by the antenna is at least half of what it is in the most powerful direction.
- For this reason it is called **half-power beam width**.
- When an antenna is used for reception, then the radiation pattern becomes **reception** pattern.

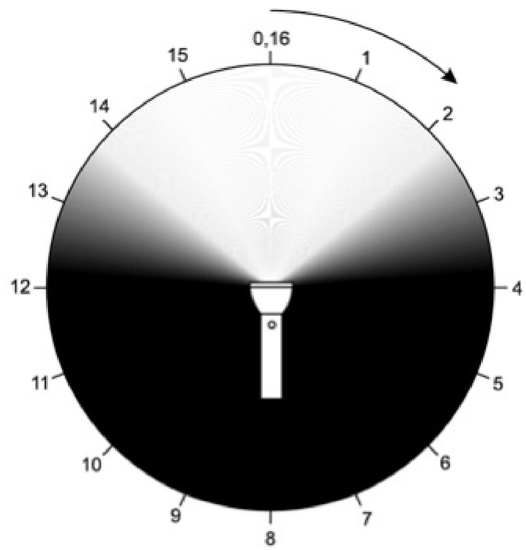
Beamwidth

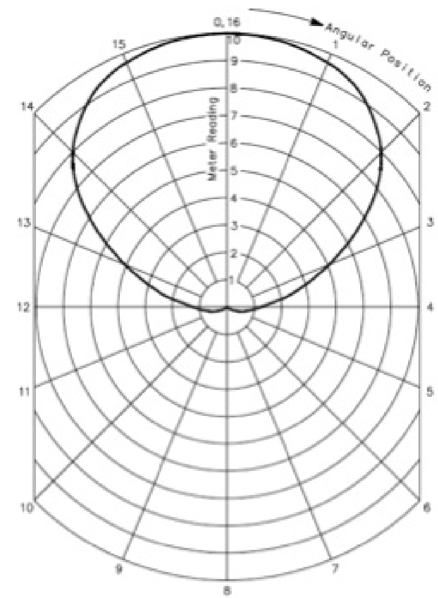


Value of θ in degree = 3 dB beamwidth.

Directivity and Gain

- Flashlight Analogy



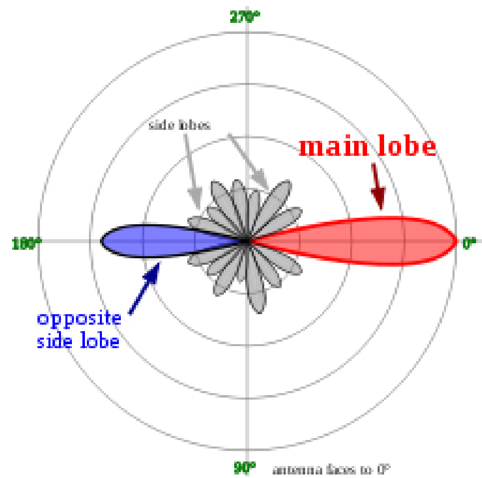


Directive Gain

- **Directive gain** compares the radiation intensity (power per unit solid angle) U that an antenna creates in a particular direction against the average value over all directions:

$$D(\theta, \phi) = \frac{U}{\text{Total radiated power}/(4\pi)},$$

where θ and ϕ are angles of the standard spherical coordinates.



- The **directivity** of an antenna is the maximum value of its directive gain.

Directive Gain

- The **directive gain signifies** the ratio of radiated power in a given direction relative to that of an isotropic radiator which is radiating the same total power as the antenna in question but uniformly in all directions.

Power Gain

- Antenna efficiency ($E_{antenna}$) is the ratio between its input power and its radiated power.
- (Power) gain is a unitless measure combining an antenna's efficiency $E_{antenna}$ and directivity D

$$G = E_{antenna} \cdot D$$

- When considering the power gain for a particular direction given by an elevation (or "altitude") θ and azimuth ϕ, then

$$G(\theta, \phi) = E_{antenna} \cdot D(\theta, \phi)$$

- The **power gain signifies** the ratio of radiated power in a given direction relative to that of an isotropic radiator which is radiating the total amount of electrical power received by the antenna in question.

Antenna Gain

- Power output, in a particular direction, compared to that produced in any direction by an isotropic antenna
- Can be expressed as a ratio of power
- Better expressed in dBi

$$10 \log_{10} \frac{P_a}{P_i}$$

iPhone Antennae

- Uses the stainless steel band around the phone as the antenna for GSM, UMTS, WiFi, GPS and Bluetooth

- Design aximizes antenna size (for better performance) and minimizes space it occupies
- The iPad is using a similar approach where the antenna is the LCD frame around the screen.

The Future: Tunable Antennae

- Technology is being pushed to its limits having to accomodate different frequencies: they need to connect via multiple cellular bands, WiFi, Bluetooth and receive GPS signals not to mention the coming of mobile TV and video which may require even more frequencies.
- Tunable antennas seem to be the upcoming technology as a single antenna might be used for all the frequencies by changing its impedance to optimize performance at various frequencies.
- Since tunable antenna are still in development, using the space around the body of the phone is an ingenious way to free up board space that would be taken up by multiple antennas.

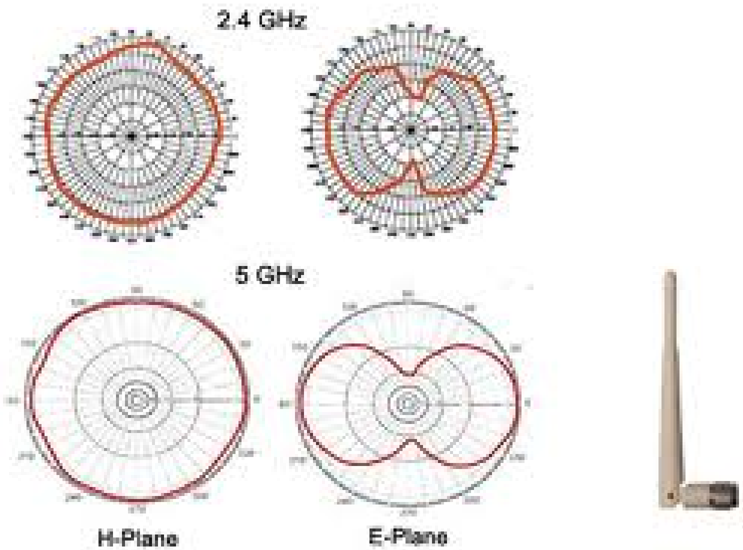
References

- The ARRL Antenna Book. R. Dean Straw, L B Cebik, Dave Hallidy, Dick Jansson. ARRL, 2007.
- Antenna theory: analysis and design. Constantine A. Balanis. John Wiley, 2005.
- Software Defined Radio: Architectures, Systems and Functions. Markus Dillinger, Kambiz Madani, Nancy Alonistioti. Wiley Series in Software Radio, 2003.

Idealized Models

Realistic Model

- Realistic models of radiation patterns are rather complex

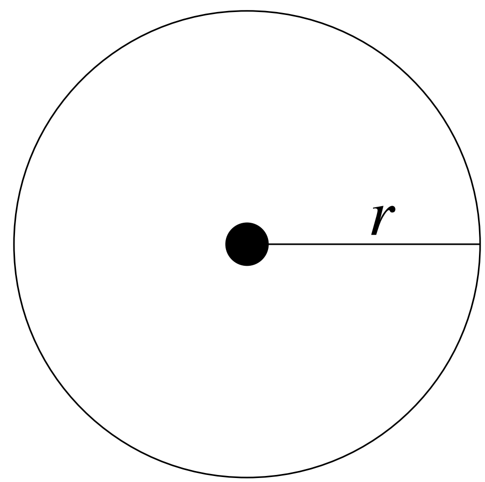


Basic (Idealized) Model

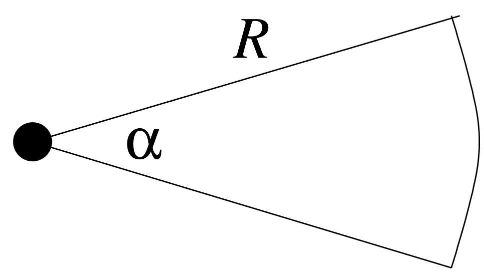
- **Isotropic Omnidirectional**
  - Idealized, point in plane/space
  - Radiates power equally in all directions
- **Isotropic Directional**
  - Idealized, point in plane/space
  - Radiates power equally in all directions within a sector/cone

2D

- Omnidirectional with range r

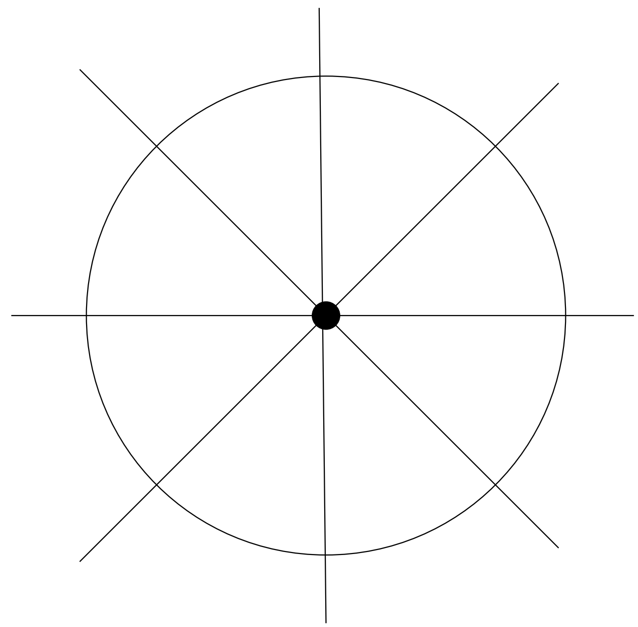


- Directional with range R and angular spread α



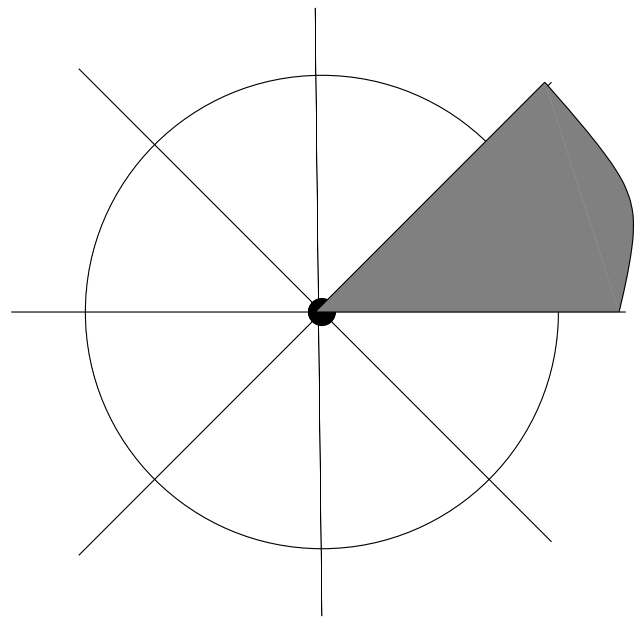
Omnidirectional as Directional Antennae (1/2)

An omnidirectional antenna consists of directional antennae each covering a different sector.



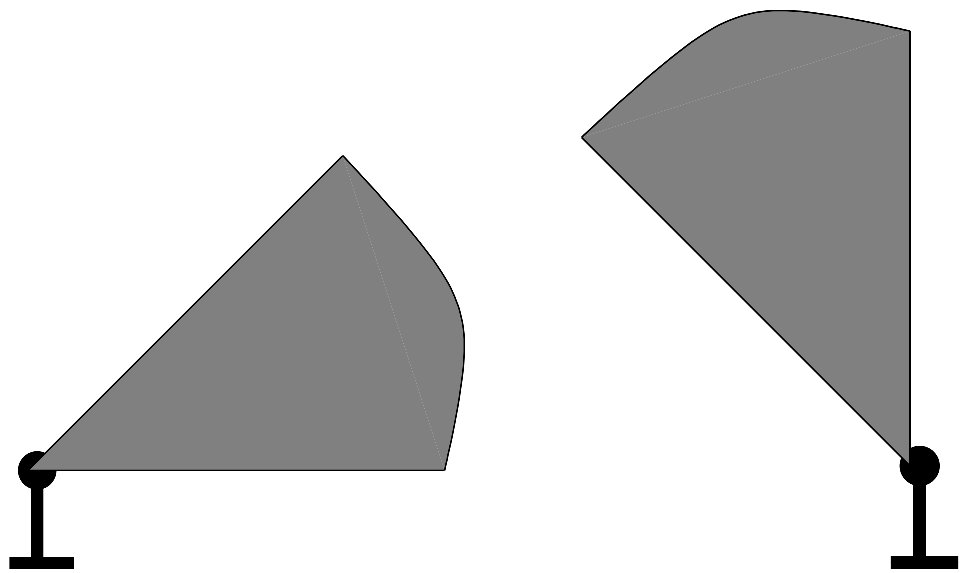
Omnidirectional as Directional Antennae (2/2)

Any of these sectors can be activated in order to connect to a neighbor.



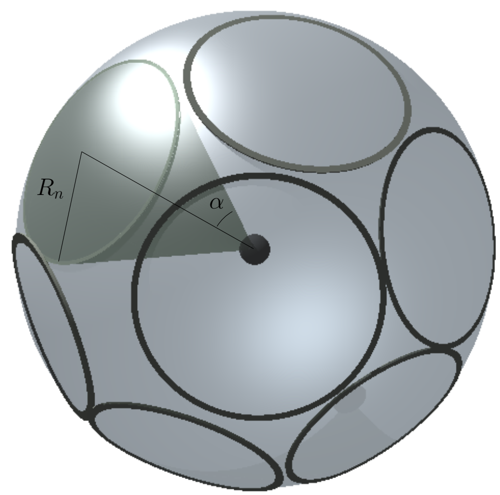
Directional Antennae on a Rotating Swivel

The sensor sits on a rotating swivel and can rotate at will in order to connect to neighbors.



3D

- Entirely analogous models and assumptions
- Omnidirectional with range r
- Directional with range R and **spherical** angular spread α



Communicating with Directional Antennae

- The range of an antenna is divided into n zones.
- Each zone has a conical radiation pattern, spanning an angle of $2\pi/n$ radians.
- The zones are fixed with non-overlapping beam directions, so that the n zones may collectively cover the entire plane.
- When a node is idle, it listens to the carrier in omni mode.
- When it receives a message, it determines the zone on which the received signal power is maximal.
- It then uses that zone to communicate with the sender.

Network Connectivity

Main Question

- Given a set of sensors with omnidirectional antennae forming a connected network:

  Question: How can omnidirectional antennae be replaced with directional antennae in such a way that the connectivity is maintained while the angle and range being used are the smallest possible?

Outline

- Motivation
- Orientation Problem
  - In 1D.
  - In 2D.
    - Complexity.
    - Optimal Range Orientation.
    - Approximate Range Orientation.
  - In 3D.
    - Complexity.
    - Optimal Range Orientation.
    - Approximate Range Orientation.
- Variations of the Antenna Orientation Problem.

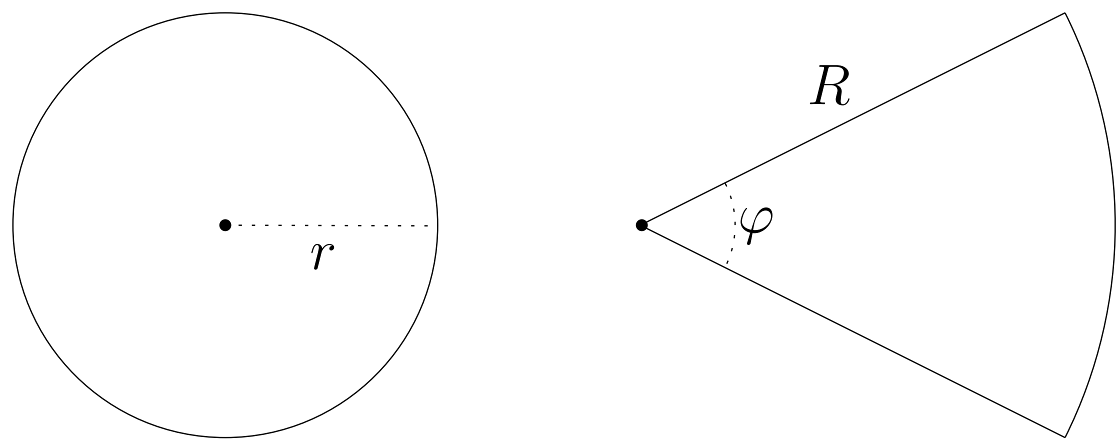
Motivation

Reasons for Replacing Antennae

- Energy Consumption
- Network Capacity

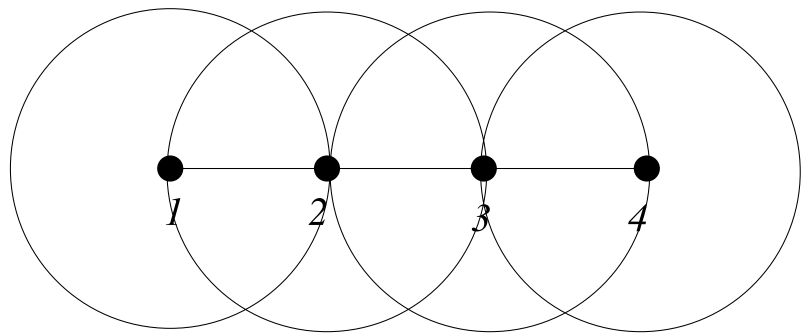
Energy

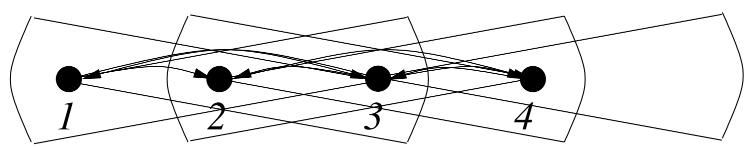
- The energy necessary to transmit a message is proportional to the coverage area.
  - An omnidirectional antenna with range r consumes energy proportional to πr^2.
  - A directional antenna with angle φ and range R consumes energy proportional to $\varphi R^2/2$.



Connectivity

- With the same amount of energy, a directional antenna with angle α can reach further.





Capacity of Wireless Networks

- Consider a set of sensors that transmit W bits per second with antennae having transmission beam of width α and a receiving beam width of angle β.

Sender	**Receiver**	
	Omnidirectional	Directional (β)
Omnidirectional	$\sqrt{\frac{1}{2\pi}}W\sqrt{n}$ [1]	-
Directional (α)	$\sqrt{\frac{1}{\alpha}}W\sqrt{n}$ [2]	$\sqrt{\frac{2\pi}{\alpha\beta}}W\sqrt{n}$ [2]

- **References:**

1. *Gupta and Kumar*. The capacity of wireless networks. 2000.
2. *Yi, Pei and Kalyanaraman*. On the capacity improvement of ad hoc wireless networks using directional antennas. 2003.

Capacity with Directional Antennae

- Consider a set of sensors that transmit W bits per second with antennae having transmission beam of width α and a receiving beam width of angle β.
- Assume that
  - sensors are placed in such a way that the interference is minimum, and
  - traffic patterns and transmission ranges are optimally chosen.
- Then the network capacity (amount of traffic that the network can handle) is at most $\sqrt{\frac{2\pi}{\alpha\beta}} W \sqrt{n}$ per second.

Enhancing Security with Directional Antennae

- The use of directional antennae enhances the network security since the radiation is more restricted.
  - Hu and Evans[a] designed several authentication protocols based on directional antennae.
  - Lu et al[b] employed the average probability of detection to estimate the overall security benefit level of directional transmission over the omnidirectional one.
  - Imai et al[c] examined the possibility of key agreement using variable directional antennae.

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[a] *Hu and Evans.* Using directional antennas to prevent wormhole attacks. 2004

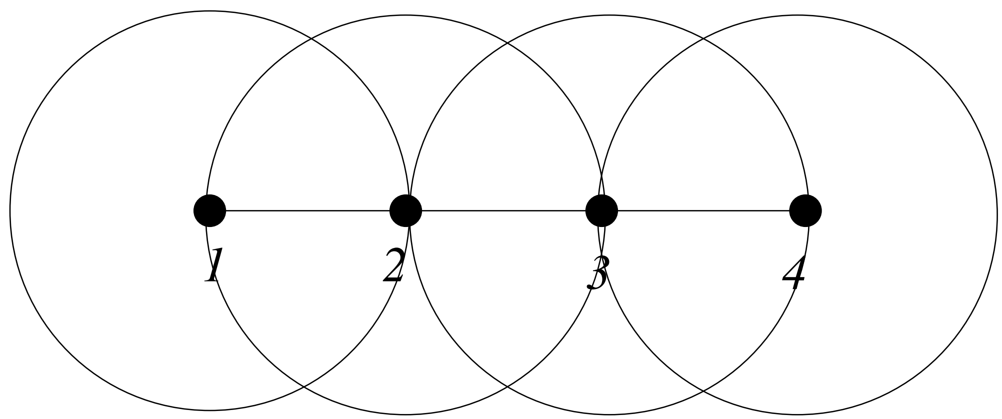
[b] *Lu, Wicker, Lio, and Towsley.* Security Estimation Model with Directional Antennas. 2008

[c] *Imai, Kobara, and Morozov.* On the possibility of key agreement using variable directional antenna. 2006

Antenna Orientation Problem in the Line

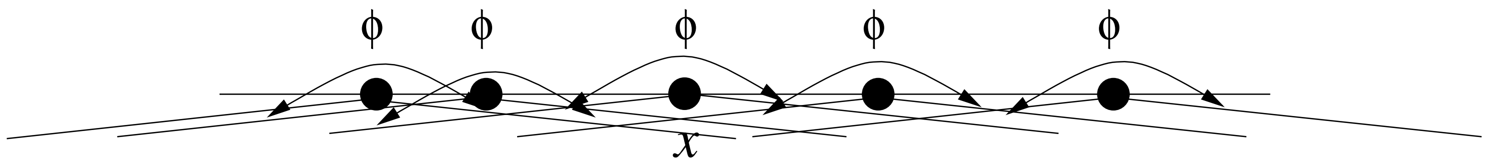
Antenna Orientation Problem in the Line

- Given a set of sensors in the line equipped with one directional antennae each of angle at most $\varphi \geq 0$.
- Compute the minimum range r required to form a strongly connected network by appropriately rotating the antennae.

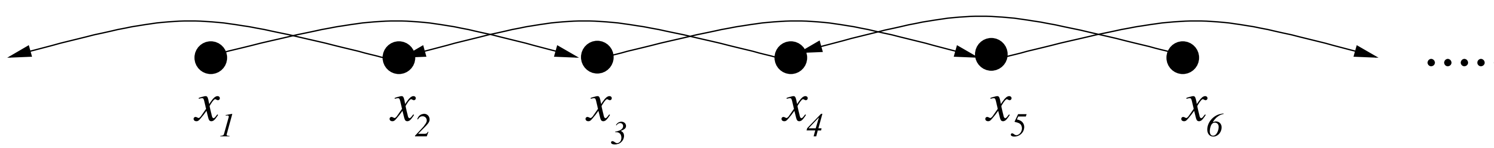


Antenna Orientation Problem in the Line

- Given $\varphi \geq \pi$. The orientation can be done trivially with the same range required when omnidirectional antennae are used.



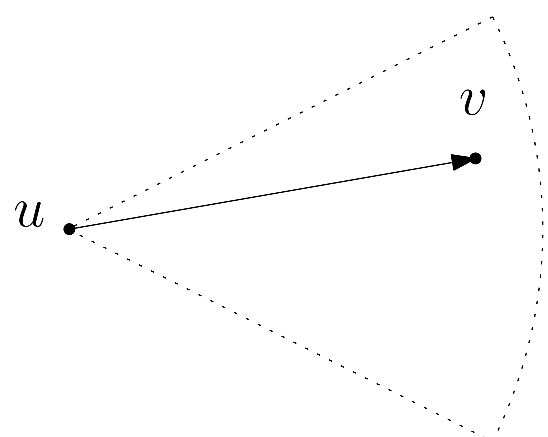
- Given $\varphi < \pi$. The strong orientation can be done with range bounded by two times the range required when omnidirectional antennae are used.



