Some Ideas on Coverage and Routing

Outline

- Coverage
  - Static case
  - Dynamic case
- Routing
  - Stretch factor

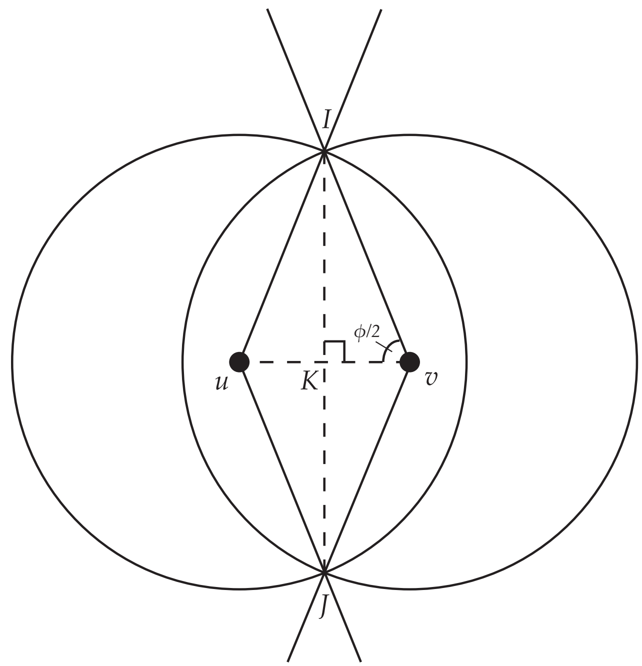
Coverage: Static Case

Outline

- How do you replace omnidirectional antennae with directional antennae?
- What are the range/angle/coverage tradeoffs?

From Omnidirectional to Directional Antennae (1/4)

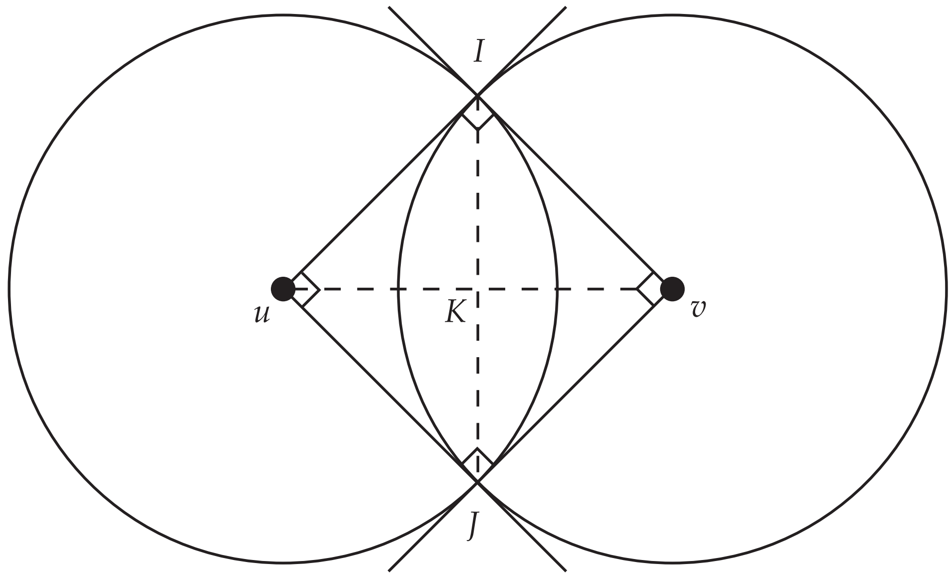
- Should we consider two points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (2/4)