Antenna Orientation Problem in the Plane

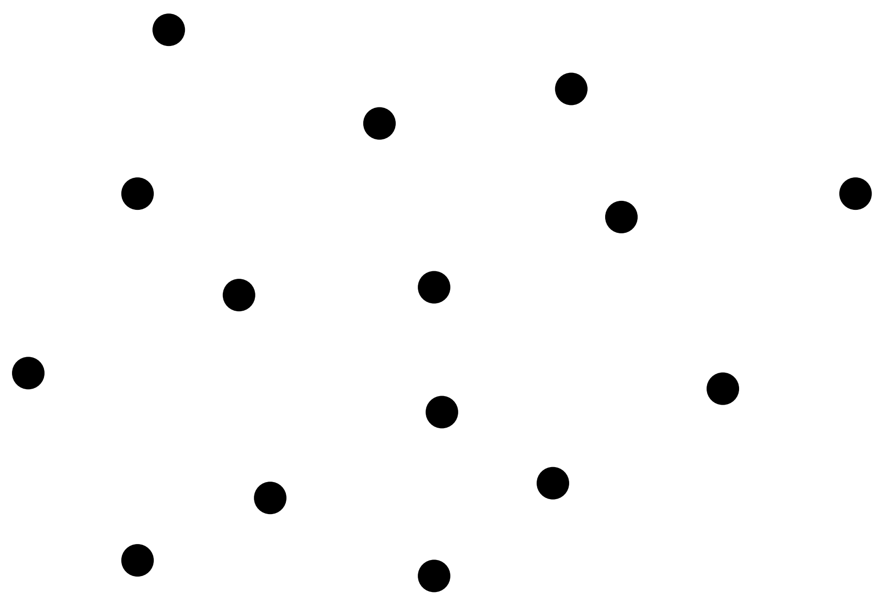
Antenna Orientation Problem

- Given a set of **identical** sensors in the plane equipped with one directional antenna each of angle at most φ.
- Compute the minimum range such that by appropriately rotating the antennae, a directed, strongly connected network on S is formed.

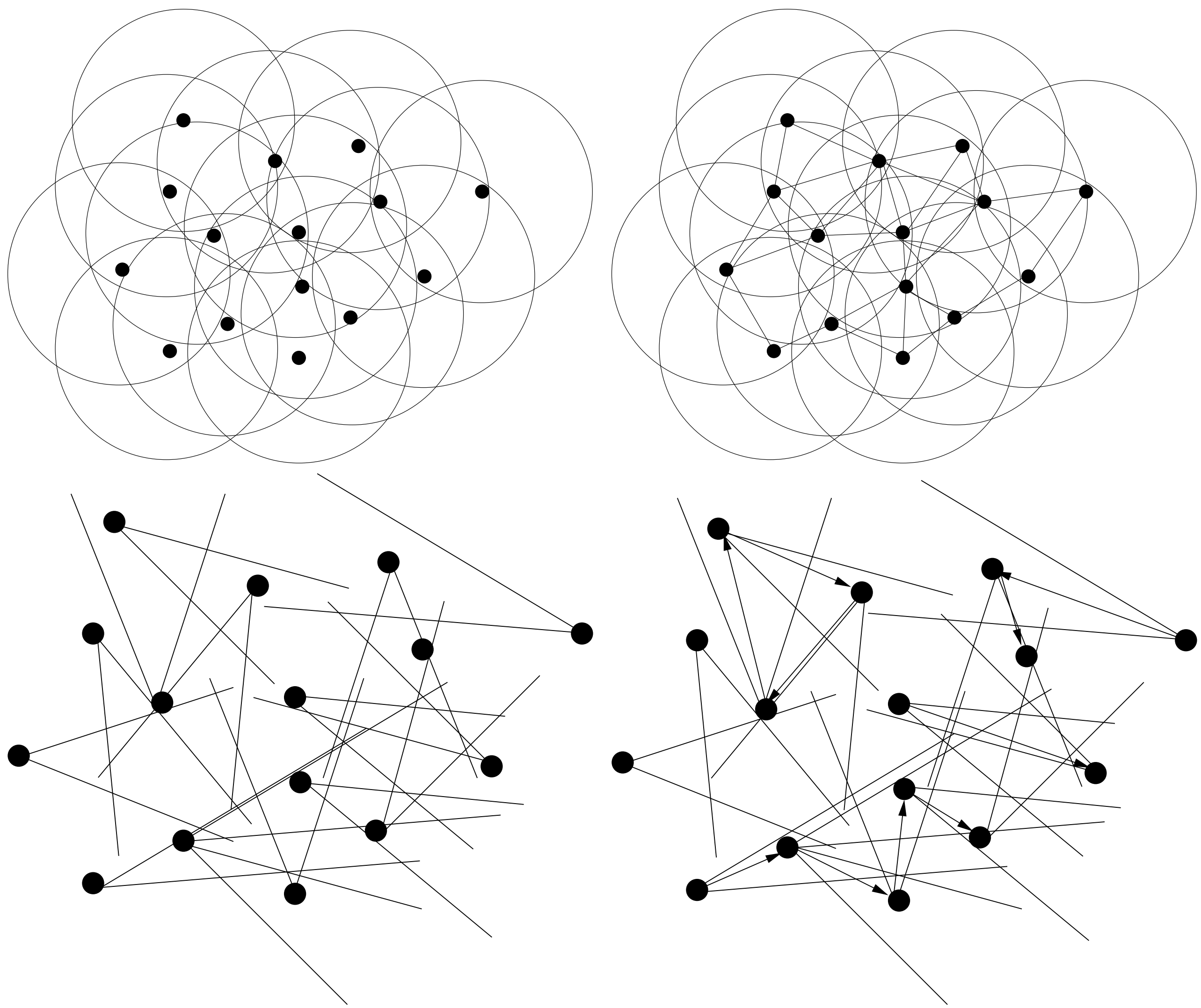


Example: Sensors in the Plane

Consider n sensors in the plane.



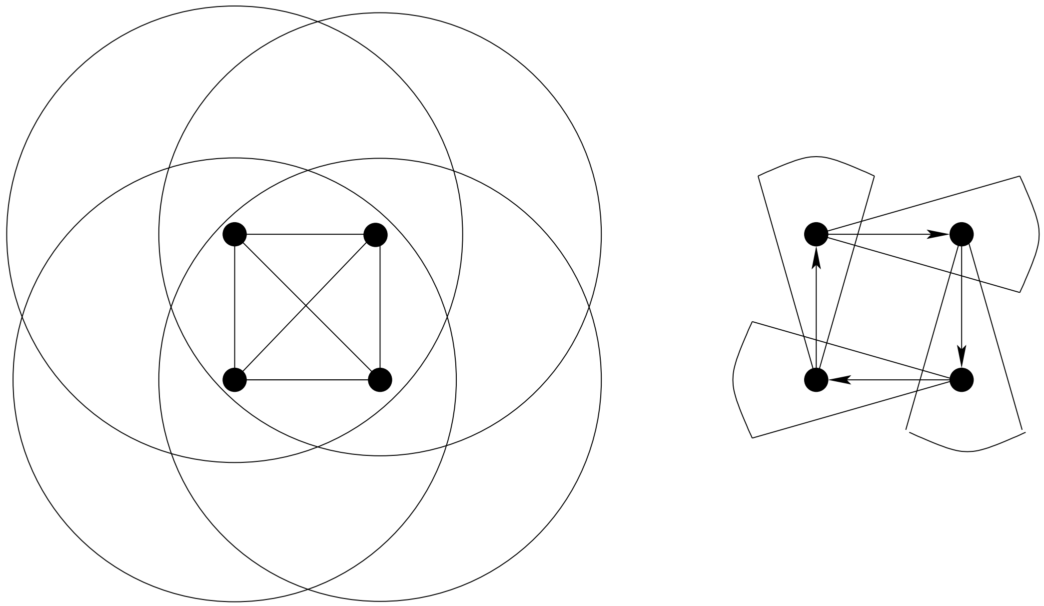
Example: Directional Antennae Affect Connectivity



Connectivity Issues

- When replacing omnidirectional with directional antennae the network topology changes!
- How do you maintain connectivity in a wireless network when the network nodes are equipped with directional antennae?
- Nodes correspond to points on the plane and each uses a directional antenna (modeled by a sector with a given angle and radius).
- The connectivity problem is to decide whether or not it is possible to orient the antennae so that the directed graph induced by the node transmissions is strongly connected.

Four sensors: Connectivity Example



Left: using omnidirectional antennae they form an underlying complete network on four nodes.

Right: using directional antennae they form an underlying cycle on four nodes.

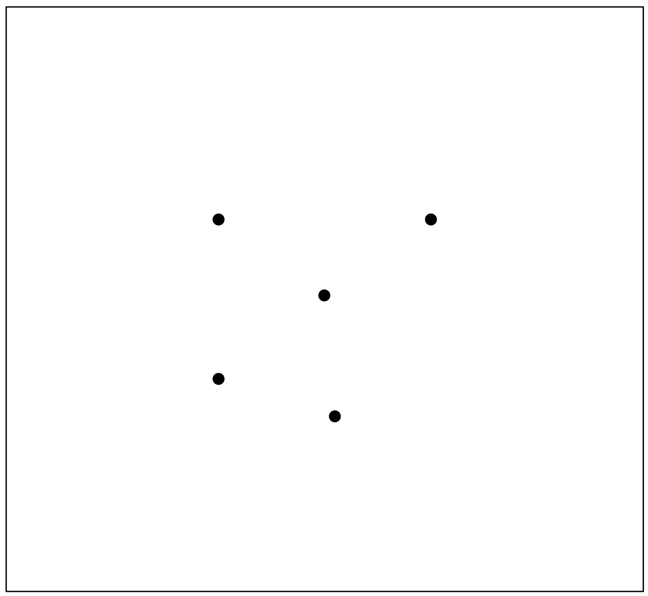
Connectivity Problem

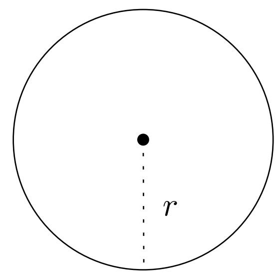
- We consider the problem of maintaining connectivity using the minimum possible range for a given angular spread.
- More specifically,

  For a set of sensors located in the plane at established positions and with a given angular spread we are interested in providing an algorithm that minimizes the range required so that by an appropriate rotation of each of the antennae the resulting network becomes strongly connected.

Antenna Orientation Problem: Distances

- Given n (identical) sensors in the plane with omnidirectional antennae, the optimal range can be computed in polynomial time.

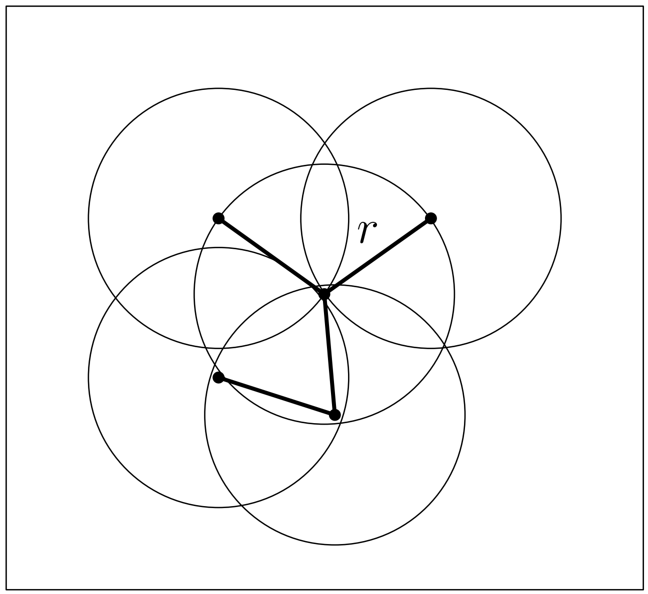


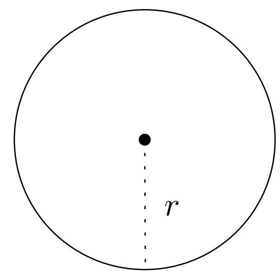


- Why?
- Try all possible (at most n^2) distances.

Antenna Orientation Problem: MST

- The sensors already form an omnidirectional network.

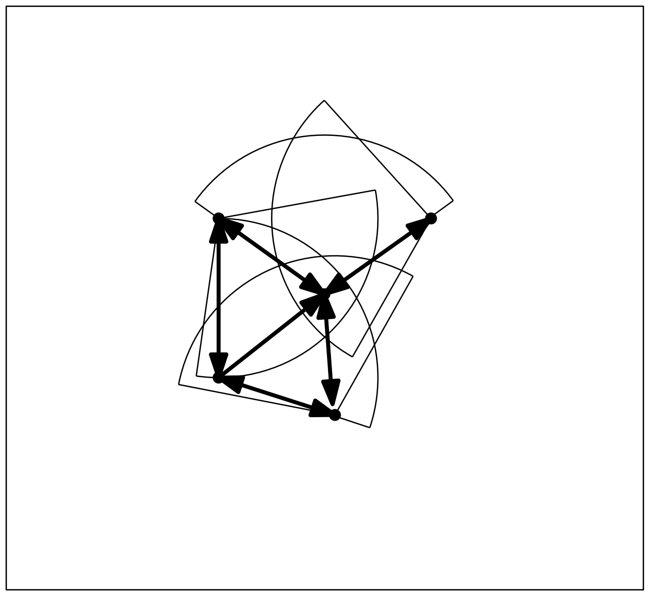


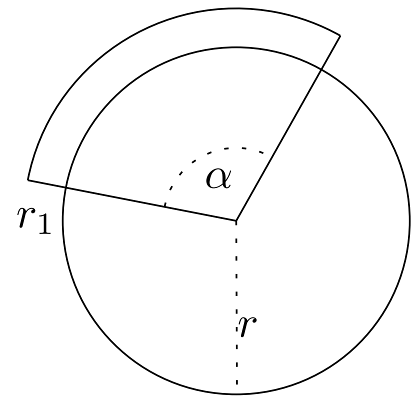


- Actually, the longest edge of the MST is the optimal range.
- Why?

Antenna Orientation Problem: Angle (1/2)

- Given a directional antenna with angle α.

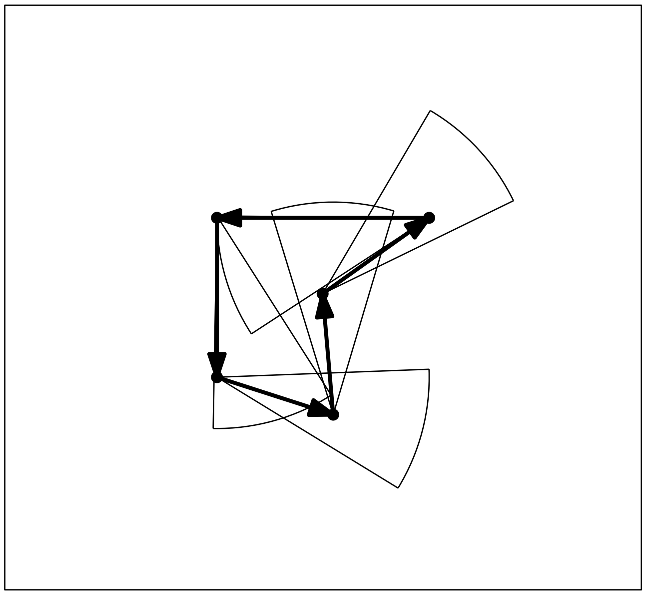


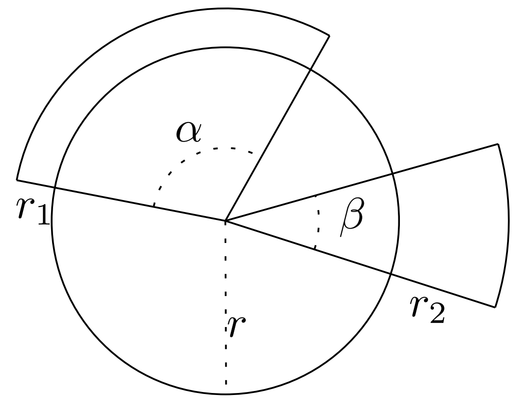


- What is the minimum radius r_1 to create a strongly connected network?

Antenna Orientation Problem: Angle (2/2)

- Given a directional antenna with angle β.



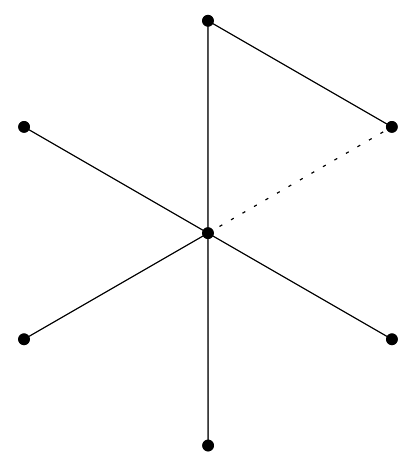


- What is the minimum radius r_2 to create a strongly connected network?

Upper Bound

Optimal Range Orientation (1/3)

- What is the minimum angle necessary to create a strongly connected network if the range of the directional antennae is the same as the omnidirectional antenna?
- Consider an MST T on the set of points.



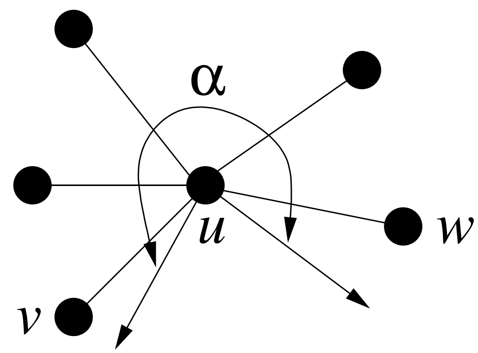
- If the maximum degree of T is 6, by a simple argument we can find an MST with the same weight and maximum degree 5.

Optimal Range Orientation (2/3)

- **If the proximity graph is not connected**, then clearly no orientation of the sectors that defines a strongly connected transmission graph can be found.
- **If the proximity graph is connected**, consider a MST.
- Since the edge costs are Euclidean, each node on this spanning tree has degree at most 5.

Optimal Range Orientation (3/3)

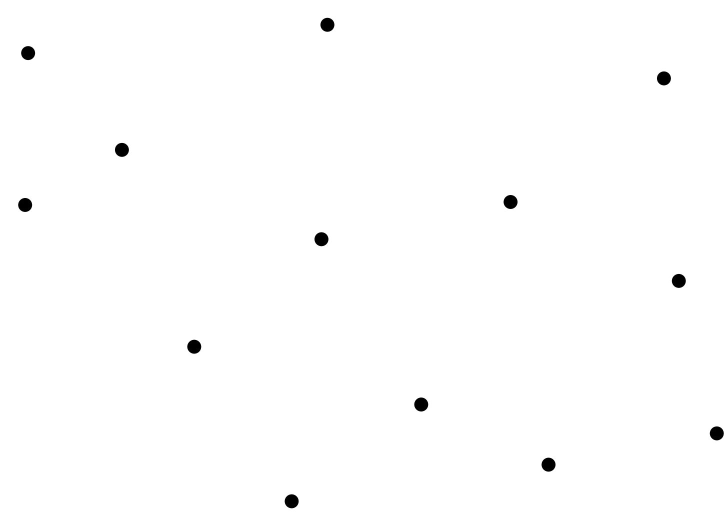
- For each node u, there are two consecutive neighbors v, w in the spanning tree so that the angle $\angle(vuw)$ is at least $2\pi/5$.



- **Theorem 2.** There exists an orientation of the directional antennae with optimal range when the angles of the antennae are at least $8\pi/5$.

Antenna Orientation With Approximation Range

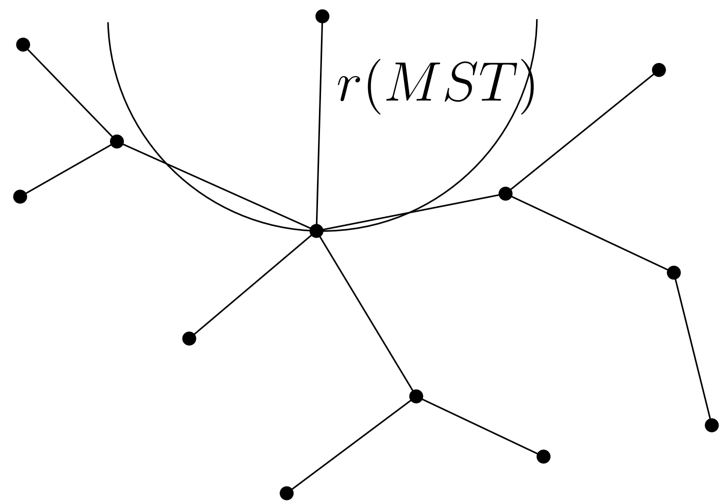
- **Theorem 3.** (Caragiannis et al[a].) There exists a polynomial time algorithm that given an angle φ with $\pi \leq \varphi < 8\pi/5$ and a set of points in the plane, computes a strong orientation with radius bounded by $2\sin(\varphi/2)$ times the optimal range.



[a] *Caragiannis, Kaklamanis,Kranakis, Krizanc and Wiese.* Communication in Wireless Networks with Directional Antennae. 2008

Proof (1/10)

- Consider a Minimum Spanning Tree on the Set of Points.

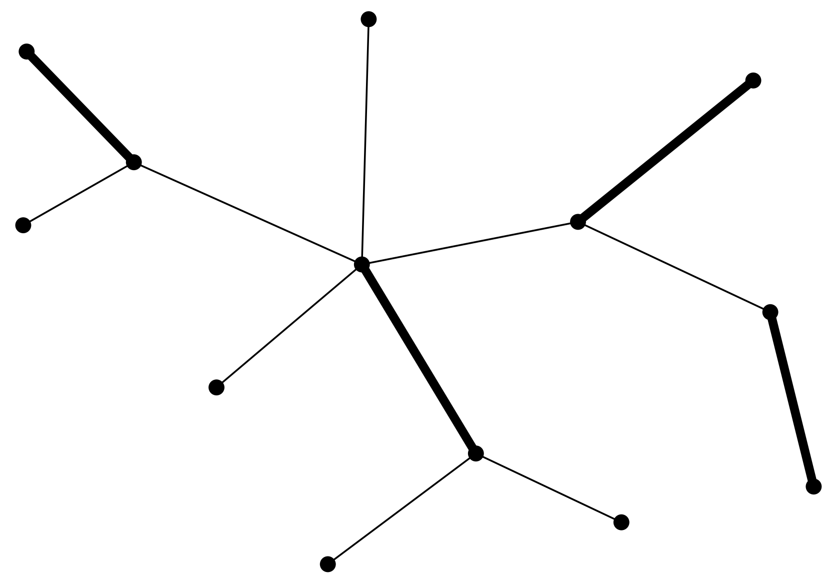


Proof (2/10)

- Let $r^*(\varphi)$ be the optimal range when the angle of the antennae is at most φ.
- Let $r(MST)$ be the longest edge of the MST on the set of points.
- Observe that for $\varphi \geq 0$, $r^*(\varphi) \geq r(MST)$.

Proof (3/10)

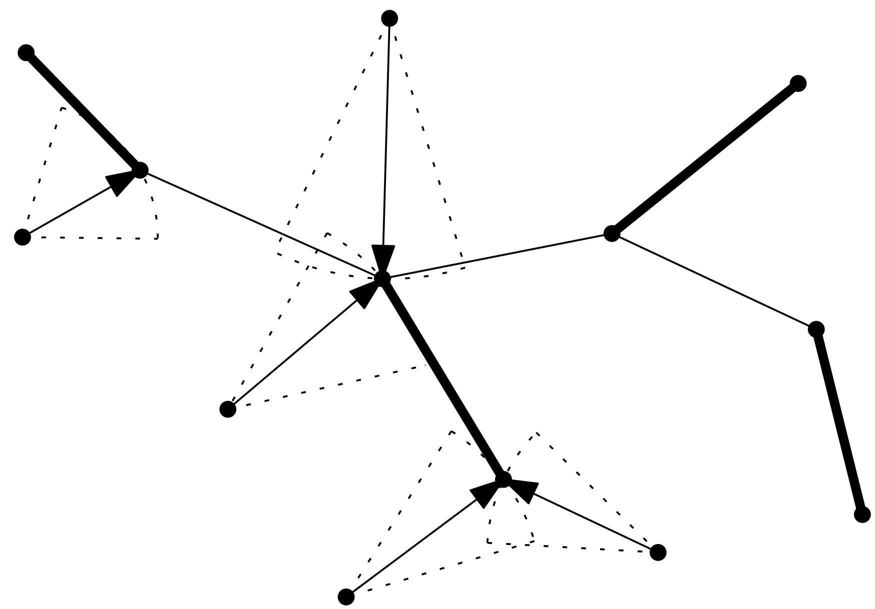
- Find a maximal matching such that each internal vertex is in the matching.



- This can be done by traversing T in BFS order.

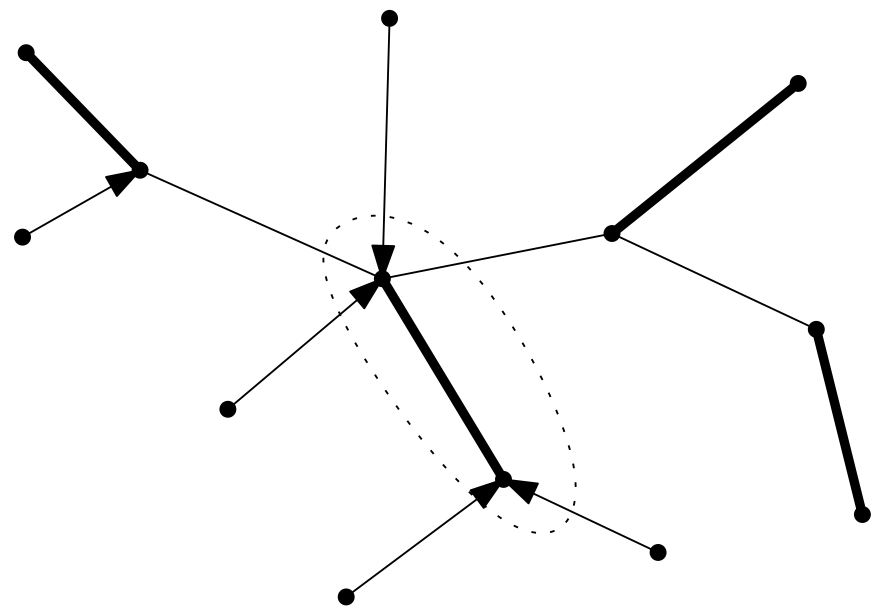
Proof (5/10)

- Orient unmatched leaves to their immediate neighbors.



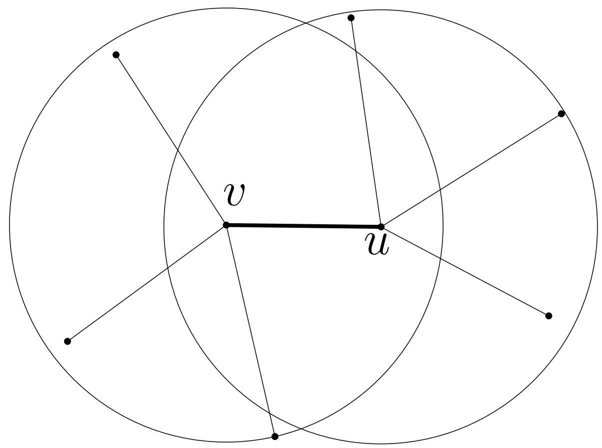
Proof (6/10)

- Consider a pair of matched vertices



Proof (7/10)

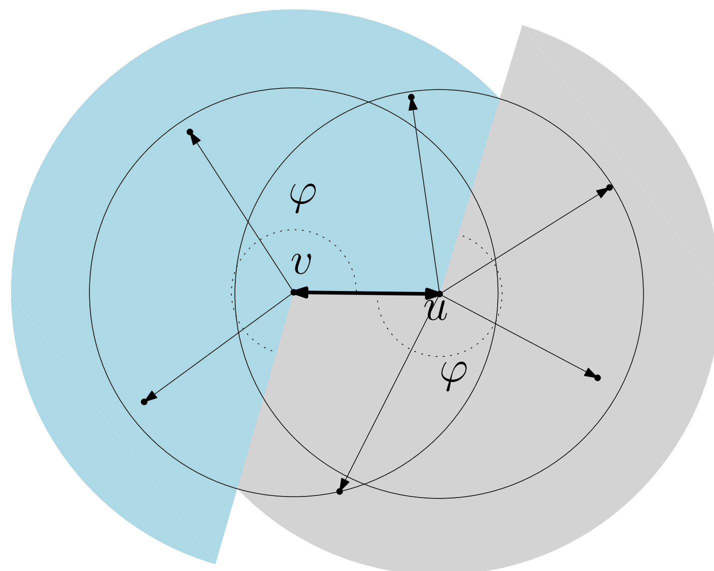
- Let $\{u, v\}$ be an edge in the matching.



- Consider the smallest disks of same radius centered at u and v that contain all the neighbors of u and v in the MST.

Proof (8/10)

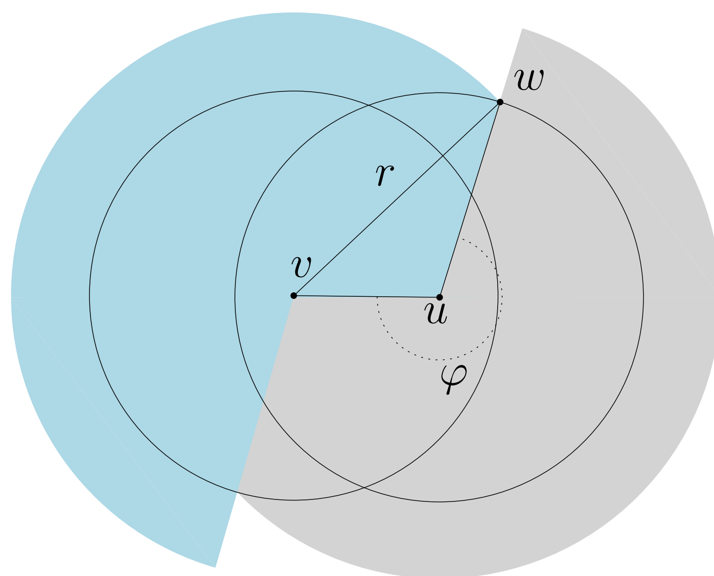
- Orient the directional antennae at u and v with angle φ in such a way that both disks are covered.



- What is the smallest radius necessary so that the union of the discs centered at u, v is covered "completely" by the directional antennae at u, v, respectively?

Proof (9/10)

- To calculate this smallest radius necessary to cover both disks, consider the triangle uvw.



- What is an upper bound on r?
- Observe that without loss of generality we can assume $|uv| = |uw| = 1$.

Proof (10/10)

- Recall the trigonometric identity

$$\sin(\alpha) = \sqrt{\frac{1 - \cos(2\alpha)}{2}} \tag{1}$$

- From the law of cosines we can determine an upper bound on r.

$$\begin{aligned} r &\leq \sqrt{|uv|^2 + |uw|^2 - 2|uv||uw|\cos(2\pi - \varphi)} \\ &= \sqrt{2 - 2\cos(2\pi - \varphi)} \text{ (since } |uv| = |uw| = 1) \\ &= 2\sin(\tfrac{2\pi - \varphi}{2}) \text{ (by Equation (1))} \\ &= 2\sin(\pi - \varphi/2) \\ &= 2\sin(\varphi/2) \end{aligned}$$

Lower Bound

Related Work

- When the angle is small, the problem is equivalent to the bottleneck traveling salesman problem (**BTSP**) of finding the Hamiltonian cycle that minimizes the longest edge.
- A 2-approximation (on the antenna length) is given by Parker and Rardin[a].
- For which angles are the two problems equivalent?

[a] *Parker and Rardin.* Guaranteed performance heuristics for the bottleneck traveling salesman problem. 1984

Complexity

- **HCBPG**
  Hamiltonian Circuit Bipartite Planar Grid:
  - **Input:** Bipartite planar grid graph G of degree at most 3.
  - **Output:** Does G have a Hamiltonian circuit?
- HCBPG is NP-Complete[a].
- By reduction to the problem HCBPG, it can be proved that the problem is NP-Complete when the angle is less than $\pi/2$ and an approximation range less than $\sqrt{2}$ times the optimal range.
- We can prove something stronger.

[a] *Itai, Papadimitriou, and Szwarcfiter.* Hamilton Paths in Grid Graphs. 1982

Computational Complexity

- **Theorem 1 (Caragiannis et al[a].)** Deciding whether there exists an orientation of one antenna at each sensor with angle less that $2\pi/3$ and optimal range is NP-Complete. The problem remains NP-complete even for approximation range less than $\sqrt{3}$ times the optimal range.

- By reduction to the problem of finding Hamiltonian circuit in bipartite planar graphs of maximum degree 3. [b]

- Given a bipartite planar graph $G = (V_0 \cup V_1, E)$ of degree ≤ 3 with n nodes, we construct an ϵ-hexagon graph H (together with its embedding) which has a hamilton circuit if and only if G has a hamilton circuit.

---

[a] *Caragiannis, Kaklamanis,Kranakis, Krizanc and Wiese.* Communication in Wireless Networks with Directional Antennae. 2008

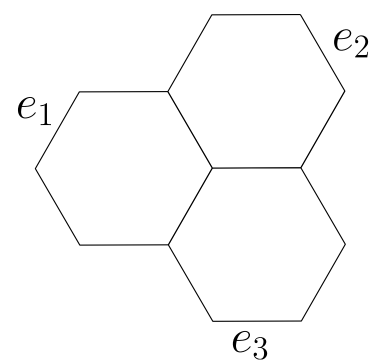
[b] *Itai, Papadimitriou, and Szwarcfiter.* Hamilton Paths in Grid Graphs. 1982

Main Idea: ϵ-Hexagon Graphs

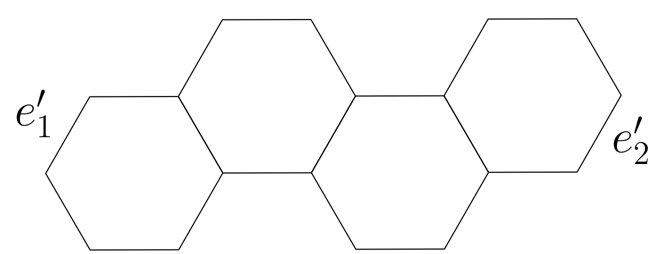
- Let $\epsilon > 0$. An ϵ-hexagon graph $G = (V, E)$ is a bipartite planar graph of maximum degree 3 which has an embedding on the plane with the following properties:
  1. Each node of the graph corresponds to a point in the plane.
  2. The euclidean distance between the points corresponding to two nodes v_1, v_2 of G is in $[1-\epsilon, 1]$ if $(v_1, v_2) \in E$ and larger than $\sqrt{3} - 3\epsilon$ otherwise.
  3. The angle between any two line segments corresponding to edges adjacent to the same node of G is at least $2\pi/3 - \epsilon/2$.
- An ϵ-hexagon graph is the proximity graph for an instance of the problem and any orientation of sector of radius 1 and angle $\phi = 2\pi/3 - \epsilon$ that induces a strongly connected transmission graph actually corresponds to a hamiltonian circuit of the proximity graph, and vice versa.

Meta Vertices/Edges

- (**Meta vertex:**) Replace every vertex by a diamond (three hexagons)

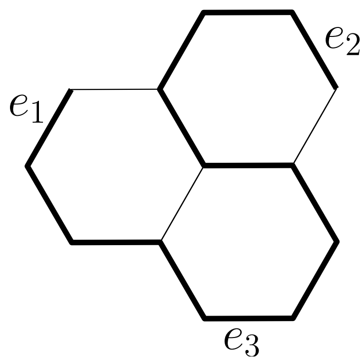


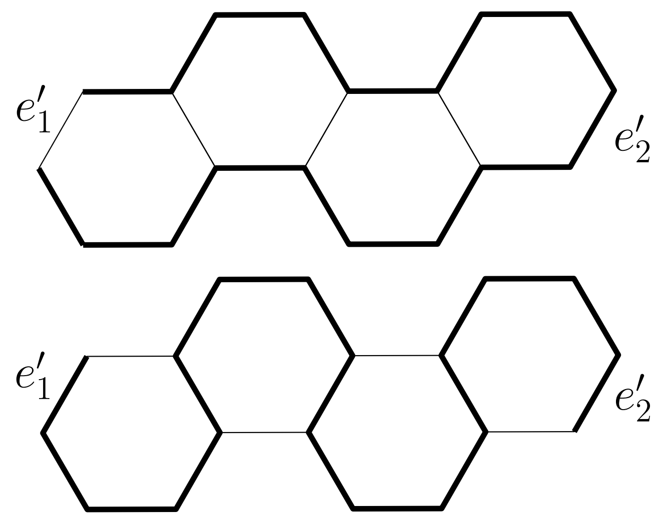
- (**Meta edge:**) Replace every edge by a necklace (path of hexagons)



Hamiltonian Paths

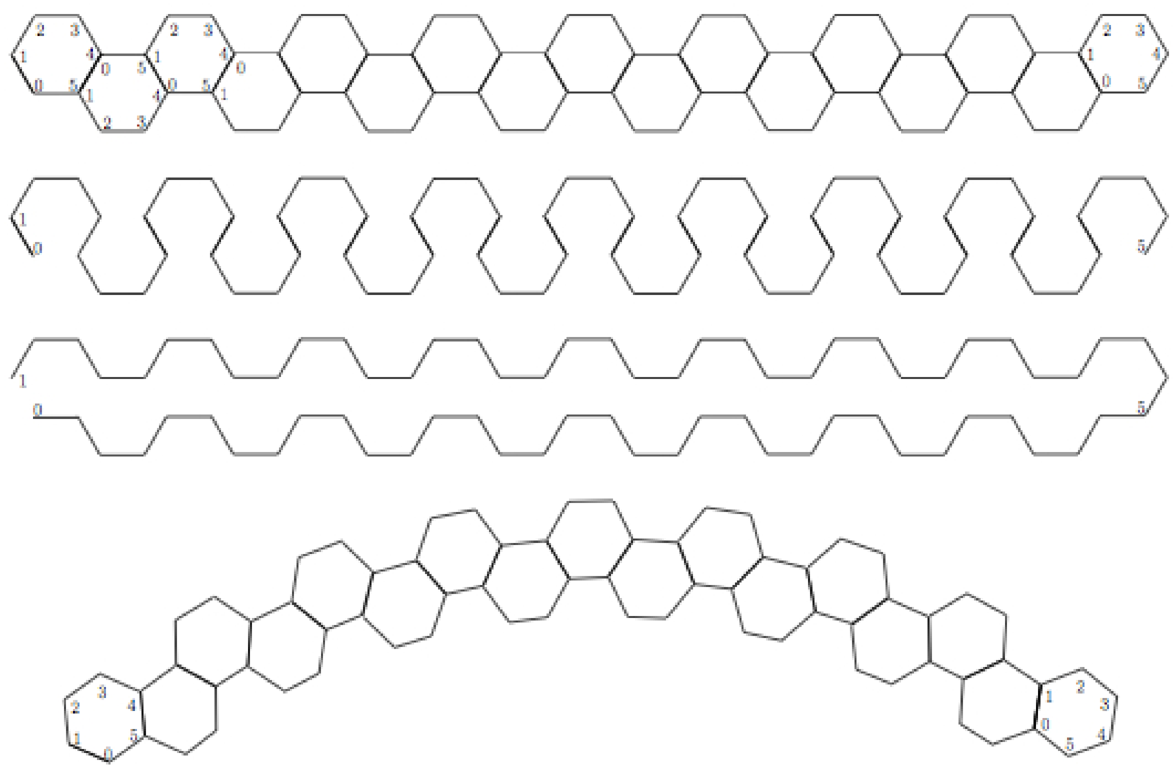
- The meta vertices and necklaces have the following Hamiltonian paths.





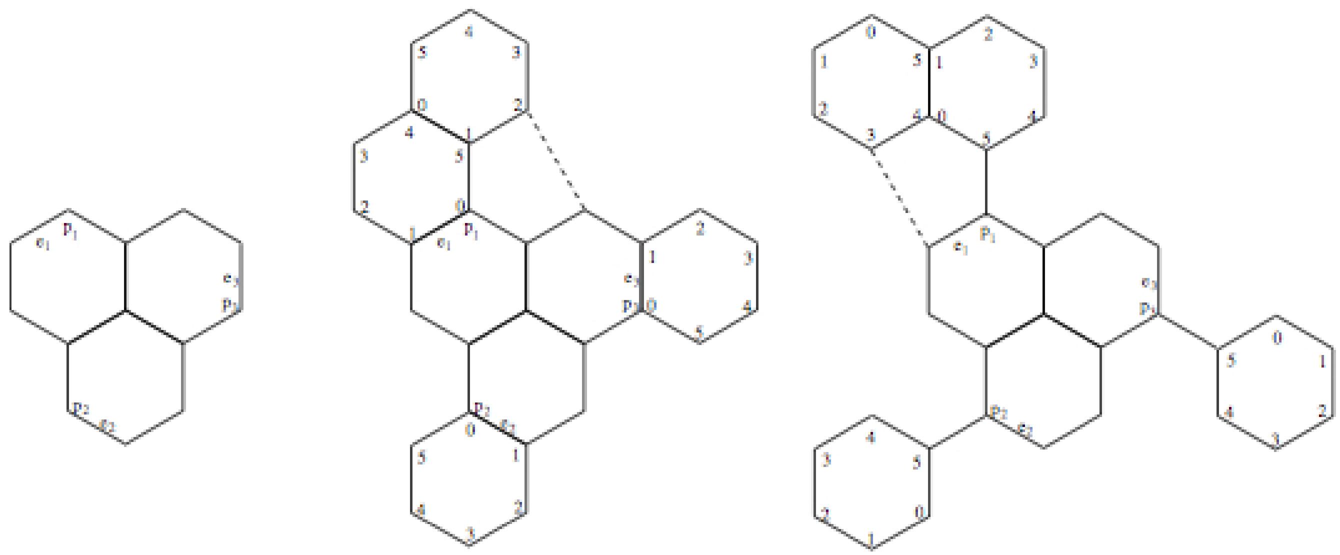
Necklaces, Cross and Return Paths (Examples)

- Top to bottom: 1) Orientation of a necklace, 2) cross path, 3) return path, and 4) representation of the necklace using irregular hexagons of sides between 0.95 and 1 and with angles between sides from 115^o to 125^o.



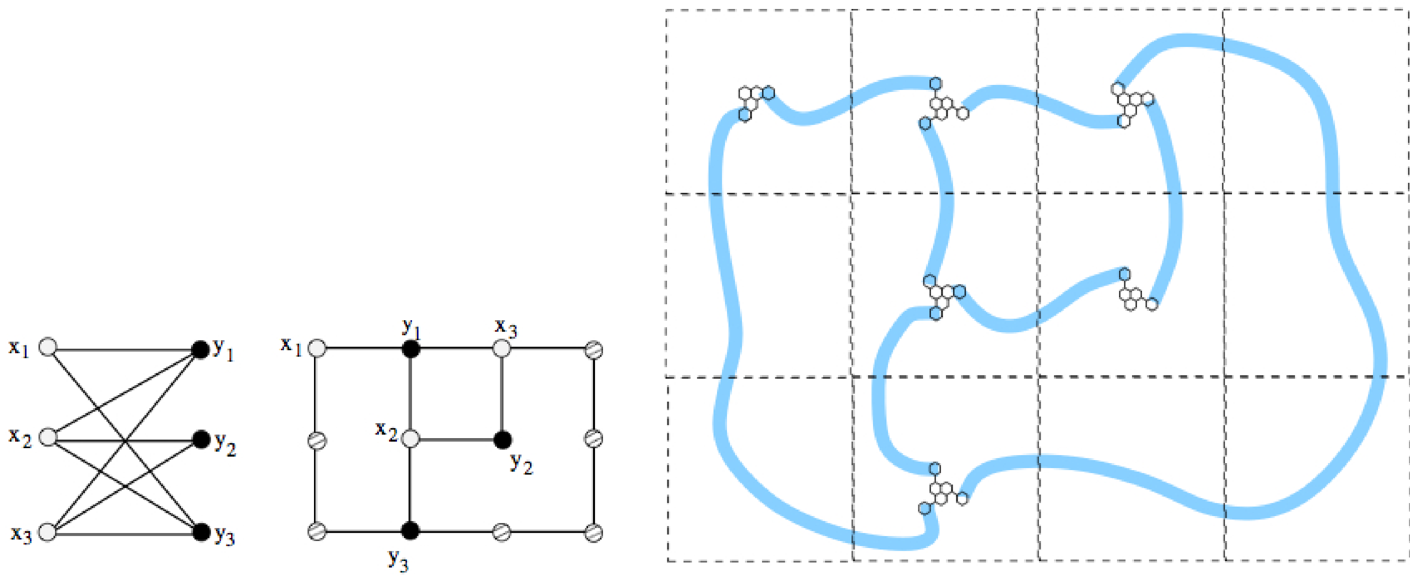
Diamonds and Necklaces

- Left to Right: A diamond (left) and its connection to necklaces when it corresponds to a node of V_0 (middle) or V_1 (right).



Embedding

- A bipartite planar graph of maximum degree 3, its embedding on the rectangular grid, and corresponding ϵ-hexagon graph.



Summary

- We can summarize known antenna angle/range tradeoffs as follows:

Angle	Approximation	Complexity	Reference
$\phi < \frac{2\pi}{3}$	$\sqrt{3} - \epsilon$	NP-C	This talk
$\frac{\pi}{2} \leq \phi \leq \frac{2\pi}{3}$	$4\cos(\phi/2) + 3$	Polynomial	To appear
$\frac{2\pi}{3} \leq \phi \leq \pi$	$2\cos(\phi/2) + 2$	Polynomial	To appear
$\frac{2\pi}{3} \leq \phi \leq \frac{4\pi}{3}$	$2\sin(\phi/2)$	Polynomial	This talk
$\frac{4\pi}{3} \leq \phi$	1 (optimal)	Polynomial	To appear

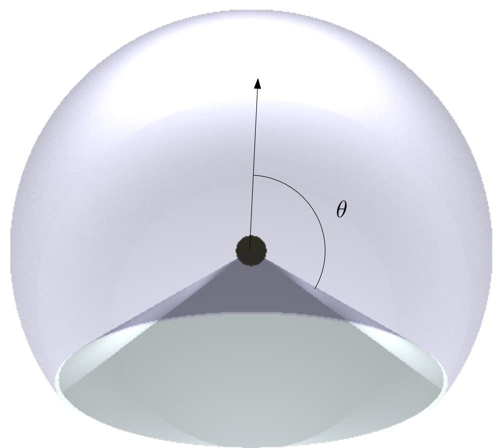
Antenna Orientation Problem in 3D Space

Sensors in 3D Space

- Due to the fact that sensors may lie in distinct altitudes, the previous algorithms do not work correctly in 3D space.
- We model an antenna in 3D space with solid angle Ω as a spherical sector of radius one.
- An omnidirectional antenna has solid angle 4π.

Sensors in 3D Space

- The **apex angle** θ of a spherical sector (with **solid angle** Ω) is the maximum planar angle between any two generatrices of the spherical sector.