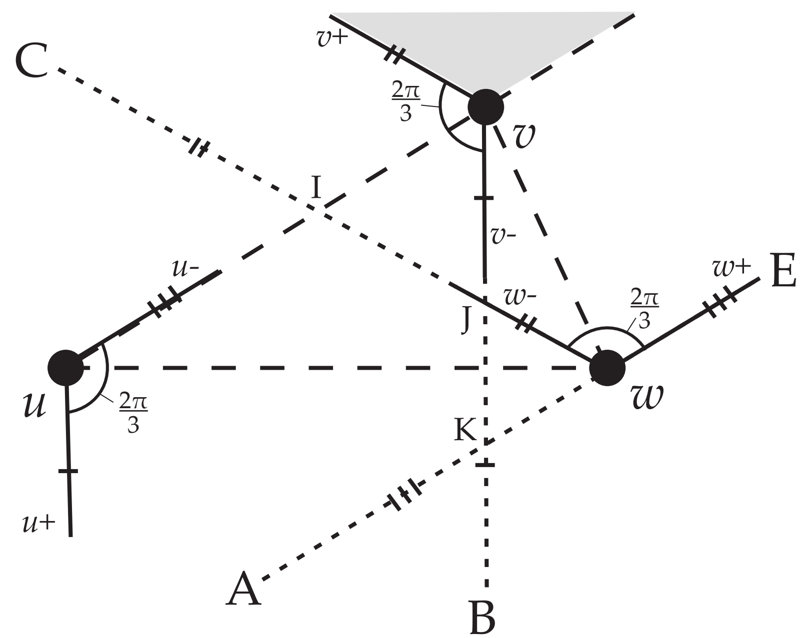
- Should we consider two points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (3/4)

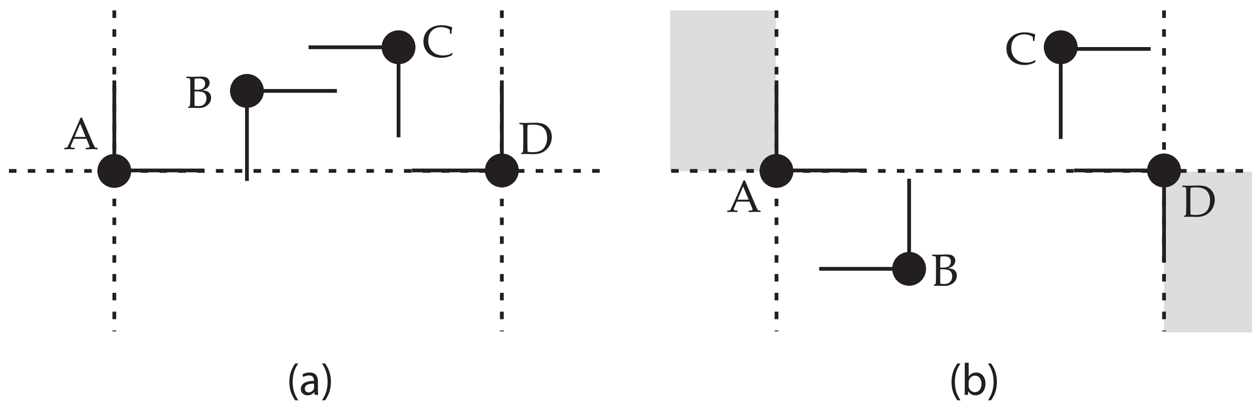
- Should we consider three points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

Omnidirectional to Directional (4/4)

- Should we consider four points at a time?
- What is the appropriate range for directional antennae?



- Distance and Angle Matter!

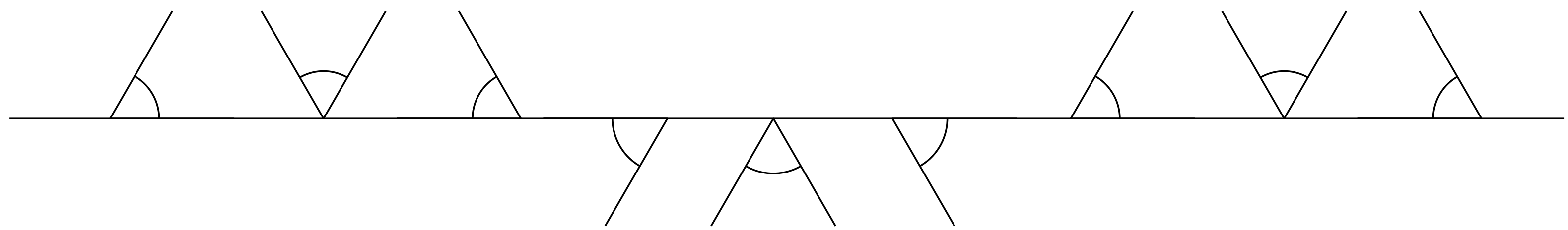
Coverage: Dynamic Case

Outline

- Antennae themselves may rotate
- Antennae rotate at a constant speed
- How do you cover a given domain under continuous rotation?