- Their relation is given by Archimedes formula

$$\Omega = 2\pi(1 - \cos\theta)$$

Complexity of the Antenna Orientation Problem in 3D Space

- **Theorem 4.** Deciding whether there exists a strong orientation when each sensor has one directional antenna with solid angle less than π and optimal range is NP-Complete.[a]

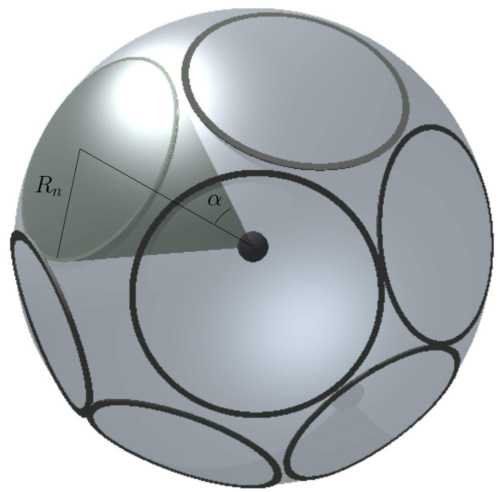
[a]E. Kranakis, D. Krizanc, A. Modi, O. Morales Ponce. Connectivity Trade-offs in 3D Wireless Sensor Networks Using Directional Antennae. In proceedings of IPDPS 2011, May 16-20, 2011.

Proof

- Consider a set S of n points in the plane.
- From Archimedes relation, any plane that cuts the coverage area of any 3D directional antennae through the apex with angle Ω has plane angle that satisfies $\cos(\theta) \leq 1 - \frac{\Omega}{2\pi}$.
- Therefore $\theta < 2\pi/3$ if and only if $\Omega < \pi$.
- A strong orientation of the directional antennae with angle less than $2\pi/3$ in 2D implies a strong orientation of directional antennae with angle less than π in 3D.
- The opposite is also true.

Tammes' Radius

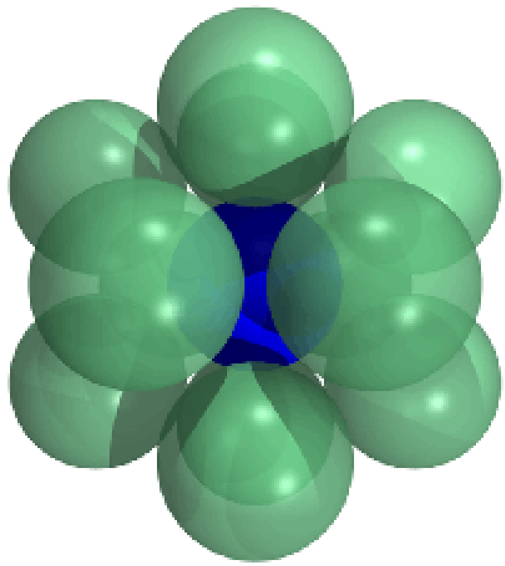
- The Tammes radius is the maximum radius of n equal non-overlapping circles on the surface of the sphere.



- We denote it by R_n.

Kissing Number and Tammes' Radius

- The Kissing number is the number of balls of equal radius that can touch an equivalent ball without any intersection,



Kissing Number and Tammes' Radius

- In particular, the Tammes' Radius is equivalent to the kissing number when all the balls have the same radius.
- The maximum degree of an MST is equal to the kissing number.
- In 3D it is 12.

Optimal Range Orientation in the Space

- **Theorem 5.** There exists an orientation of the directional antennae in 3D with optimal range when the solid angles of the antennae are at least $18\pi/5$.

Proof (1/4)

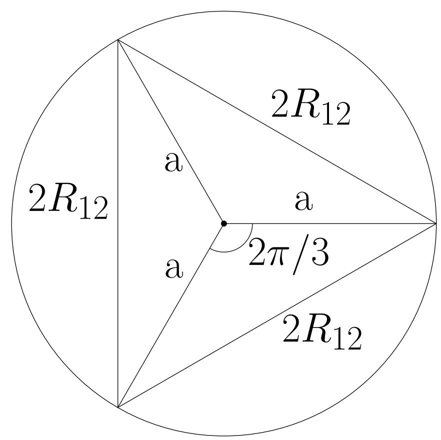
- Let T be an MST on the points.
- Let B_p be the sphere centered at p of minimum radius that covers all the neighbors of p in T.
- For each neighbor u of p in T, let u' be the intersection point of B_p with the ray emanating from p toward u

Proof (2/4)

- Thus, we have an unit sphere with at most 12 points.
- Compute the Delauney Triangulation on the points of the sphere.
- Orient the antenna in opposite direction of the center of largest triangle.

Proof (3/4)

- Observe that every edge of the Delaunay Triangulation has length at least twice the Tammes' Radius $R_{12} = \sin \frac{63^o 26}{2}$.
- Thus, every triangle is greater than the equilateral triangle of side $2R_{12}$.



Proof (4/4)

- It follows that

$$\begin{aligned} R_{12} &= \sin\left(\frac{63^o 26'}{2}\right) \\ a &= R_{12}/\sqrt{3} \\ \alpha &\leq \arcsin(a) \end{aligned}$$

- and therefore

$$\begin{aligned} \Omega &\geq 4\pi - 2\pi(1-\cos(\alpha)) \\ &= 2\pi(1+\cos(\alpha)) \\ &= 2\pi\left(1+\cos\left(\arcsin\left(\frac{2R_{12}}{\sqrt{3}}\right)\right)\right) \\ &\geq \frac{18\pi}{5} \end{aligned}$$

Antenna Orientation With Approximation Range

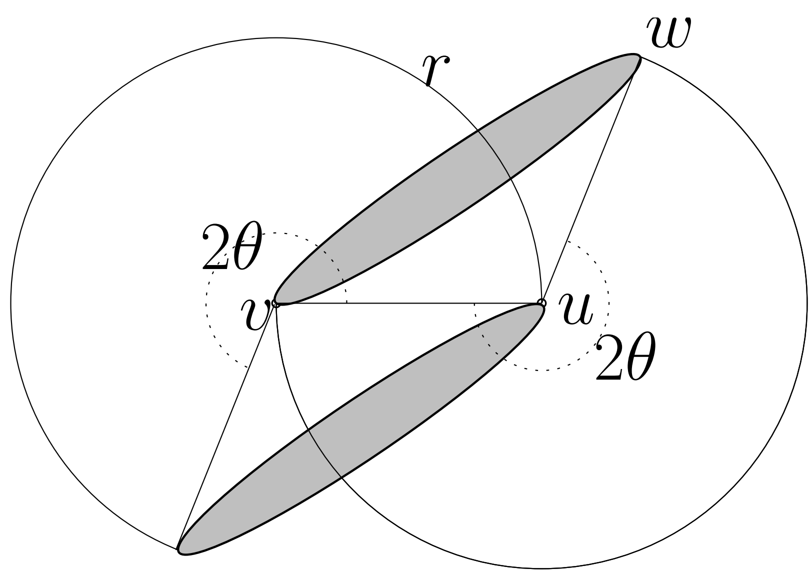
- **Theorem 6.** Given a solid angle φ with $2\pi \leq \varphi < 18\pi/5$ and a set of points in the space, there exists a polynomial time algorithm that computes a strong orientation with radius bounded by $\frac{\sqrt{\Omega(4\pi - \Omega)}}{\pi}$ times the optimal range.

Proof (1/3)

- Let T be the MST on the set of points.
- Consider a maximal matching such that each internal vertex is matched.
- Orient unmatched leaves to their immediate neighbors.
- Let $\{u, v\}$ be an edge in the matching. Consider the smallest sphere of same radius centered at u and v that contain all the neighbors of u and v in the MST.

Proof (2/3)

- Orient the directional antennae at u and v with plane angle 2θ in such a way that both spheres are covered.



Proof (3/3)

- From the law of cosine we can determine r.
- Let θ be the apex angle of Ω.
- Observe that

$$
\begin{aligned}
r &= \sqrt{|uv|^2 + |uw|^2 - 2|uv||uw|\cos(2\theta)} \\
&\leq \sqrt{2 - 2\cos(2\theta)} \\
&= 2\sin(\theta) \\
&= 2\sqrt{1 - \cos^2(\theta)} \\
&= 2\sqrt{1 - (1 - \tfrac{\Omega}{2\pi})^2} \\
&= \frac{\sqrt{\Omega(4\pi - \Omega)}}{\pi}
\end{aligned}
$$

Summary of the Antenna Orientation Problem

2D		3D	
Angle	Range	Solid Angle	Range
$\varphi < \frac{2\pi}{3}$	NP-C	$\Omega < \pi$	NP-C
$\frac{2\pi}{3} \leq \varphi < \pi$	Open	$\pi \leq \Omega < 2\pi$	Open
$\pi \leq \varphi < \frac{8\pi}{5}$	$2\sin(\varphi/2)$	$2\pi \leq \Omega < \frac{18\pi}{5}$	$\frac{\sqrt{\Omega(4\pi-\Omega)}}{\pi}$
$\varphi \geq \frac{8\pi}{5}$	1	$\Omega \geq \frac{18\pi}{5}$	1

Multiple Antennae

Overview

- Introduction
- Multiple Antennae Orientation Problem: Angle/Range Tradeoffs
  - Upper Bounds
  - Lower Bounds/NP-Hardness
  - Toughness of UDGs and Robust Antennae Range
- Minimum Number of Antennae Orientation Problem
- Conclusions/Open Problems

Introduction

Orientation Problem

- Given a set S of sensors. Assume that each sensor has $k > 1$ directional antennae such that the sum is at most φ.

  What is the minimum range necessary to create a strongly connected network by appropiatly rotating the antennae?

- Two variants: Transmission angle (spread) is limited to φ, where φ is
  - either the **sum of angles** for antennae in the same node, or
  - the **maximum transmission angle** of the antennae.

The Setting

- Set of sensors represented as a set of points S in the 2D plane.
- Each sensor has k directional antennae.
- All antennae have the same transmission range r.
- Each antenna has a max transmission angle, forming a coverage sector up to distance r.
- Typically, we fix k and φ and try to minimize r for a given point set S.

Transmission Range

- $r_{(k,\varphi)-OPT}(S)$ denotes the optimal (shortest) range for which a solution exists.
- $r_{MST}(S)$ is the shortest range r such that UDG(S, r) is connected.
  - obviously, $r_{MST}(S) \leq r_{(k,\varphi)-OPT}(S)$
- As establishing $r_{(k,\varphi)-OPT}$ might be NP-hard, we will compare the radius r produced by a solution to r_{MST}.
  - for simplicity, we re-scale S to get $r_{MST} = 1$
  - later, we will discuss comparing to $r_{(k,\varphi)-OPT}$

Angle/Range Tradeoffs: Minimize Sum of Angles

Basic Observations

- Angle between (any two) incident edges of an MST is $\geq \pi/3$.
- For every point set there exists an MST of maximal degree 5.
- All angles incident to a vertex of degree 5 of the MST are between $\pi/3$ and $2\pi/3$ (included).
- **Observation:** with $k \geq 5$ antennae, each of spread 0, there exists a solution with range 1.
- **Main method:** Locally modify the MST, using various techniques when k is smaller than the degree of the node in the MST to (locally) ensure strong connectivity: Use
  1. antenna spread to cover several neighbors by one antenna,
  2. neighbour's antennae to locally ensure strong connectivity

Upper Bounds: Sum of Angles

#	Antennae Spread	Antennae Range	Paper
1	$0 \leq \varphi < \pi$	2	[4]
1	$\pi \leq \varphi < 8\pi/5$	$2\sin(\varphi/2)$	[2]
1	$8\pi/5 \leq \varphi$	1	[2]
2	$2\pi/3 \leq \varphi < \pi$	$2\cos(\varphi/4)$	[1]
2	$\pi \leq \varphi < 6\pi/5$	$2\sin(2\pi/9)$	[1]
2	$6\pi/5 \leq \varphi$	1	[1]
3	$4\pi/5 \leq \varphi$	1	[1]
4	$2\pi/5 \leq \varphi$	1	[1]

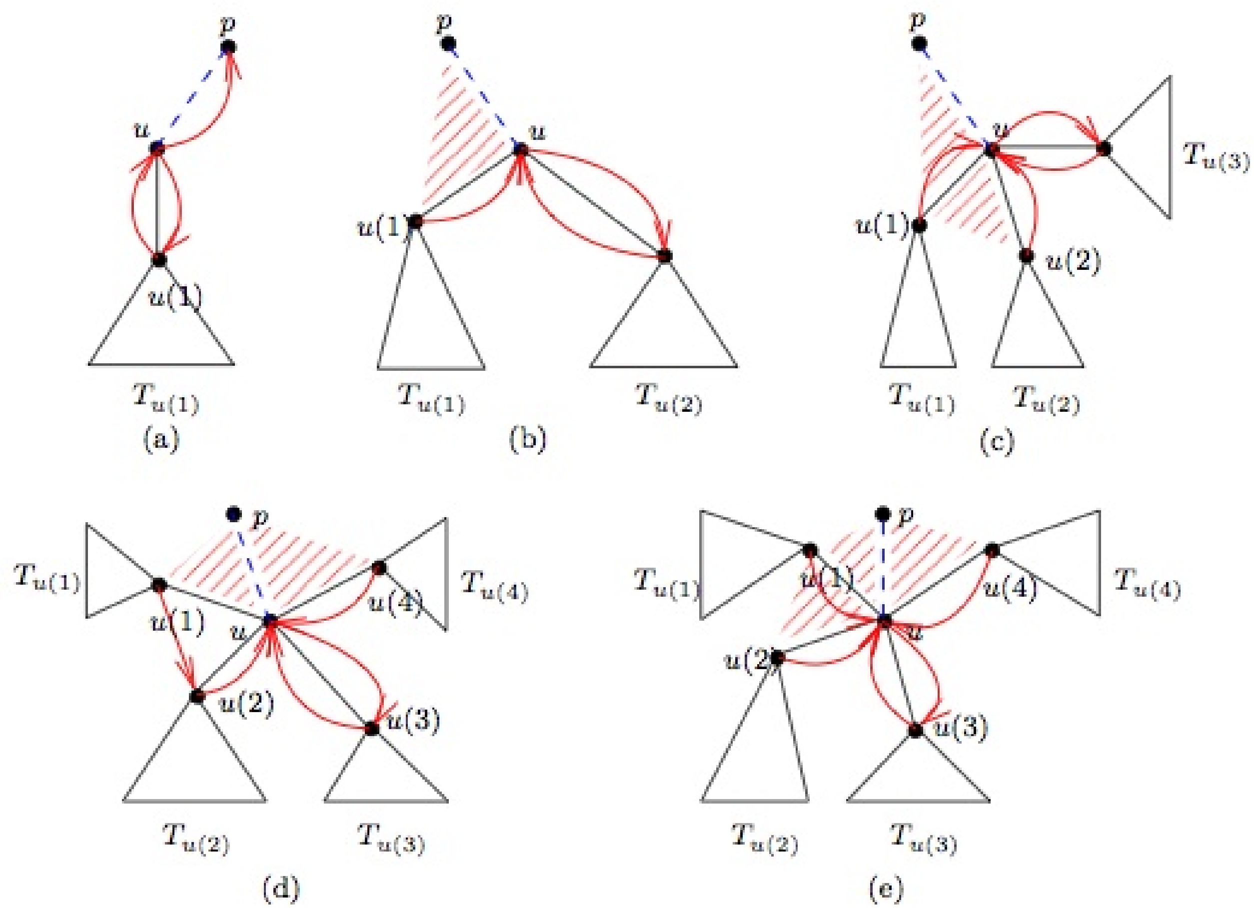
Antenna Range 1

- **Theorem.** For any $1 \leq k \leq 5$, there exists a solution with
  1. range 1, and
  2. sum of angles $\leq \frac{2(5-k)\pi}{5}$.
- Why?
- Here is the reason, briefly:
  - Consider the MST.
  - Take any vertex of degree 5 (other cases are similar).
  - Exclude k incident (consecutive) angles with sum $\leq 2k\pi/5$.
  - What is left can be covered with an antenna of angle $\leq \frac{2(5-k)\pi}{5}$ and $k-1$ antennae of angle 0 each.

Two Antennae, $\varphi \geq \pi$, Range $2\sin(2\pi/9)$

- A vertex p is a *nearby target vertex* to a vertex $v \in T$ if $d(v,p) \leq 2\sin(2\pi/9)$ and p is either a parent or a sibling of v in T.
- A subtree T_v of T is *nice* iff for any nearby target vertex p the antennae at vertices of T_v can be set up so that the resulting graph (over vertices of T_v) is strongly connected and p is covered by an antenna from v.
- **Theorem.** There is a way to set up 2 antennae per vertex, with antenna spread (i.e., sum of antenna angles) of π and range $2\sin(2\pi/9)$ in such a way that the resulting graph is strongly connected.
  - **Proof:** By proving that T_v is nice for all v, by induction on the depth of T_v.

Induction: Case Analysis on the Number of Children of u



The length $2\sin(2\pi/9)$ arises from the fact that
$\min\{\angle(u(1)uu(2)), \angle(u(2)uu(3)), \angle(u(3)u(4))\} \leq \frac{4\pi}{9}$

Angle/Range Tradeoffs: Minimize Max Range

Main Theorem (Upper Bound)

- Consider a set S of n sensors in the plane and suppose each sensor has k, $1 \leq k \leq 5$, directional antennae.
  - Then the antennae can be oriented at each sensor so that the resulting spanning graph is strongly connected and the range of each antenna is at most
$$2 \cdot \sin\left(\frac{\pi}{k+1}\right)$$
times the optimal.
  - Moreover, given a MST on the set of points the spanner can be constructed with additional $O(n)$ overhead.

Main Steps: Angle 0

- The more antennae per sensor the easier the proof.
- Algorithm is in three steps.
  1. 4 Antennae: Spread 0, Range $2\sin(\pi/5)$
  2. 3 Antennae: Spread 0, Range $2\sin(\pi/4)$
  3. 2 Antennae: Spread 0, Range $2\sin(\pi/3)$
- Details of complete algorithm too technical to present here!
- Lets outline the ideas for the proof of Item 1.

Main Idea: Angle 0

- **Idea:**
  The basic antenna orientation algorithm;
  - By induction on the depth of the MST T;
  - We avoid connecting child solutions to the parent, instead
    1. remove all leaves,
    2. apply induction hypothesis to the resulting tree,
    3. return back the leaves and show how to connect them to the original structure.
- **NB:**
  Since the spread is 0, a solution can be represented as a directed graph $\overrightarrow{G}$ with maximum out-degree k and edge lengths at most $2\sin(\frac{\pi}{k+1})$.

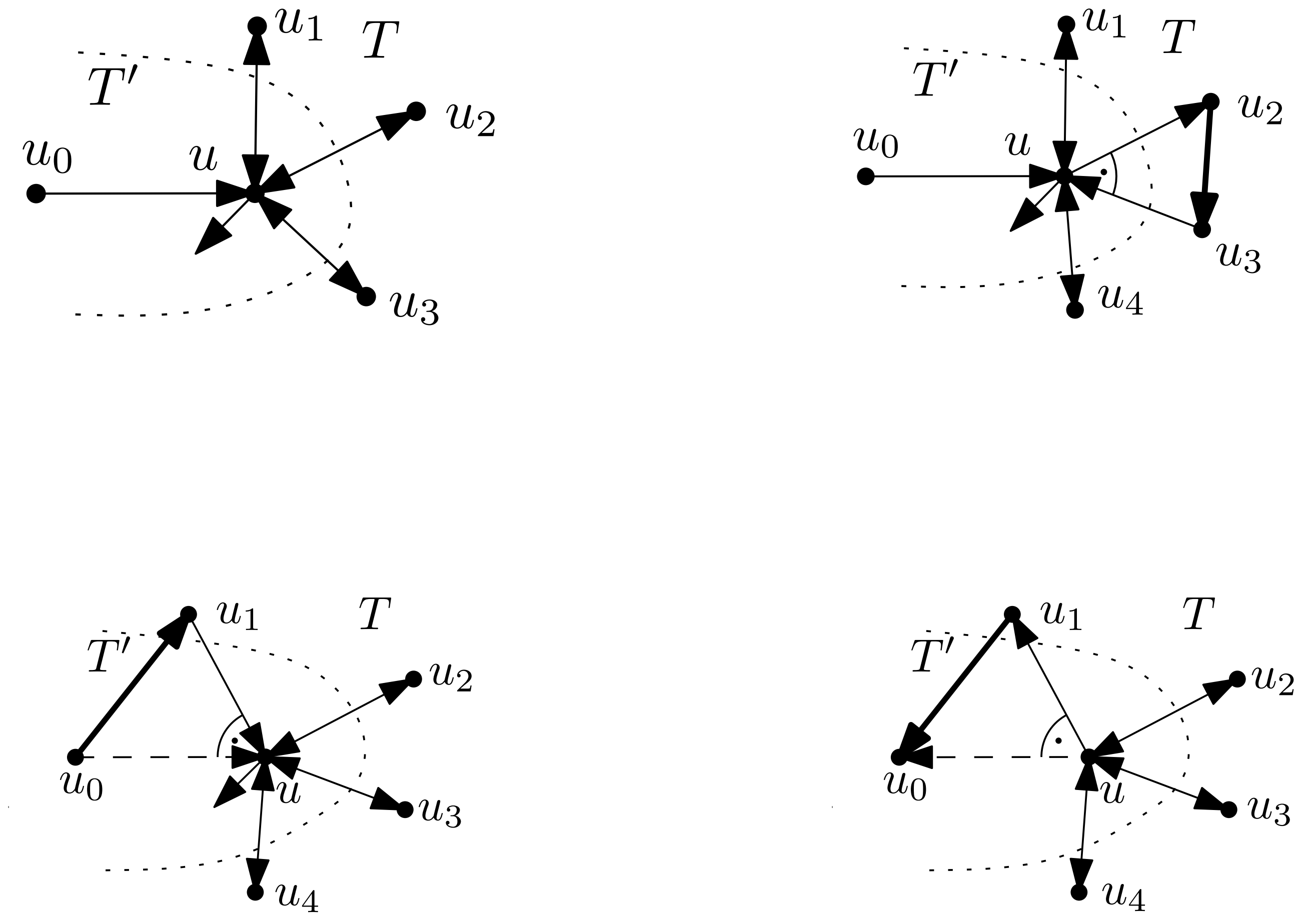
Example: 4 **Antennae, Spread** 0**, Range** $2\sin(\pi/5)$

Induction hypothesis: Let T be an MST of a point set of radius at most x. Then, there exists a solution $\overrightarrow{G}$ for T such that:

- the out-degree of u in $\overrightarrow{G}$ is one for each leaf u of T
- every edge of T incident to a leaf is contained in $\overrightarrow{G}$

Base step

Inductive Step: 4 antennae, spread 0



Summary of Complete Picture: Upper Bounds

#	Antennae Spread	Antennae Range	Paper
1	$0 \leq \varphi < \pi$	2	[4]
1	$\pi \leq \varphi < 8\pi/5$	$2\sin(\pi - \varphi/2)$	[2]
1	$8\pi/5 \leq \varphi$	1	[2]
2	$0 \leq \varphi < 2\pi/3$	$\sqrt{3}$	[3]
2	$2\pi/3 \leq \varphi < \pi$	$2\sin(\pi/2 - \varphi/4)$	[1]
2	$\pi \leq \varphi < 6\pi/5$	$2\sin(2\pi/9)$	[1]
2	$6\pi/5 \leq \varphi$	1	[1]
3	$0 \leq \varphi < 4\pi/5$	$\sqrt{2}$	[3]
3	$4\pi/5 \leq \varphi$	1	[1]
4	$0 \leq \varphi < 2\pi/5$	$2\sin(\pi/5)$	[3]
4	$2\pi/5 \leq \varphi$	1	[1]

Lower Bounds

Is the Result Optimal?

- Consider a regular $k+1$-star.
  - With angle less then $\frac{2\pi}{k+1}$, the central vertex cannot reach all leaves using k antennae, hence a leaf must connect to another leaf, using range at least $2\sin(\frac{\pi}{k+1})$.
  - Hence results for spread 0 are optimal ...
  - ... with respect to r_{MST}.
- But what about $r_{(k,\varphi)-OPT}$?
- In regular $k+1$-star also $r_{(k,\varphi)-OPT}$ is large!