On a Line

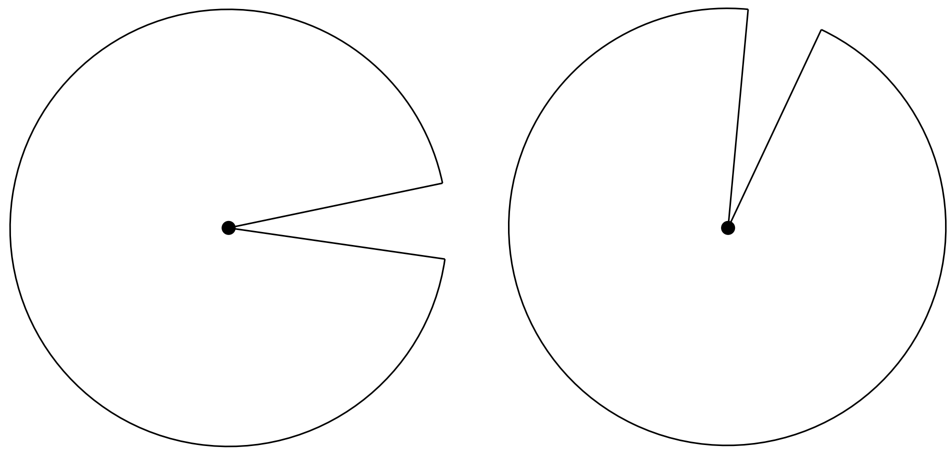
- n directional antennae on a line rotate at constant identical speeds



- What are the angle/range tradeoffs?

Two Directional Antennae

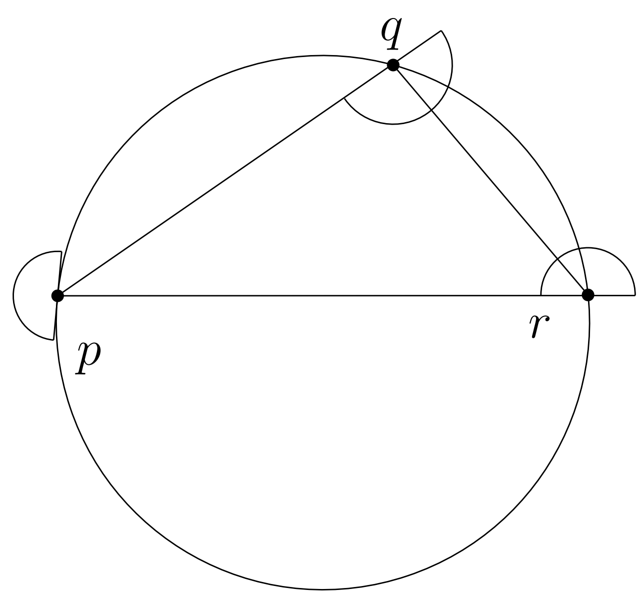
- 2 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Three Directional Antennae

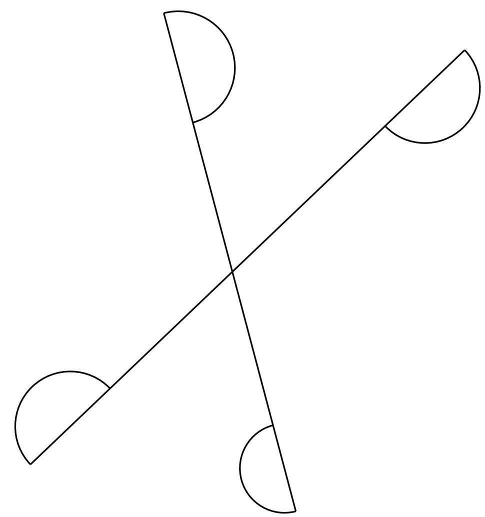
- 3 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Four Directional Antennae