Main Theorem (Lower Bound)

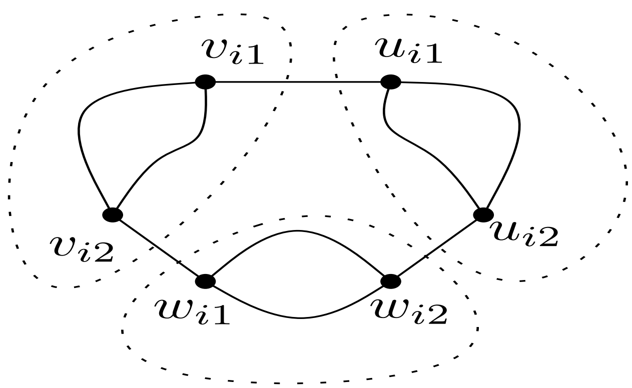
- For $k = 2$ antennae.
  - Let x and α be the solutions of equations

    $$x = 2\sin(\alpha) = 1 + 2\cos(2\alpha)$$

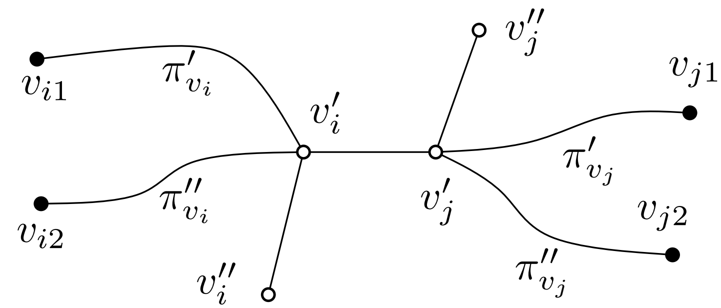
    (**Note:** $x \approx 1.30, \alpha \approx 0.45\pi$.)
  - If the angular sum of the antennae is less then α then it is NP-hard to approximate the optimal radius to within a factor of x.
- The proof is by reduction from the problem of finding Hamiltonian cycles in degree three planar graphs.

Key Gadgets

Take a degree three planar graph $G = (V, E)$ and replace each vertex v_i by a vertex-graph (meta-vertex) G_{v_i} shown in Figure 1a. Furthermore, replace each edge $e = \langle v_i, v_j \rangle$ of G by an edge-graph (meta-edge) G_e shown in Figure 1b.



(a) Vertex graph (The dotted ovals delimit the three parts.)



(b) Edge graph (The connecting vertices are black.)

Figure 1: Meta-vertex and meta-edge for the NP completeness proof

Embed Resulting Graph in the Plane:

1) Distance (in the embedding) between neighbours in G' is ≤ 1, 2) the distance between non-neighbours in G' is $\geq x$, and 3) the smallest angle between incident edges in G' is $\geq \alpha$.

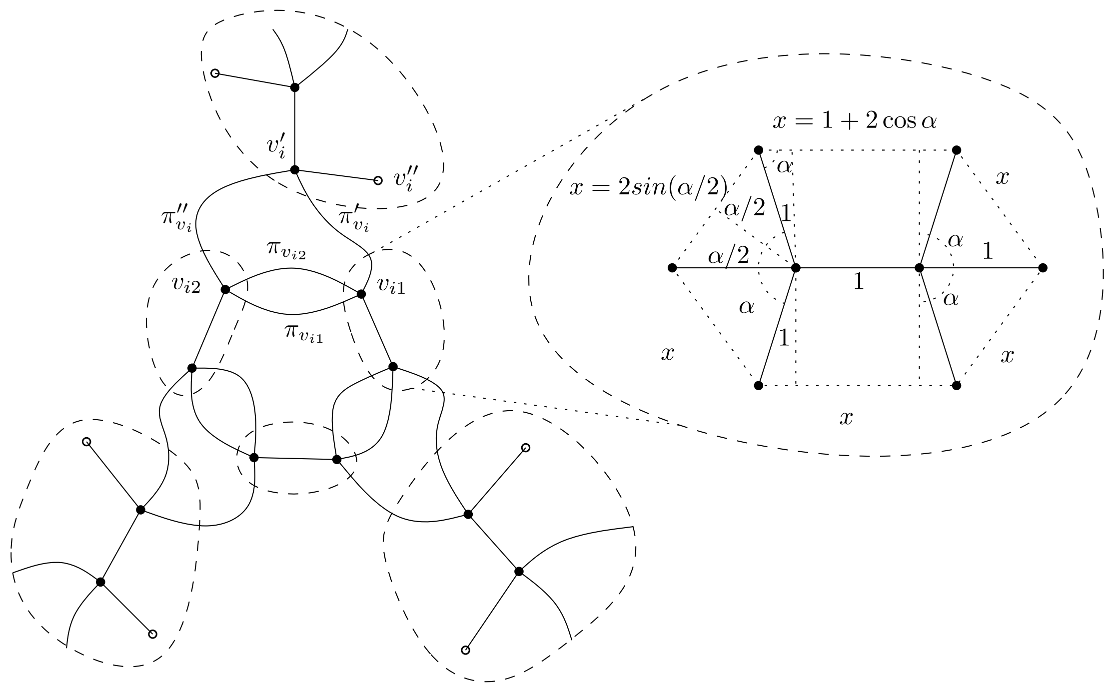


Figure 2: Connecting meta-edges with meta-vertices (The dashed ovals show the places where embedding is constrained.)

Key Observations

- Each meta-vertex must have at least incoming and one outgoing meta-edge
- Each meta-vertex can have at most one outgoing meta-edge
- Hence each meta-vertex has exactly one outgoing and one incoming meta-edge

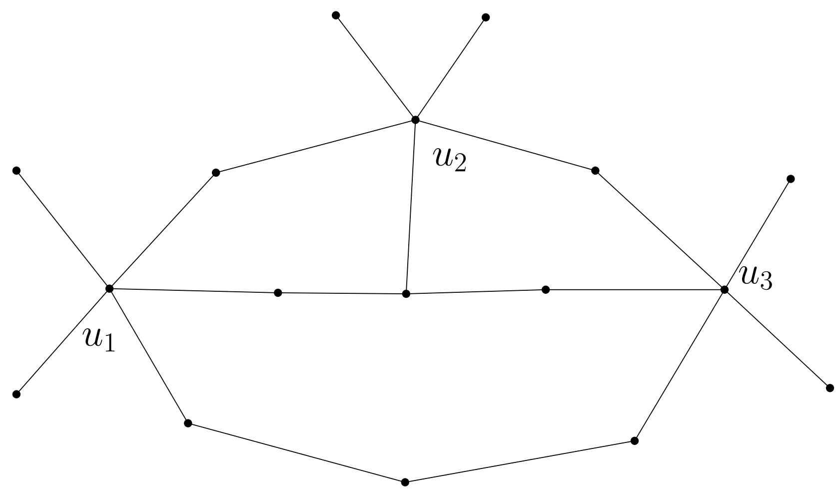
What we know so far

Out degree	Lower Bound	Upper Bound	Approx. Ratio	Complexity
4	r_{MST}	$2\sin(\pi/5)r_{MST}$	$2\sin(\pi/5)$	Polynomial
3	r_{MST}	$2\sin(\pi/4)r_{MST}$	$\sqrt{2}$	Polynomial
2	r_{MST}	$2\sin(\pi/3)r_{MST}$	$\sqrt{3}$	Polynomial
2	-	-	≤ 1.3	NP-Complete

Toughness of Antennae and Robust Range (Cases $k = 3, 4$)

Lower Bounds for $k = 3$ and $k = 4$: Main Idea

- For a pointset P: How robust is the radius r to point deletions?
- For $S \subseteq P$, let $r_k(S) :=$ "smallest radius r s.t, $UDG(P \setminus S, r)$ does not contain a $(k+1)|S|$ connected components".
- Obviously, $r_k(S) \leq r_{(k,0)-OPT}(S)$. Is $r_k(S) = r_{(k,0)-OPT}(S)$?
- $r_3(S) < r_{(3,0)-OPT}(S)$! E.g., take $S = \{u_1, u_2, u_3\}$.



- How about $r_4(S) = r_{(4,0)-OPT}(S)$?

Tougness of UDGs

- The concept of toughness of a graph as a measure of graph connectivity has been extensively studied in the literature.
- Intuitively, graph toughness measures the resilience of the graph to fragmentation after subgraph removal.
- A graph G is t-tough if $|S| \geq t\omega(G \setminus S)$, for every subset S of the vertex set of G with $\omega(G \setminus S) > 1$.
- The toughness of G, denoted $\tau(G)$, is the maximum value of t for which G is t-tough (taking $\tau(K_n) = \infty$, for all $n \geq 1$).
- We are interested in the toughness of UDGs over a given point set P, and in particular how does the toughness of $U(P, r)$ depends on the radius r.

New Concept: Robust Range

Definition 1 *[Strong and Weak t-robustness for UDG radius] Let P be a set of points in the plane.*

1. *A subset $S \subseteq P$ is called t-tough if $\omega(U(P \setminus S; r)) \leq |S|/t$. Similarly, a point u is called t-tough if the singleton $\{u\}$ is t-tough.*
2. *The strong t-robustness of the set of points P, denoted by $\sigma_t(P)$, is the infimum taken over all radii $r > 0$ such that for all $S \subseteq P$, the set S is t-tough for the radius r.*
3. *The weak t-robustness of the set of points P, denoted by $\alpha_t(P)$, is the infimum taken over all radii $r > 0$ such that for all $u \in P$, the point u is t-tough for the radius r.*

Main Result

- **Theorem.** We have
  1. $\sigma_{1/k}(P) \leq r_k(P)$, for all k.
  2. For any set P of points, $\alpha_{1/4}(P) = \sigma_{1/4}(P)$.
  3. For every point of P, weak $1/i$-robustness, for $1 \leq i < 5$, can be computed in time $O(|P| \log |P|)$.
- In particular,
  1. the optimal range for the 4 antennae orientation problem (strong connectivity) can be solved in $O(n \log n)$ time,
  2. a $2 \sin(2\pi/9)$ approximation to the optimal range for the 3 antennae orientation problem can be solved in $O(n \log n)$ time.

Summary of Results

Out degree	Lower Bound	Upper Bound	Approx. Ratio	Complexity
4	$\sigma_{1/4}$	$\alpha_{1/4}$	1	$O(n \log n)$
3	$\sigma_{1/3}$	$2\sin(2\pi/9)\alpha_{1/3}$	$\leq 2\sin(2\pi/9)$	$O(n \log n)$

Conclusions/Open Problems

- There are still gaps between the lower and upper bounds, especially for non-zero φ
- The x and φ in the NP-hardness results might possibly be improved
- Consider different model variants
  - directional receivers
  - temporal aspects (antennae steering, ...)
- and different problems...

Minimum Number of Antennae

Antenna Orientation Problem

- Given a connected network formed by a set of sensors with omnidirectional antennae and an angle $\varphi \geq 0$.

  Compute the minimum number of arcs in the network in such a way that the resulting network is strongly connected and the stretch factor does not depend on the size of the network.

- Two variants:
  - Notice that you must respect the underlying network.
  - Can consider angle/range tradeoffs.

Orienting Edges of Undirected Graph with Original Range

- Orient every edge in both directions
  - stretch factor 1 but $2|E|$ arcs
- Orient edges along a Hamiltonian cycle (if it exists)
  - $|V|$ arcs but unbounded stretch factor
- **(Roberts, 1935)** Strong Orientation Procedure
  1. label vertices $1..n$ according to DFT T
  2. orient ij as $i \rightarrow j$ iff $ij \in T$ and $i < j$
  3. orient ij as $i \rightarrow j$ iff $ij \notin T$ and $i > j$
- **(Robbins, 1939)** G has a strong orientation iff it is connected and 2-edge connected.
- **(Nash-Williams, 1960)** Every G has an orientation D so that $\forall u, v \in V$, $\lambda_D(u, v) \geq \lfloor \frac{1}{2} \lambda_G(u, v) \rfloor$, where $\lambda(u, v)$ is the number of $u - v$ paths

Strong Orientation Algorithms

Can give algorithms to strongly orient a given (planr) graph $G = (V, E)$ for[a]

- More than $|E|$ edges
- Exactly $|E|$ edges
- Less than $|E|$ edges

[a] E. Kranakis, O. Morales Ponce, L. Stacho. Strong Orientations of Planar Graphs with Bounded Stretch Factor. In proceedings of 17th SIROCCO (Colloquium on Structural Information and Communication Complexity) 2010. LNCS 6058 Springer.

Orientation Algorithms (More than $|E|$ Edges)

- **Theorem.** Let $G = (V, E)$ be a plane 2-edge connected graph with a face λ-coloring. Then it has a strong orientation with at most

$$\left(2 - \frac{4\lambda - 6}{\lambda(\lambda - 1)}\right) \cdot |E|$$

arcs and stretch factor at most $\phi(G) - 1$, where $\phi(G) =$ max number of edges of a face of G.

Orientation Algorithms (Exactly $|E|$ Edges)

- **Theorem.** Let $G = (V, E)$ be a plane 2-edge connected graph with a face λ-coloring. Then it has a strong orientation with exactly $|E|$ arcs and stretch factor at most

$$(\phi(G) - 1)^{\lceil \frac{\lambda+1}{2} \rceil}.$$

Orientation Algorithms (Less than $|E|$ Edges)

- **Theorem.** Let $G = (V, E)$ be a plane 3-edge connected graph. Then it has a strong orientation with at most

$$\left(1 - \frac{k}{10(k+1)}\right) \cdot |E|$$

arcs and stretch factor at most $\phi(G)^2 \cdot (\phi(G) - 1)^{2k+4}$, for any $k \geq 1$.

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Neighbor Discovery with Directional Antennae

Outline

- Introduction
- Deterministic Algorithms
- Randomized Algorithms
- Conclusions

Introduction

Goals

- Investigate the complexity of discovering neighbors in a setting of rotating antennae:
  - What knowledge is required?
  - How long does it take?
  - What protocols are possible?
  - How does it compare to the omnidirectional setting?

Communication Models with Directional Antennae

- (O, O) model: two sensors can communicate if they are within transmission range of each other,
- (D, O) (respectively, (O, D)) model: the sender (respectively, receiver) must turn its antenna so as to reach its neighbor, and
- (D, D) model: both sender and receiver must direct their antennae towards each other at the same time.
  This is the model we look at!

Neighbor Discovery Process

- Usually entails the exchange of identities (e.g., MAC addresses) between two adjacent nodes.
- It will be sufficient to assume that this is a one step process whereby one sensor sends its identity and the other acknowledges by sending back its own.
- We assume that the sensors have distinct identities but their corresponding locations (i.e., (x, y)-coordinates) in the plane are not known to each other.
- There is a vertex coloring $\chi : V \to \{0, 1, \ldots, c-1\}$

Parameters of the Directional Antennae Model

- For simplicity, for each node u assume an angle (or beam width) $\phi_u = \frac{2\pi}{k_u}$, for some integer k_u.

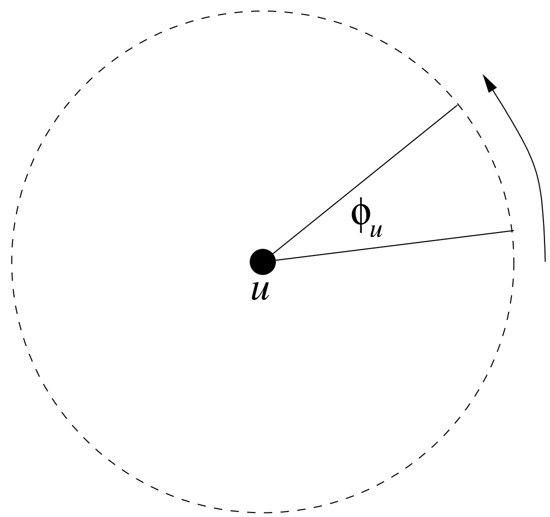


Figure 3: An antenna at u rotating counter-clockwise.

- Sensor network is synchronous

Deterministic Algorithms

Deterministic Algorithms

- Lower Bound: $\Omega(k_u k_v)$ time steps, for two sensors u, v within communication range of each other.
- Upper Bounds

Antenna at u	Knowledge	Running Time	Theorems
$2\pi/k$	Identical	$O(k^{c-1})$	Theorem 1
$2\pi/k$	Identical	$O(k(c \ln c)^3)$	Theorem 2

Table 1: Theorems and running times of deterministic algorithms.

- Recall our basic assumption that there is a coloring $\chi : V \to \{0, 1, \ldots, c-1\}$ of the vertices of the sensor network using c colors.

Lower Bound

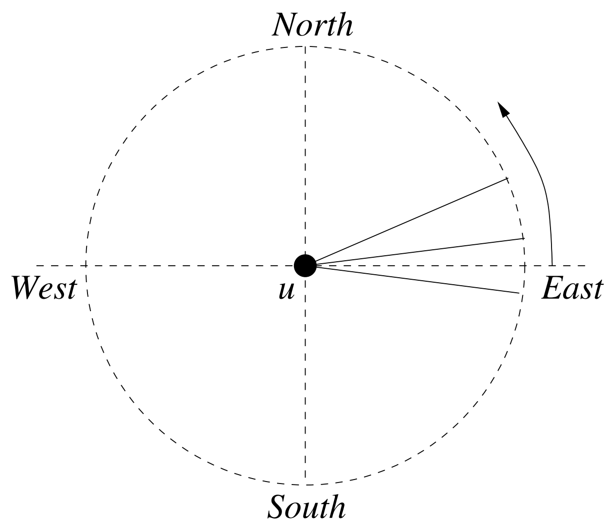
- Consider two sensors u, v within communication range of each other and respective antenna beam widths $\frac{2\pi}{k_u}$ and $\frac{2\pi}{k_v}$, respectively. If the sensors do not know each other's location then any algorithm for solving the neighbor discovery problem in the (D, D) communication model requires at least $\Omega(k_u k_v)$ time steps.
- This is because, for a successful communication to occur each sensor must be within the beam of the other sensor's antenna at the same time. Since the sensors do not know each other's location they must attempt transmissions in all their respective sectors.

Communicating Position

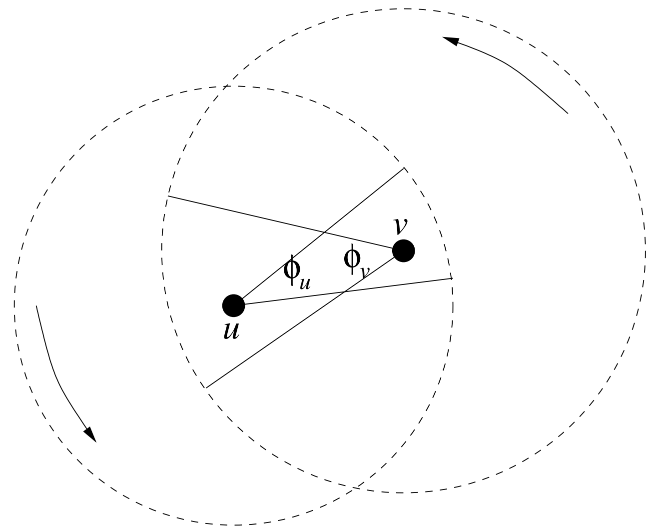
1. Sensors must be within range of each other.

Figure 4: Directional antennae in communicating position.

(a) An antenna at u with sectors counted counter-clockwise.



(b) Neighbor discovery for sensors u, v.



2. Directional antennae must be facing each other.

Communication Failure of Deterministic Algorithms

- Not every deterministic algorithm would work!
- Example: Sensor u employs delay $d_u = 2$ and sensor v delay $d_v = 1$, under which sensors with directional antennae will never be able to communicate as illustrated in Figure 5.

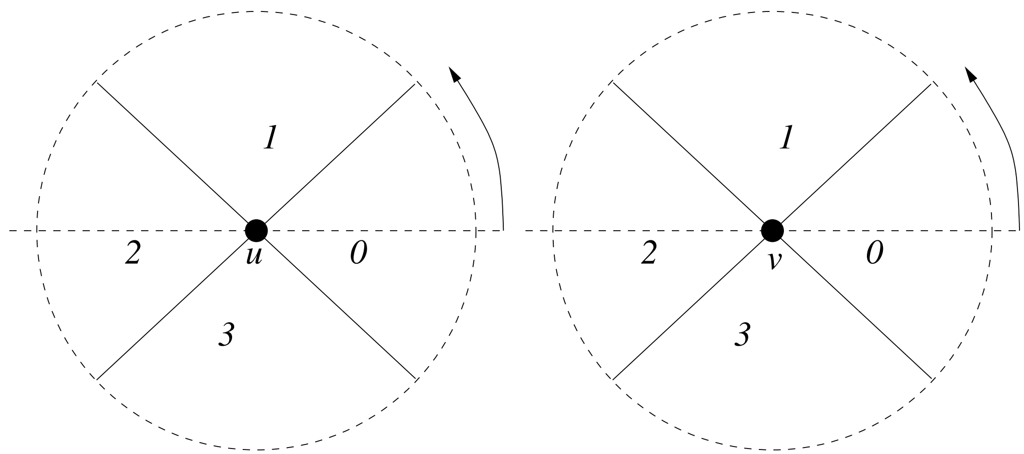


Figure 5: Neighbor discovery for sensors u, v is not possible.

(Basic) Antenna Rotation Algorithm (with Delay)

- For each sensor u, let d_u be an integer delay parameter and k be defined so that $\phi = \frac{2\pi}{k}$

Algorithm 1: Antenna Rotation Algorithm $ARA(d_u, k_u)$

Start at a given orientation;
while *true* **do**
    for $i \leftarrow 0$ **to** $d_u - 1$ **do**
        `//For` d_u `steps stay in chosen sector`
        Send message to neighbor(s);
        Listen for messages from neighbor(s) (if any);
    Rotate antenna beam one sector counter-clockwise;
    `//rotate by an angle equal to` ϕ

A Simple Choice of Delays

- A simple theorem is the following:
- **Theorem 1** *Consider a set of sensors in the plane with identical antenna beam widths equal to* $\phi = \frac{2\pi}{k}$*. For each sensor* u *let the delay be defined by* $d_u := k^{\chi(u)}$*. If each sensor* u *executes algorithm* $ARA(d_u, k)$ *then every sensor in the network will discover all its neighbors in at most* k^{c-1} *time steps.*
- Running time can be improved by choosing delays appropriately!

Improving on Delay

Theorem 2 *Consider a set of sensors in the plane such that the antenna beam width of sensor u is equal to $\phi = \frac{2\pi}{k}$. Assume the sensor network is synchronous. Suppose that the delays d_u at the nodes are chosen so that*

1. $\gcd(k, d_u) = 1$, *for all u, and*
2. *if u, v are adjacent then* $\gcd(d_u, d_v) = 1$.

If each sensor u executes algorithm $ARA(d_u, k)$ then every sensor in the network will discover all its neighbors in at most $O\left((k(\max_u d_u)^3\right)$ time steps.

In addition, the delays d_u can be chosen so that every sensor in the network will discover all its neighbors in at most $O(k(c \log c)^3$ time steps.

In particular, this is at most $O\left((c \ln c)^3\right)$ time steps, if $k \in O(1)$.

Proof (1/2)

- Without loss of generality assume that
  1. u and v are in horizontal position and sensor u is to the left of sensor v, and
  2. that both antennae orientations are initially set to *East*.
- u, v can communicate when v's antenna is facing *West* which is sector $\lfloor \frac{k}{2} \rfloor$.
- Since $\gcd(d_u, d_v) = 1$, by Euclid's algorithm there exist integers $0 < a_u < d_u, 0 < a_v < d_v$ such that
$$a_u d_u = a_v d_v + 1. \tag{2}$$
- Lets look at sensor u first. After $d_u k$ steps sensor u will be in its starting position and, clearly, the same applies for any time duration that is a multiple of $d_u k$. Thus sensor u is in its initial position (facing East) at time $j a_u d_u k$, for any $j > 0$.

Proof (2/2)

- Multiply both sides of Equation $a_u d_u = a_v d_v + 1$ by jk to obtain $ja_u d_u k = ja_v d_v k + jk$
- So at time $t = ja_u d_u k$ sensor u is facing *East*. If there is a j such that $jk = \lfloor \frac{k}{2} \rfloor d_v + r$ for $0 \leq r < d_v$, then sensor v is facing West and therefore the sensors u, v can discover each other.
- Find a j such that,

$$jk \leq \left\lfloor \frac{k}{2} \right\rfloor d_v < jk + k \tag{3}$$

  which means that $jk + k = \lfloor \frac{k}{2} \rfloor d_v + r$, with $r \leq k < d_v$.
- A simple modification of the proof will prove the result when the two sensors are not necessarily on a horizontal line.
- # of rotations required is $ja_u d_u k$, where j satisfies Inequality (3).

Randomized Algorithms

Randomized Neighbor Discovery Algorithms

- **Upper Bounds**

Antenna at u	Knowledge	Running Time	Theorems
$2\pi/k$	Identical	$kn^{O(1)}$	Theorem 3
$2\pi/k$	Identical	$O(k^2 \log n)$	Theorem 4
$2\pi/k_u$	$\max_u k_u \leq k$	$O(k^4 \log n)$	Theorem 5

Table 2: Theorems and running times of randomized algorithms.

Randomized Neighbor Discovery Algorithm (1/4)

Algorithm 2: Randomized Antenna Rotation Algorithm $RARA(d_u, k)$

1 Select $d_u \leftarrow RANDOMPRIME(k..R)$;

2 Execute $ARA(d_u, k)$;

Theorem 3 *Consider a set of sensors in the plane such that the antenna beam width of sensor u is equal to $\phi = \frac{2\pi}{k}$. Assume the sensor network is synchronous. If each sensor u executes algorithm $RARA(k; R)$, where $R = n^{O(1)}$ and n is an upper bound on the number of sensors, then every sensor in the network will discover all its neighbors in at most $kn^{O(1)}$ expected time steps, with high probability.*

Randomized Neighbor Discovery Algorithm (2/4)

- For every node u, let $N(u)$ denote the neighborhood of u and $\deg(u)$ the degree of u.
- Let $D = \max_u \deg(u)$ denote the maximum degree of a node of the sensor network.
- By the prime number theorem, the number of primes $\leq R$ and $> k$ is approximately equal to $\frac{R}{\ln R} - \frac{k}{\ln k}$ and therefore the probability that the primes chosen by two adjacent nodes, say u and v, are different is $1 - \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}}$.
- Let E_u be the event that the prime chosen at u is different from all the primes chosen by its neighbors.

Randomized Neighbor Discovery Algorithm (3/4)

- It is easily seen that

$$\begin{aligned}
\Pr[E_u] &= 1 - \Pr[\neg E_u] \\
&= 1 - \Pr\left[\exists v \in N(u)(d_u = d_v)\right] \\
&\geq 1 - \sum_{v \in N(u)} \Pr\left[d_u = d_v\right] \\
&\approx 1 - \deg(u) \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}} \\
&\geq 1 - D \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}}.
\end{aligned}$$

Randomized Neighbor Discovery Algorithm (4/4)

- Similarly, we can prove that

$$
\begin{aligned}
\Pr\left[\bigcap_u E_u\right] &= 1 - \Pr\left[\bigcup_u \neg E_u\right] \\
&\geq 1 - \sum_u \Pr[\neg E_u] \\
&\geq 1 - nD\frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}} \\
&\geq 1 - \frac{1}{n}.
\end{aligned}
$$

- By choosing R in $n^{O(1)}$ and recalling that $D \leq n$ we see that all the primes chosen by all the nodes in the network are pairwise distinct, with high probability.

Additional Algorithms

- **Theorem 4** *Consider a set of n sensors in the plane with identical antenna beam width equal to $\phi = \frac{2\pi}{k}$. Assume the sensor network is synchronous. There is an algorithm so that every sensor in the network will discover all its neighbors in at most $O(k^2 \log n)$ expected time steps, with high probability.*

- **Theorem 5** *Consider a set of n sensors in the plane such that sensor u has antenna beam width equal to $\phi_u = \frac{2\pi}{k_u}$. Assume the sensor network is synchronous and that an upper bound k is known to all sensors so that $\max_u k_u \leq k$. There is an algorithm so that every sensor in the network will discover all its neighbors in at most $O(k^4 \log n)$ expected time steps, with high probability.*

Conclusions

- **Interesting Problem:** Efficiency of broadcasting
  1. in the single channel UDG model, i.e., there is a single send/receive channel and multiple transmissions on the same node produce packet collisions, and
  2. a link between two sensors u, v exists if and only if $d(u, v) \leq 1$.
- If broadcasting time with omnidirectional antennae without collisions is B then the result of Theorem 3 indicates that broadcasting in the directional antennae model can be accomplished in time $O(B(c \ln c)^3)$, where c is the number of colors of a vertex coloring of the sensor network. The main question arising is whether we can improve on this time bound when using directional antennae.

Additional Work

- J. Du, E. Kranakis, O. Morales Ponce, S. Rajsbaum, Neighbor Discovery in a Sensor Network with Directional Antennae. In proceedings of Algosensors 2011, Saarbruecken, Germany, September 08-09, 2011.

Wormhole Attacks in Sensor Networks

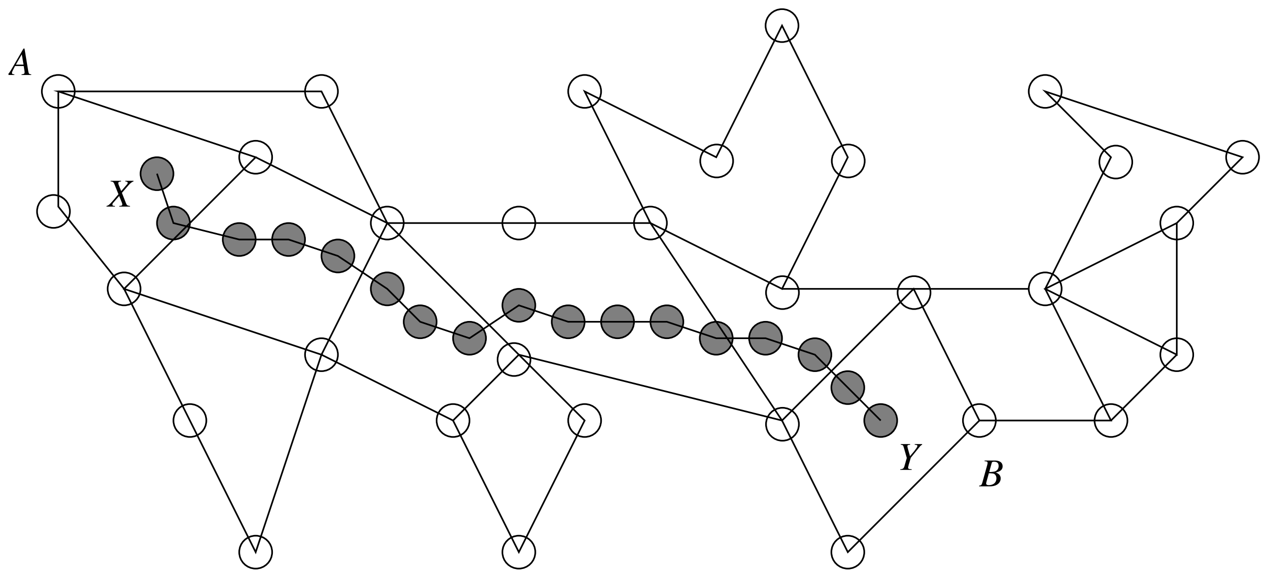
Introduction

- Wormhole Attacks
- Detecting Wormholes
- Preventing Wormholes with Directional Antennae
- Protocols

Wormhole Attacks

Wormhole Attacks

- A and B are not neighbors.
- The attacker can make A and B believe they are neighbors.



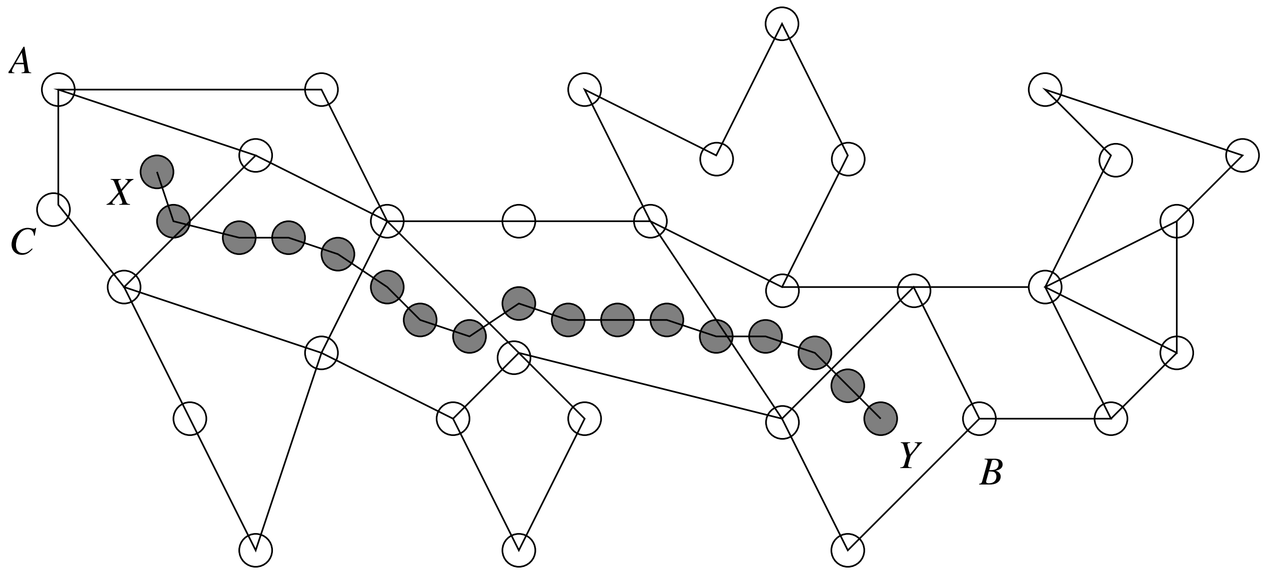
- The attacker replays packets received by X at node Y, and vice versa.

Wormhole Attacks

- In a wormhole attack, an attacker forwards packets through a high quality out-of-band link and replays those packets at another location in the network.
- The attacker replays packets received by X at node Y, and vice versa.
- If it would normally take several hops for a packet to traverse from a location near X to a location near Y, packets transmitted near X traveling through the wormhole will arrive at Y before packets traveling through multiple hops in the network.
- The attacker can make A and B believe they are neighbors by forwarding routing messages, and then selectively drop data messages to disrupt communications between A and B.

Impact on Routing Protocols: Beyond the Neighborhood

- For most routing protocols, the attack has impact on nodes beyond the wormhole endpoints' neighborhoods.



Impact on Routing Protocols: One-Hop Tunneling

- Node A will advertise a one-hop path to B so that C will direct packets towards B through A.
- For example, in on-demand routing protocols (DSR and AODV) or secure on-demand routing protocols (SEAD, Ariadne, SRP), the wormhole attack can be mounted by tunneling ROUTE REQUEST messages directly to nodes near the destination node.
- Since the ROUTE REQUEST message is tunneled through high quality channel, it arrives earlier than other requests.

Impact on Routing Protocols: Sinkholes and More

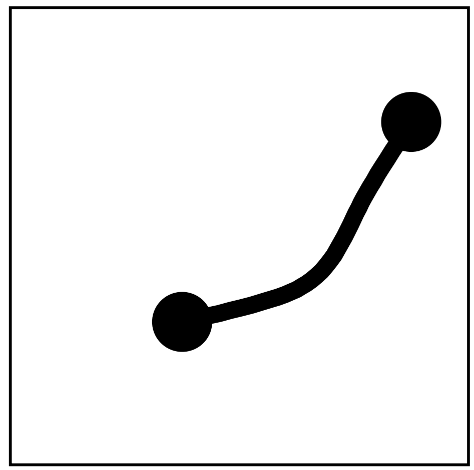
- Wormhole attacks prevent other routes from being discovered.
- The wormhole will have full control of the route.
- The attacker can discard all messages to create a denial-of-service attack, or more subtly, selectively discard certain messages to alter the function of the network.
- An attacker with a suitable wormhole can easily create a sinkhole that attracts (but does not forward) packets to many destinations.
- An intelligent attacker may be able to selectively forward messages to enable other attacks.

Impact on Sensor Networks: Disrupting Strategically

- An intelligent attacker may be able to place wormhole endpoints at particular locations.
- Strategically placed wormhole endpoints can disrupt nearly all communications to or from a certain node and all other nodes in the network.
- In sensor network applications, where most communications are directed from sensor nodes to a common base station, wormhole attacks can be particularly devastating.

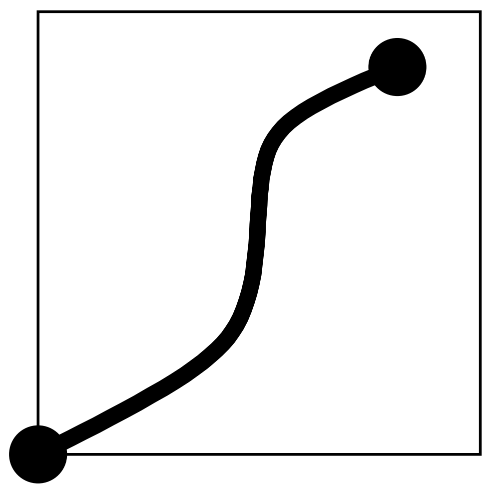
Impact on Sensor Networks: Location Matters

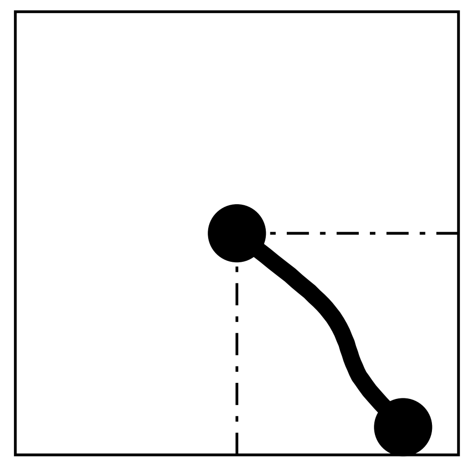
- In sensor networks traffic is directed from sensors to a base station.
- A wormhole can disrupt traffic depending on its location



Impact on Sensor Networks: Location Matters

- If the base station is at the corner of the network, a wormhole with one endpoint near the base station and the other endpoint one hop away (from base station) will be able to attract nearly all traffic from sensor nodes to the base station.
- If the base station is at the center of the network, a single wormhole will be able to attract traffic from a quadrant of the network.

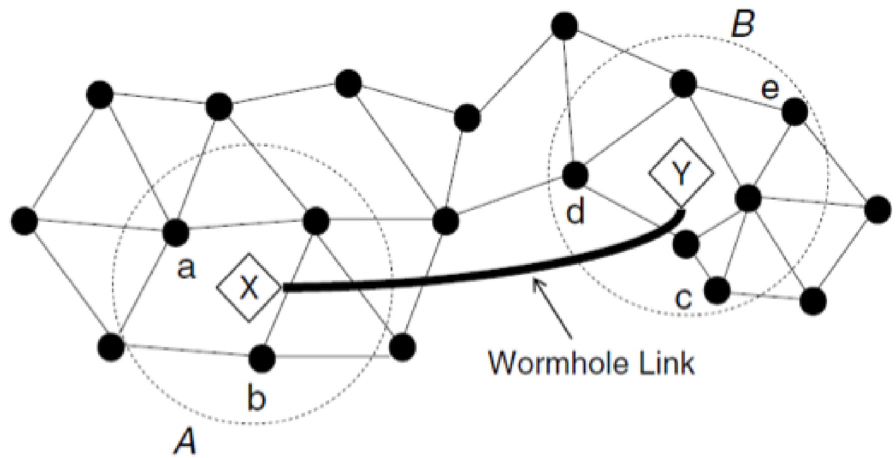




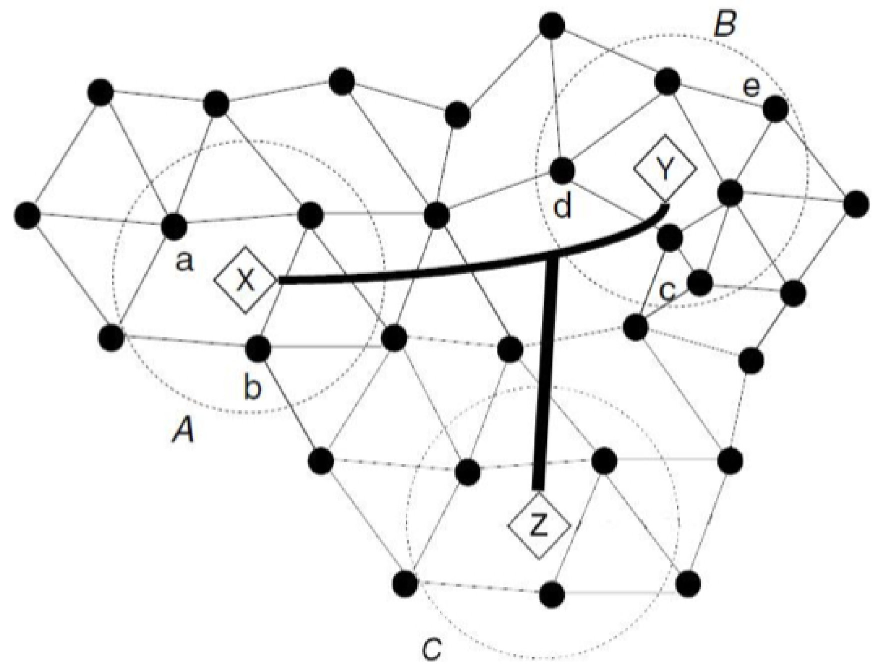
Detecting Wormholes

Wormhole: Example

- Let the network be represented by a graph, G.

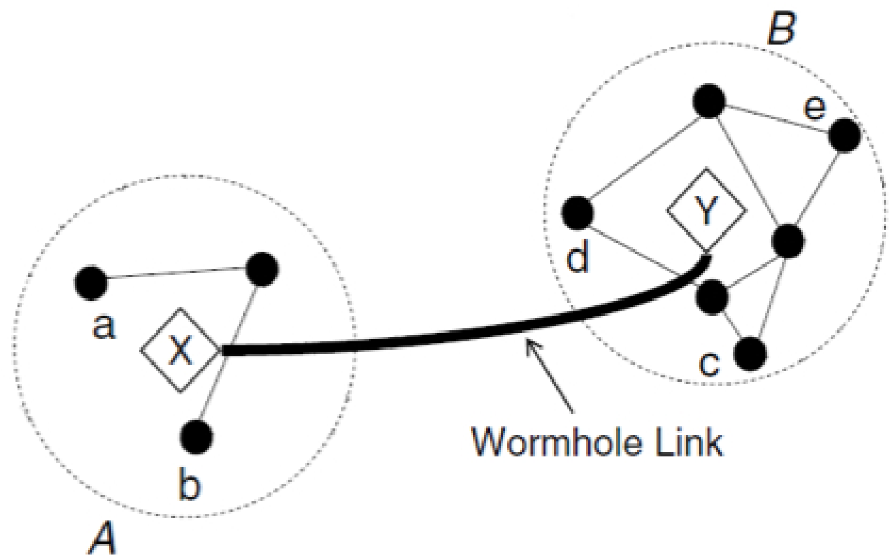


Multiple Wormholes: Example



Wormhole Subgraph

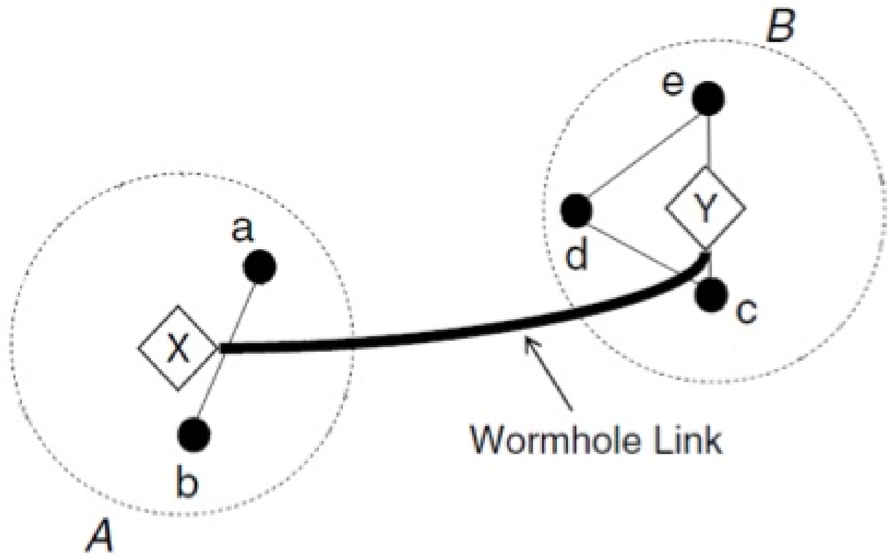
- Now consider the subgraph of G containing only nodes connected via a wormhole.



- Label 5 nodes in this subgraph $\{a, b, c, d, e\}$.

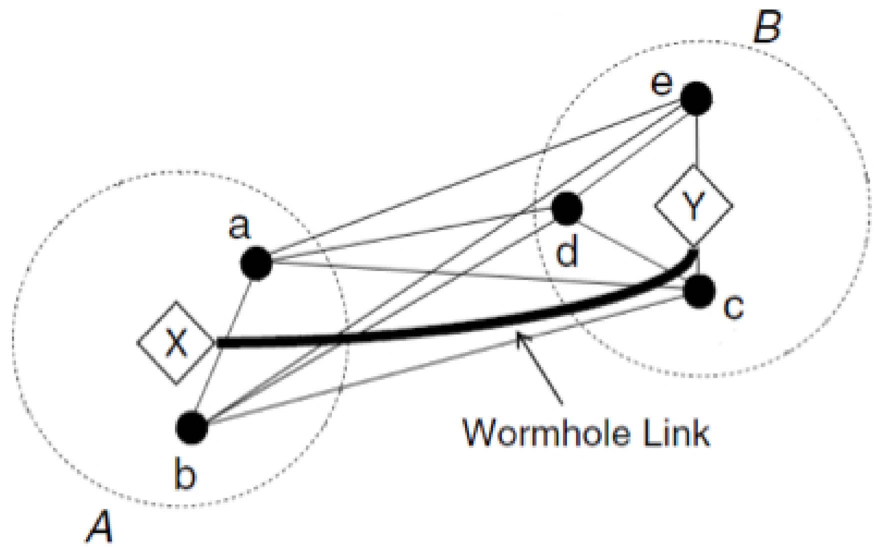
Edge Contraction

- Now contract the edges of the subgraph so that only the labelled nodes remain



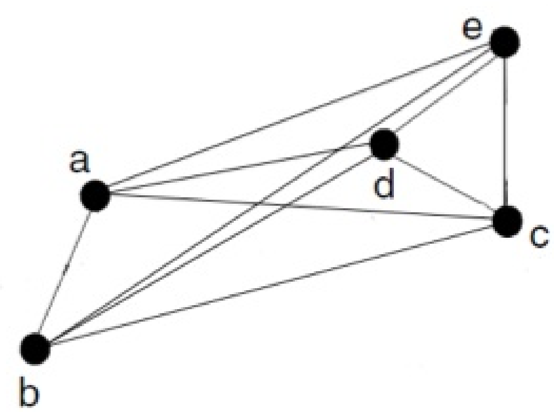
K_5

- If we now add the edges due to the wormhole connection we get K_5.



Contradicting Planarity

- But by Kuratowski's Theorem, this means that G is not planar.



- Therefore the existence of the wormhole has made the connectivity graph non-planar (and the routing algorithm, which requires planarity, will no longer work).

Altering the Topology: Impossible Graphs

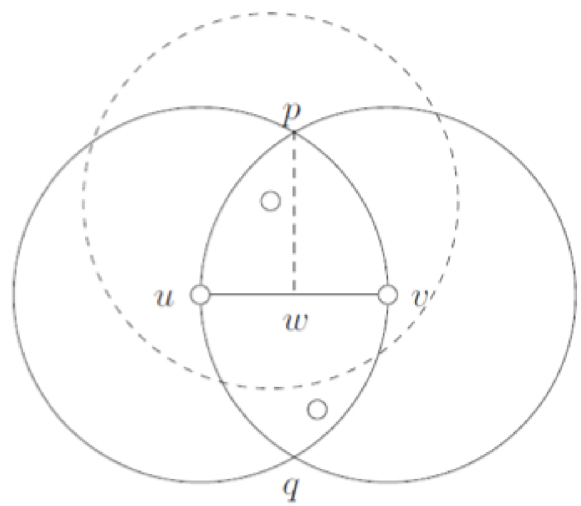
- Core problem in discovering wormholes: identifying neighbours who would not be if the wormhole did not exist.
- Two main approaches to accomplish this task:
  1. those that attempt to make the determination based solely on connection information, and
  2. those using in part location awareness of the nodes (even if only within a neighbourhood) and determine if arrangement of nodes is possible.

Test for Impossible Graphs

- Perform neighbour discovery (ND) and run a planarization algorithm.
- If we discover there are wormhole links and removed these links, the graph could become disconnected.
- This is the reason wormhole discovery is performed during ND, and we will refer to such an algorithm as secure neighbour discovery (SND).

Hop-1 Test

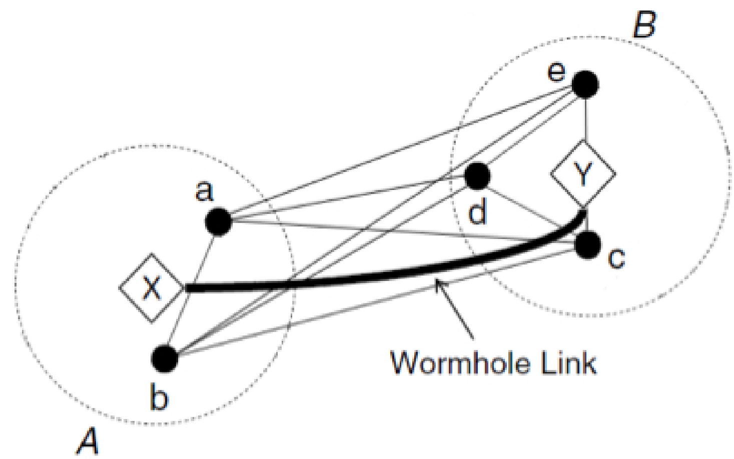
- Consider the process of ND between two neighbours u and v



- If u and v are neighbours, and not connected by a wormhole, they can have at most 2 independent neighbours in common

An Impossible Graph

- In the presence of a wormhole, two nodes (a and b if they were connected) can have three independent neighbours (c, d and e).



Hop-k Tests, $k \geq 2$

- Just because two nodes do not have 3 or more independent neighbours in common does not mean that they are not being affected by a wormhole.
- Therefore if no wormhole is detected within a 1-hop neighbourhood, we must examine the 2-hop neighbourhood and determine how many 2-hop independent neighbours these nodes have in common.
- Such tests depend on the density of the wireless network and may not always be feasible. [a]

[a] Detecting wormhole attacks in wireless networks using connectivity information R. Maheshwari, J. Gao, Samir Das, INFOCOM 2007

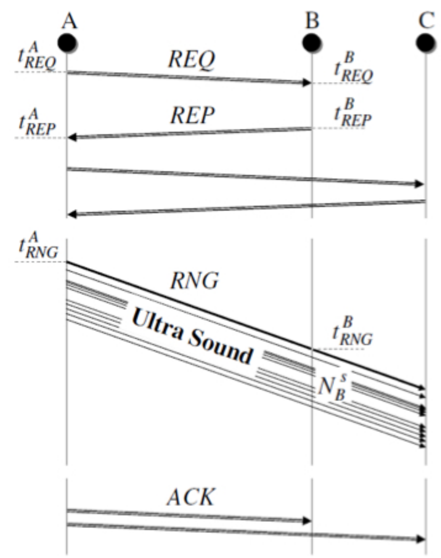
Using Time Difference of Arrival

- The SND proposed in [a] requires that nodes are equipped with microsecond precise clock (which is likely to be required in the node anyways) and an ultrasonic (UF) transceiver, which is not an especially expensive or power-consuming device.
- The algorithm proposed is localized to a 1-hop neighbourhood.
- It is assumed that all node share keys so that each node can identify and authenticate itself.

[a] A Practical Secure Neighbor Verification Protocol for Wireless Sensor Networks Reza Shokri, Marcin Poturalski, Gael Ravot, Panos Papadimitratos, and Jean-Pierre Hubaux Proceedings of the second ACM conference on Wireless network security, 2009

Neighbor Discovery (1/2)

- Each node (A) will broadcast a probe message (REQ).
- Neighbours of A will respond to this message and identify and authenticate themselves.



- This stage is used to eliminate attacking nodes (but will not prevent a wormhole attack).

Neighbor Discovery (2/2)

- After a suitable amount of time, A will broadcast a UF message and then begin sending messages to each verified neighbour indicating the time of events as recorded by A ($t^A_{REQ}, t^A_{REP}, t^A_{RNG}$) and encrypt this message specifically for the recipient.
- Each message will contain different t^A_{REP} values depending on when the response arrived from the specific neighbour. With this information, the neighbours of A can estimate their distance from A by the time difference of arrival. Since all the neighbours of A will be doing the same, A will eventually have an estimate for its distance to all its neighbours.
- Once it has all the estimates, it broadcasts its 1-hop neighbourhood–including its distance estimates–to all its neighbours.

Three Tests: A Validates Link (A, B) for each Neighbor B

1. **Symmetry:** A confirms that $d(A, B) = d(B, A)$.
2. **Maximum Range Test:** Assuming nodes know their range R. If a neighbour lies beyond this range, the link must be across a wormhole, so $d(A, B) \leq R$
3. **Quadrilateral Test:** For any link (A, B) find two nodes D and C, such that A, B, C, D form a 4-clique. If there is no wormhole, then is should be possible to arrange all four nodes so that they form a quadrilateral.

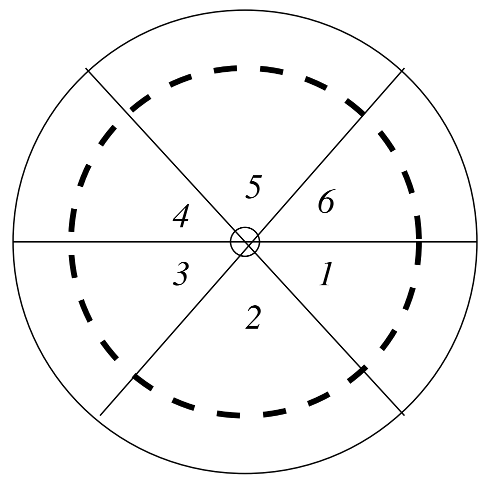
Preventing Wormholes: Directional Antenna Model

Directional Sensors and Zones

- The range of an antenna is divided into n zones.
- Each zone has a conical radiation pattern, spanning an angle of $2\pi/n$ radians.
- The zones are fixed with non-overlapping beam directions, so that the n zones may collectively cover the entire plane.
- When a node is idle, it listens to the carrier in omni mode.
- When it receives a message, it determines the zone on which the received signal power is maximal. It then uses that zone to communicate with the sender.

Directional Sensors and Zones

The zones are numbered 1 to 6 oriented clockwise starting with zone 1 facing east.



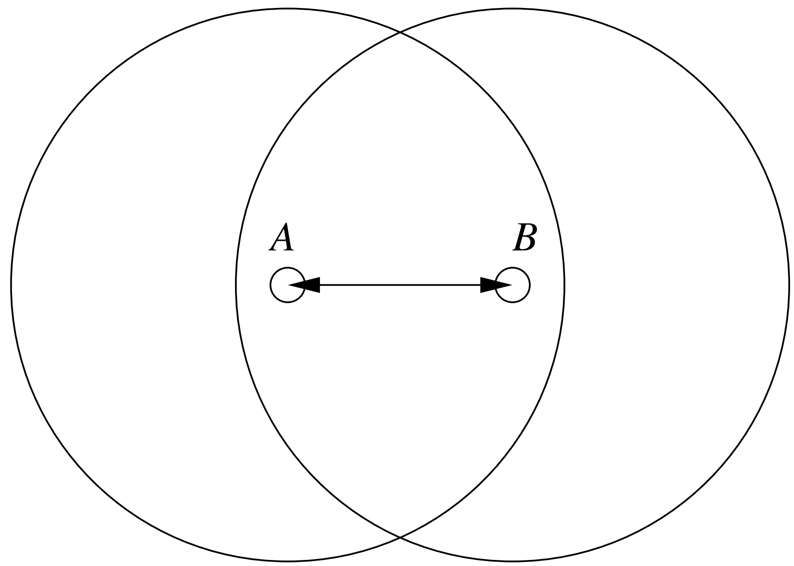
This orientation is established with respect to the earth's meridian regardless of a node's physical orientation. This is achieved in modern antennas with the aid of a magnetic needle that remains collinear to the earth's magnetic field. It ensures that a particular zone always faces the same direction.

Sending/Receiving

- **Receiving:**
  a node can receive messages from any direction.
- **Sending:**
  a node can work in omni or directional mode.
  - In omni mode signals are received with a gain G^o, while in directional mode with a gain of G^d.
  - Since a node in directional mode can transmit over a longer distance, $G^d > G^o$.

Assumptions on Security

- All communication channels are bidirectional: if A can hear B, then B can hear A.



- A mechanism is available to establish secure links between all pairs of nodes and that all critical messages are encrypted.
- Sensor network must be "relatively" dense.

Notations

- $A, B, C, \ldots$: Legitimate nodes
- X, Y: Wormhole endpoints
- R: Nonce
- $E_{KAB}(M)$: M encrypted with key shared by nodes A and B
- $zone$: The directional element, which ranges from 1 to 6.
- $\overline{zone}$: The opposite directional element. For example, if $zone = 1$ then $\overline{zone} = 4$.
- $zone(A, B)$: Zone in which node A hears node B
- $neighbors(A, zone)$: Nodes within one (directional distance) hop in direction zone of node A.

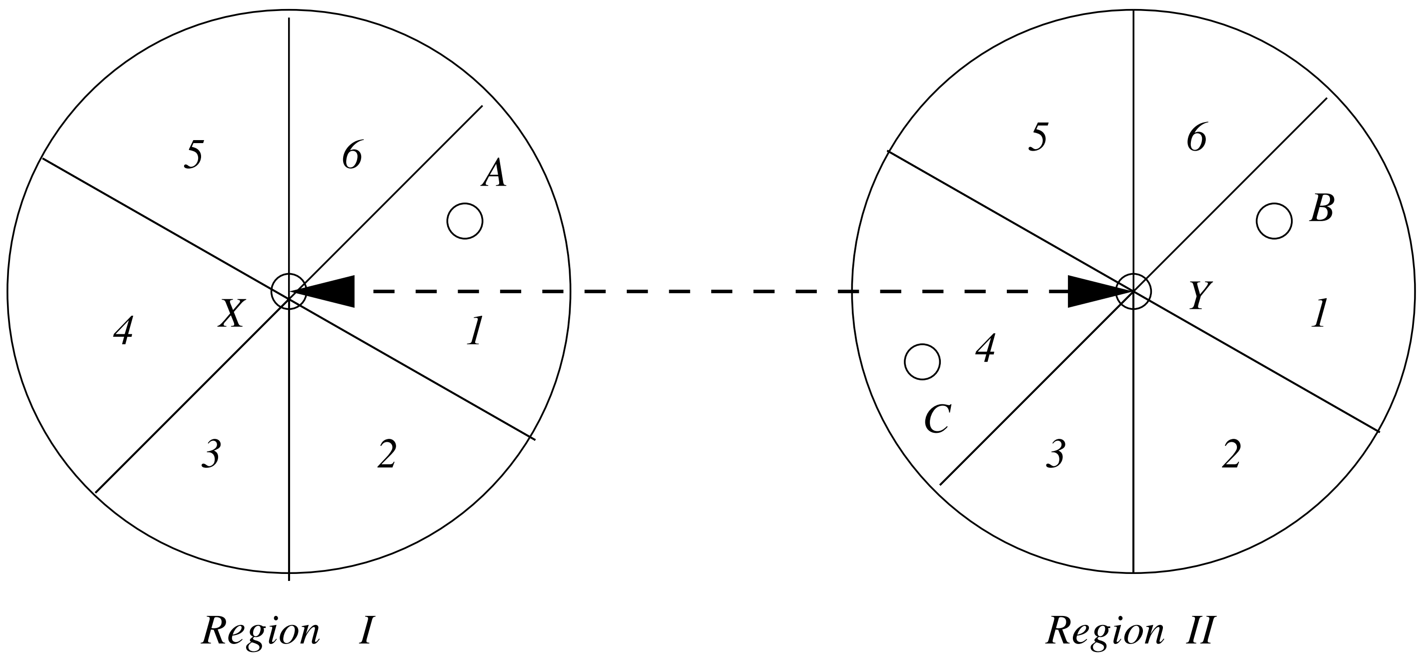
Protocols

Protocol 1: Directional Neighbor Discovery

- 1. $A \to Region$: HELLO and ID_A.
- 2. $N \to A : ID_N|E_{KNA}(ID_A|R|zone(N, A))$.
  All nodes that hear the HELLO message send their node ID and an encrypted message to the announcer. The encrypted message contains the announcer's ID, a random challenge nonce, and the zone in which the message was received.
- 3. $A \to N$: R.
  A decrypts message and verifies that it contains its node ID. It verifies $zone(A, N) = \overline{zone}(N, A)$. If correct, it adds the sending neighbor to its neighbor set for $zone(A, N)$. If message was not received in the appropriate zone, it is ignored. Otherwise, the announcer transmits the decrypted challenge nonce to the sending neighbor. Upon receiving the correct nonce, the neighbor inserts the announcer into its neighbor set.

Wormhole Vulnerability of Protocol 1

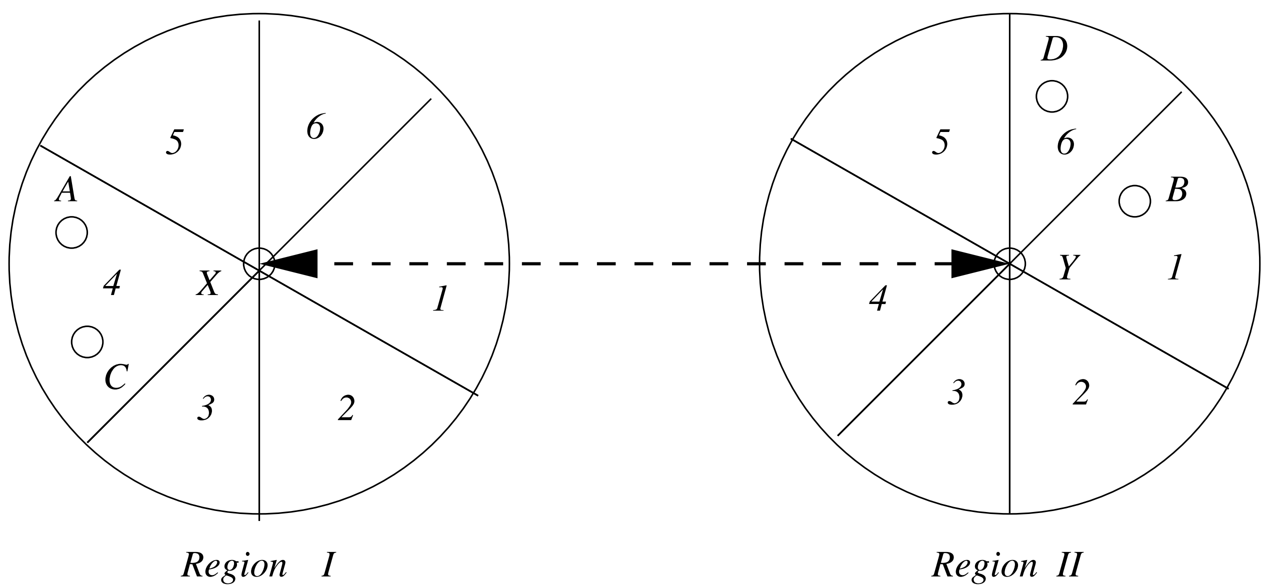
An attacker with a wormhole can establish a false distant neighbor.



The adversary establishes a wormhole between X and Y, and can trick A and C into accepting each other as neighbors by forwarding messages since they are in opposite zones relative to the respective wormhole endpoints.

Further Problems with Protocol 1

B will hear A and C from the west through the wormhole ($zone(B, A) = zone(B, C) = 4$), and C will hear A directly from the east ($zone(A, C) = \overline{zone}(C, A) = 1$) and C will hear B from the west through the wormhole ($zone(C, B) = \overline{zone}(B, C) = 4$).



Mitigating Wormhole Attacks

If nodes cooperate with their neighbors they can prevent wormholes since the attacker will only be able to convince nodes in particular regions that they are neighbors.

Assume the adversary has one transceiver at each end of the wormhole.

An adversary can only trick nodes that are in opposite directions from the wormhole endpoints into accepting each other as neighbors.

Hence, nodes in other locations can establish the announcer's legitimacy.

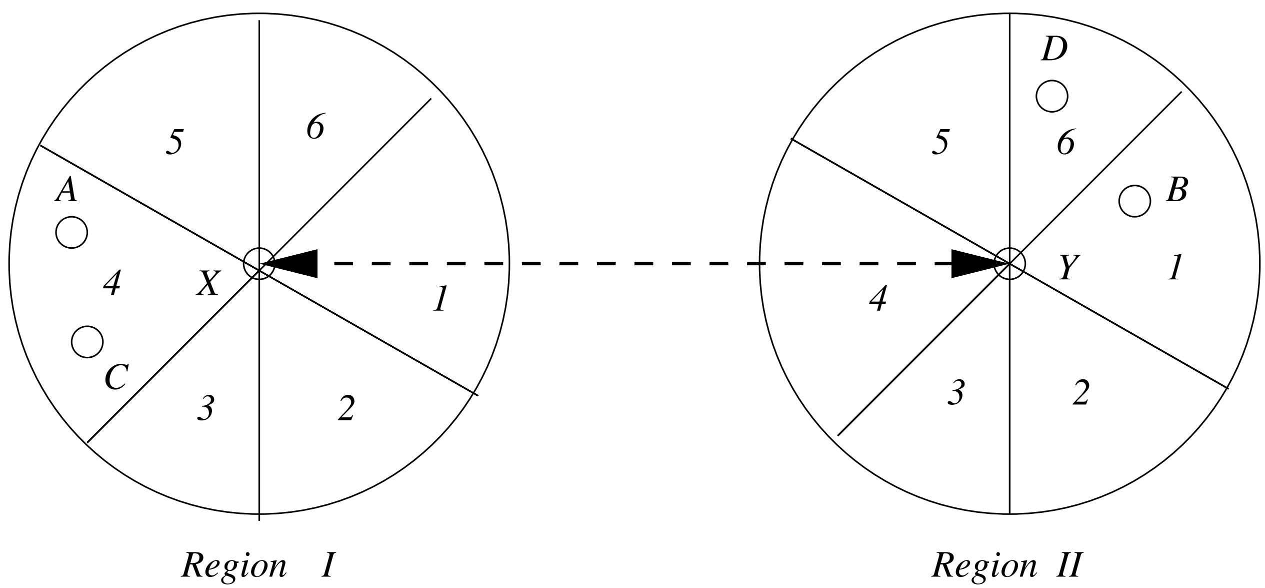
Such nodes are called verifiers.

Introducing Verifiers

How do we prevent verifiers from acting through the wormhole?

Node C cannot act as a verifier for the link AB since the wormhole attacker could make a node appear on the other end of the wormhole.

Node D could act as a verifier, since it satisfies the verifier properties.



Verifiers

A valid verifier V for the link $A \leftrightarrow B$ must satisfy the following properties:

1. $zone(B, A) \neq zone(B, V)$.
   Node B hears V in a different zone from node A, hence it knows A and V are in different locations, and both cannot be coming through a single wormhole endpoint.

2. $zone(B, A) \neq zone(V, A)$.
   Node B and V hear node A from different directions. A wormhole can deceive nodes in only one direction. So if both B and V are directionally consistent with A in different directions ($zone(B, A) = \overline{zone}(A, B)$ and zone $(V, A) = \overline{zone}(A, V)$), then they know A is not being retransmitted through a wormhole.

Protocol 2: Verified Neighbor Discovery

First three steps 1-3 are exactly as in Protocol 1.

- 4. $N \rightarrow Region$: $INQUIRY|ID_N|ID_A|zone(N, A)$
  All neighbor nodes that hear the HELLO message broadcast an inquiry in directions except for the received direction and opposite direction.

  So, if N received the announcement in zone 1, it will send inquiries to find verifiers to zones 2, 3, 5 and 6.

  The message includes $zone(N, A)$, so prospective verifiers can determine if they satisfy the verification properties by having heard A in a different zone.

Protocol 2: Verified Neighbor Discovery

- 5. $V \to N$: $ID_V | E_{KNV}(ID_A | zone(V, N))$
  Nodes that receive the inquiry and satisfy the verification properties respond with an encrypted message.

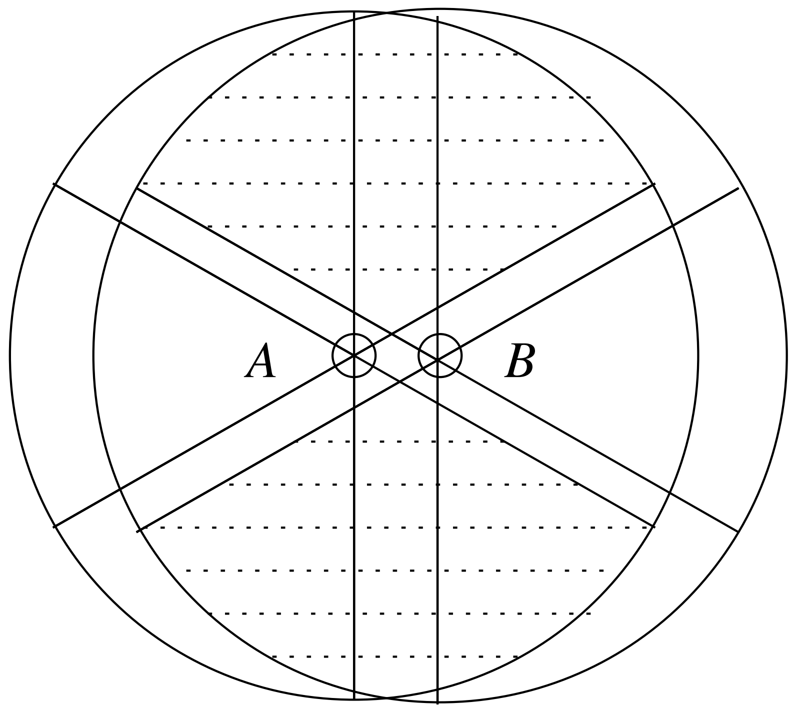
  This message confirms that the verifier heard the announcement in a different zone from N and has completed steps 1-3 for the protocol to authenticate A and its relative position.

  To continue the protocol, N must receive at least one verifier response. If it does, it accepts A as a neighbor, and sends a message to A:

- 6. $N \to A$: $ID_N | E_{KAN}(ID_A | ACCEPT)$
  After receiving the acceptance messages, the announcer adds N to its neighbor set.

Verifier Region

The shaded area is the verifier region of nodes A and B in verified neighbor discovery protocol.



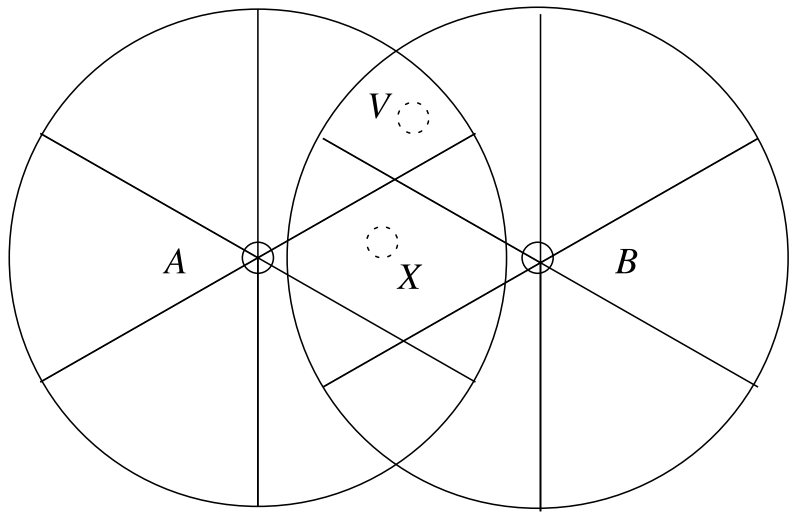
If there is a node in the shaded region, it can act as a verifier for A and B.

And Now the Density!

- Now you can see why you need the sensor network to be dense.
- It is required that with high probability there is a verifier node in the shaded region so as to enable A and B to have a successful protocol verification.
- The shaded region determines a given a area. The probability must be sufficiently high that sensors lie within this region so as to act as verifiers!
- The verifier region may still exist when two nodes are slightly out of radio range, and a smart adversary can use this to make them to be neighbors.

Worawannotai Attack: Wormhole Vulnerability of Protocol 2

Node B is located just beyond the transmission range of node A.

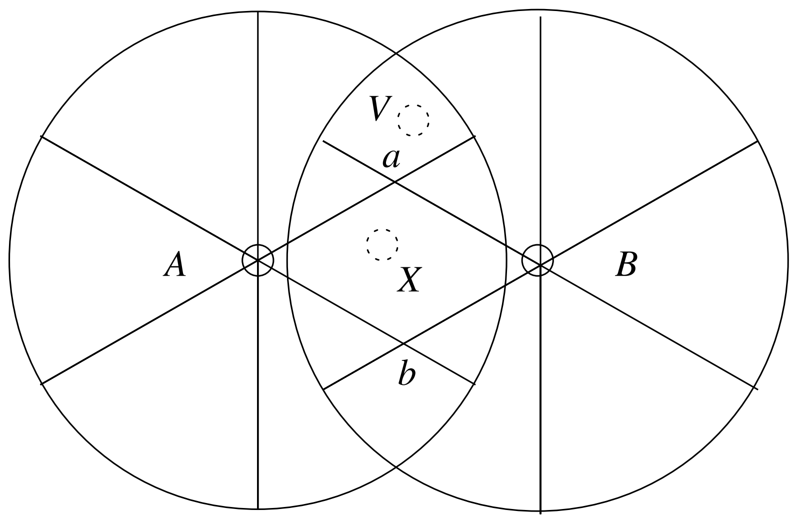


If there is a valid verifier in those areas, the attacker can just put one node in between A and B (node X) and use it to listen to and retransmit messages between A and B.

Nodes A and B will mistakenly confirm they are neighbors using verifier V, but the attacker will have control over all messages between A and B.

Preventing the Worawannotai Attack

There are two areas (a, b) that could have valid verifier for this protocol. If there is a valid verifier in those areas, the attacker can just put one node in between A and B (node X) and use it to listen to and retransmit messages between A and B.



A and B mistakenly confirm they are neighbors using verifier V, but the attacker will have control over all messages between A and B.

Protocol 3: Strict Verification Rules

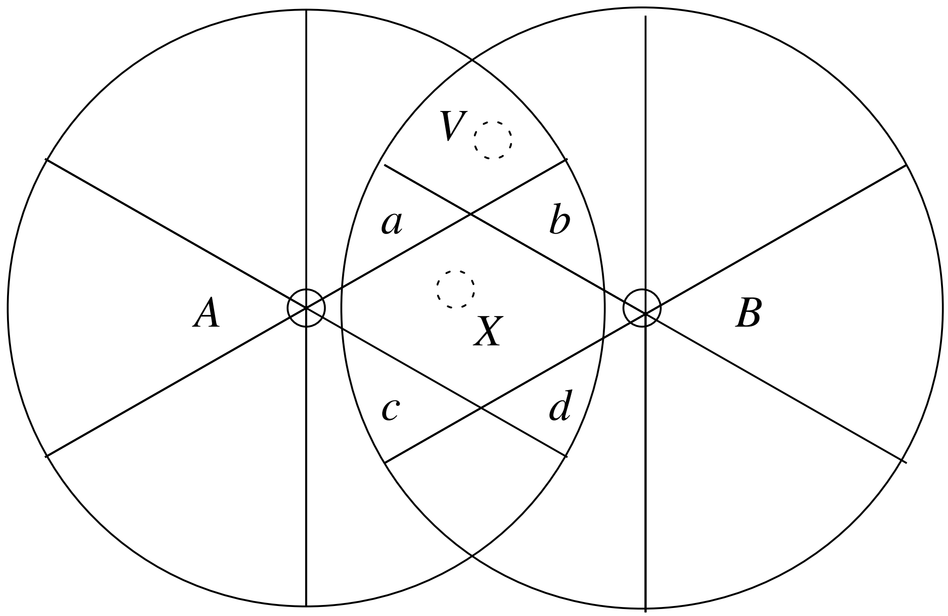
In the strict protocol, a valid verifier V for the link $A \leftrightarrow B$ must satisfy three properties:

1. $zone(B, A) \neq zone(B, V)$.
2. $zone(B, A) \neq zone(V, A)$.
3. $zone(B, V)$ cannot be both adjacent to $zone(B, A)$ and adjacent to $zone(V, A)$.

The first two conditions are the same as previous protocol, and they guarantee that the adversary cannot replay the confirmation message from verifiers. The third condition ensures that the verifier region is empty when two nodes are out of radio range, so the adversary cannot use this to conduct Worawannotai attack.

Protocol 3: Strict Neighbor Discovery

The verifier region determined by the previous three rules is depicted by the four regions a, b, c, d.



These areas are the verifier region's of node A and B in strict neighbor discovery protocol

Some Ideas on Coverage and Routing

Outline

- Coverage
  - Static case
  - Dynamic case
- Routing
  - Stretch factor

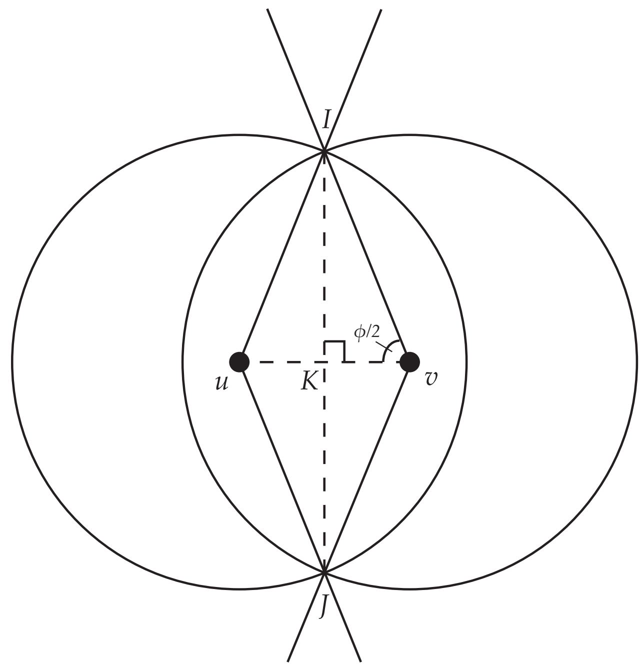
Coverage: Static Case

Outline

- How do you replace omnidirectional antennae with directional antennae?
- What are the range/angle/coverage tradeoffs?

From Omnidirectional to Directional Antennae (1/4)

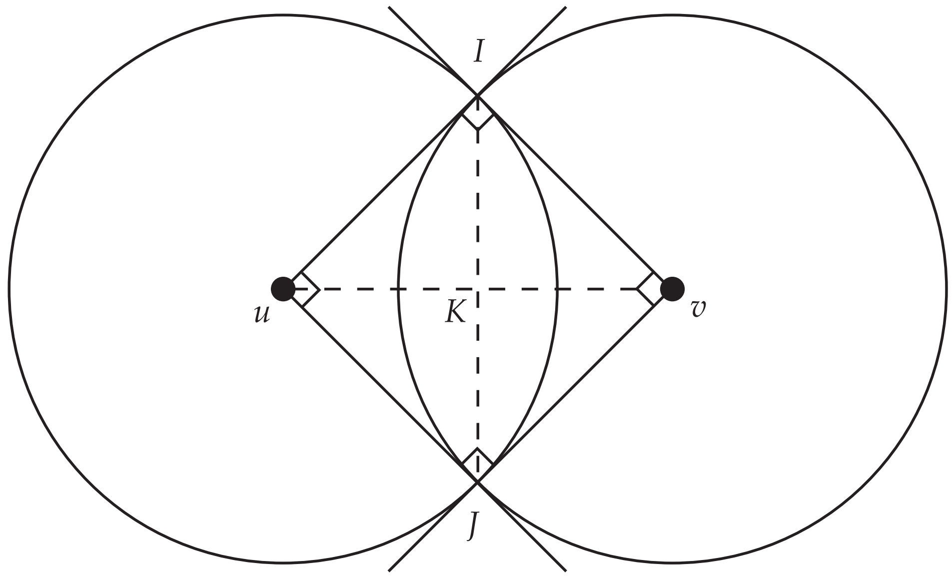
- Should we consider two points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (2/4)

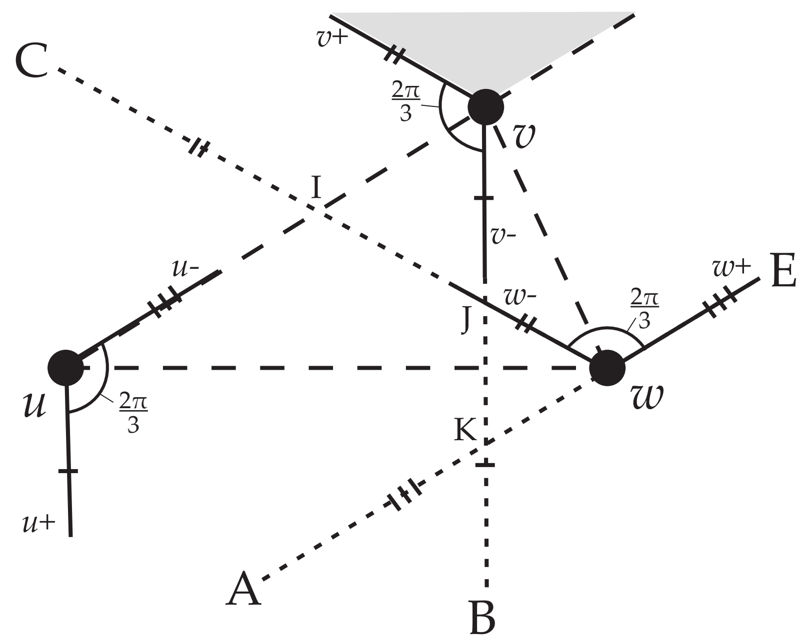
- Should we consider two points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (3/4)

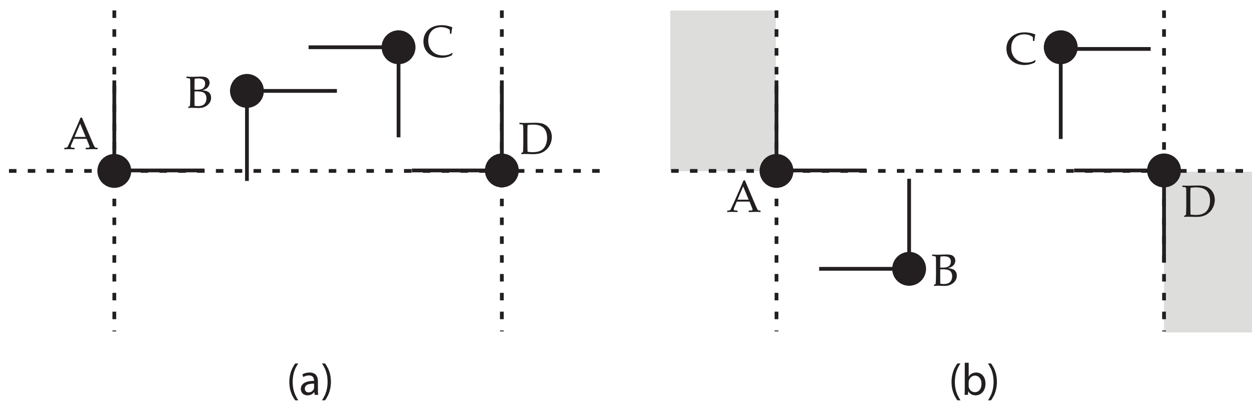
- Should we consider three points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (4/4)

- Should we consider four points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

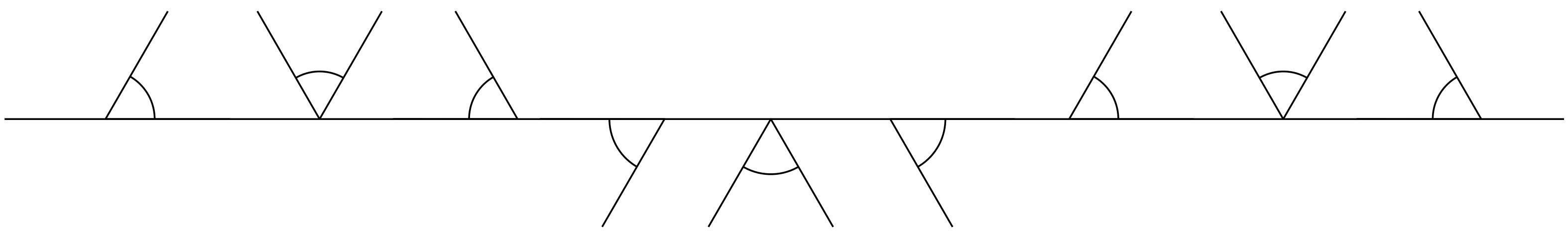
Coverage: Dynamic Case

Outline

- Antennae themselves may rotate
- Antennae rotate at a constant speed
- How do you cover a given domain under continuous rotation?

On a Line

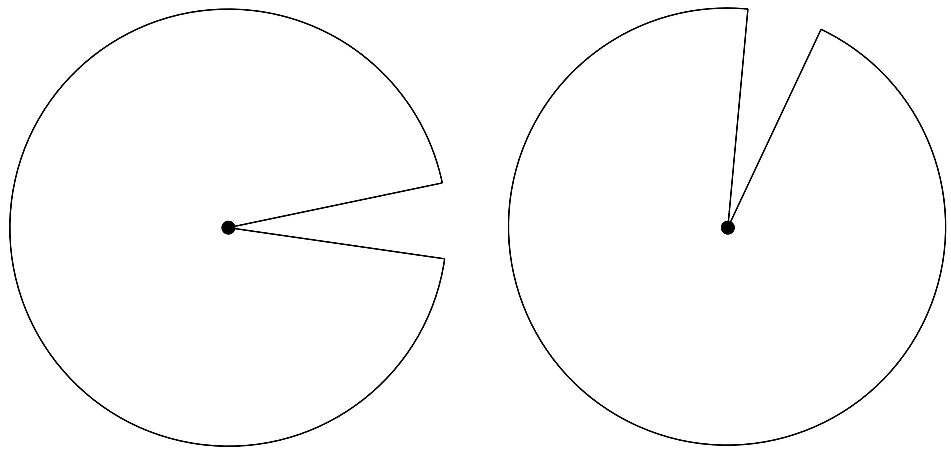
- n directional antennae on a line rotate at constant identical speeds



- What are the angle/range tradeoffs?

Two Directional Antennae

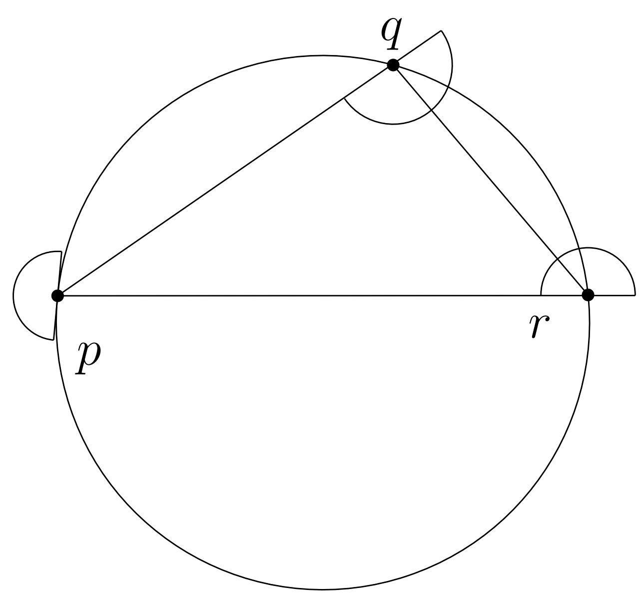
- 2 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Three Directional Antennae

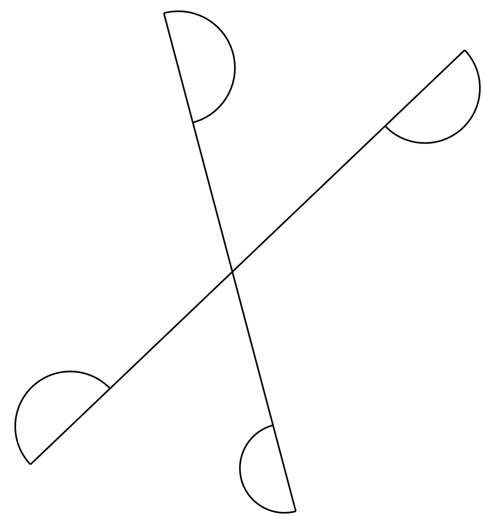
- 3 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Four Directional Antennae

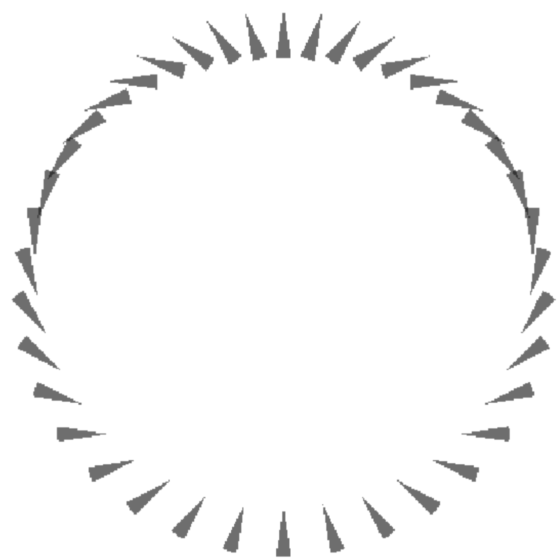
- 4 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Antennae in Convex Position

- n antennae (in convex position) rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Routing

Graphs of Directional Antennae

- Consider a set P of n points in the plane and assume that the Unit Disk Graph $U := U(P, 1)$ (with radius 1) is connected.
- Consider (ϕ, r)-directional antennae of angle ϕ and radius $r \geq 1$ and assume that k such antenna can be placed per point $p \in P$, for some $k \geq 1$.
- Let $\mathcal{G}(k, \phi, r)$ be the class of all possible directed strongly connected graphs arising under all possible rotations of the antennae.
- Note that $\mathcal{G}(k, \phi, r)$ may be empty for a given integer $k \geq 1$, angle ϕ and radius r.
- Similarly, since there is always a MST of max degree at most 5 on the set P of points it is easy to see that $\mathcal{G}(5, 0, 1) \neq \emptyset$.

Connectivity Range: Problem

- Given angle ϕ the connectivity range $r(\phi)$ is the smallest radius $r > 0$ such that there is an orientation of (ϕ, r)-antennae on the set P of points which results in a strongly connected graph, i.e.,

$$r(k, \phi) := \min\{r > 0 : \mathcal{G}(k, \phi, r) \neq \emptyset\}.$$

- An algorithm A which rotates the antennae so that the resulting graph is strongly connected produces a graph, say G_A, such that $G_A \in \mathcal{G}(k, \phi, r)$, for some $r \geq 1$.
- Let $r_A(k, \phi)$ be the radius of the antennae used in G_A.

Connectivity Range

- Consider the class $\mathcal{A}(k, \phi, P)$ of all such orientation algorithms on the set P of points above.

Problem 1 *We are given a set P of n points in the plane such that the Unit Disk Graph $U := U(P, 1)$ is connected. Let $\phi \geq 0$ be any angle and $k \geq 1$ an integer.*

1. *Give an algorithm $A \in \mathcal{A}(k, \phi, P)$ for orienting the antennae and which achieves the optimal range $r(k, \phi)$ for antennae of angle ϕ.*
2. *If there is no algorithm attaining the optimal range, then give an algorithm $A \in \mathcal{A}(k, \phi, P)$ which attains the best approximation to $r(k, \phi)$.*

(Hop) stretch factor

- For any graph G on the set P of points and any two points $s, t \in P$ let $d_G(s,t)$ denote the (hop) distance between s and t.
- The (ϕ, r)-antenna (hop) stretch factor of a graph $G \in \mathcal{G}(k, \phi, r)$ is defined by

$$\sigma_G(\phi, r) := \max \left\{ \frac{d_G(s,t)}{d_U(s,t)} : s \neq t \right\},$$

  where $d_U(s,t)$ is the hop distance between s, t in the graph U.
- The (ϕ, r)-antenna (hop) stretch factor for k antennae per point is defined by

$$\sigma(k, \phi, r) := \min \{ \sigma_G(\phi, r) : G \in \mathcal{G}(k, \phi, r) \}$$

(Hop) stretch factor

- Clearly, $\sigma(k,\phi,r) = +\infty$ when $\mathcal{G}(k,\phi,r) = \emptyset$. The ϕ-antenna (hop) stretch factor for k antennae per point is defined by

$$\begin{aligned} \sigma(k,\phi) &:= \min\{\sigma(k,\phi,r) : \mathcal{G}(k,\phi,r) \neq \emptyset, \text{ for some } r \geq 1\} \\ &= \min_{G \in \mathcal{G}(k,\phi,r)} \max_{s \neq t} \frac{d_G(s,t)}{d_U(s,t)} \end{aligned}$$

- An algorithm A which rotates the antennae so that the resulting graph is strongly connected produces a graph, say G_A, such that $G_A \in \mathcal{G}(k,\phi,r)$, for some $r \geq 1$.
- Let $d_A(s,t)$ be the hop-distance between s,t in the graph G_A.

(Hop) stretch factor

- The stretch factor of algorithm A is defined by

$$\sigma_A(\phi) := \max_{s \neq t} \frac{d_A(s,t)}{d_U(s,t)}.$$

- **Problem 2** *We are given a set P of n points in the plane such that the Unit Disk Graph $U := U(P,1)$ is connected. Let ϕ be an angle and $k \geq 1$ an integer.*

  1. *Give an algorithm $A \in \mathcal{A}(k,\phi,P)$ for orienting the antennae and which achieves the optimal stretch factor for antennae of angle ϕ.*
  2. *If there is no algorithm attaining the optimal stretch factor, then give an algorithm $A \in \mathcal{A}(k,\phi,P)$ which attains the best approximation to $\sigma(k,\phi)$.*