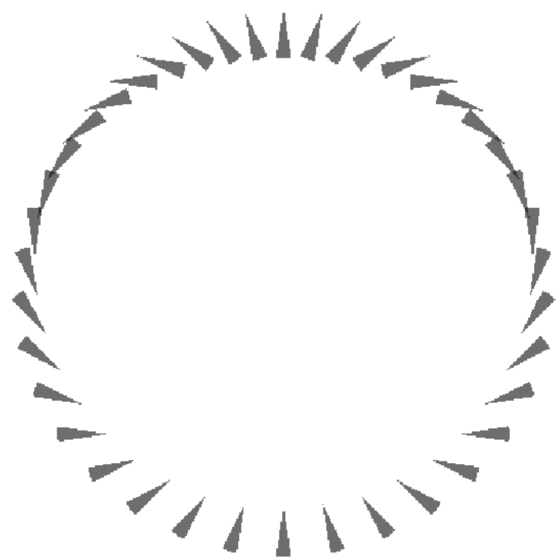
- 4 antennae rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Antennae in Convex Position

- n antennae (in convex position) rotate at constant identical speeds



- What is the min angle required to cover the whole plane?

Routing

Graphs of Directional Antennae

- Consider a set P of n points in the plane and assume that the Unit Disk Graph $U := U(P,1)$ (with radius 1) is connected.
- Consider (ϕ, r)-directional antennae of angle ϕ and radius $r \geq 1$ and assume that k such antenna can be placed per point $p \in P$, for some $k \geq 1$.
- Let $\mathcal{G}(k, \phi, r)$ be the class of all possible directed strongly connected graphs arising under all possible rotations of the antennae.
- Note that $\mathcal{G}(k, \phi, r)$ may be empty for a given integer $k \geq 1$, angle ϕ and radius r.
- Similarly, since there is always a MST of max degree at most 5 on the set P of points it is easy to see that $\mathcal{G}(5, 0, 1) \neq \emptyset$.

Connectivity Range: Problem

- Given angle ϕ the connectivity range $r(\phi)$ is the smallest radius $r > 0$ such that there is an orientation of (ϕ, r)-antennae on the set P of points which results in a strongly connected graph, i.e.,

$$r(k, \phi) := \min\{r > 0 : \mathcal{G}(k, \phi, r) \neq \emptyset\}.$$

- An algorithm A which rotates the antennae so that the resulting graph is strongly connected produces a graph, say G_A, such that $G_A \in \mathcal{G}(k, \phi, r)$, for some $r \geq 1$.
- Let $r_A(k, \phi)$ be the radius of the antennae used in G_A.

Connectivity Range

- Consider the class $\mathcal{A}(k, \phi, P)$ of all such orientation algorithms on the set P of points above.

Problem 1 *We are given a set P of n points in the plane such that the Unit Disk Graph $U := U(P, 1)$ is connected. Let $\phi \geq 0$ be any angle and $k \geq 1$ an integer.*

1. *Give an algorithm $A \in \mathcal{A}(k, \phi, P)$ for orienting the antennae and which achieves the optimal range $r(k, \phi)$ for antennae of angle ϕ.*
2. *If there is no algorithm attaining the optimal range, then give an algorithm $A \in \mathcal{A}(k, \phi, P)$ which attains the best approximation to $r(k, \phi)$.*

(Hop) stretch factor

- For any graph G on the set P of points and any two points $s, t \in P$ let $d_G(s,t)$ denote the (hop) distance between s and t.
- The (ϕ, r)-antenna (hop) stretch factor of a graph $G \in \mathcal{G}(k, \phi, r)$ is defined by

$$\sigma_G(\phi, r) := \max \left\{ \frac{d_G(s,t)}{d_U(s,t)} : s \neq t \right\},$$

  where $d_U(s,t)$ is the hop distance between s, t in the graph U.
- The (ϕ, r)-antenna (hop) stretch factor for k antennae per point is defined by

$$\sigma(k, \phi, r) := \min \left\{ \sigma_G(\phi, r) : G \in \mathcal{G}(k, \phi, r) \right\}$$

(Hop) stretch factor

- Clearly, $\sigma(k,\phi,r) = +\infty$ when $\mathcal{G}(k,\phi,r) = \emptyset$. The ϕ-antenna (hop) stretch factor for k antennae per point is defined by

$$\begin{aligned} \sigma(k,\phi) &:= \min\{\sigma(k,\phi,r) : \mathcal{G}(k,\phi,r) \neq \emptyset, \text{ for some } r \geq 1\} \\ &= \min_{G \in \mathcal{G}(k,\phi,r)} \max_{s \neq t} \frac{d_G(s,t)}{d_U(s,t)} \end{aligned}$$

- An algorithm A which rotates the antennae so that the resulting graph is strongly connected produces a graph, say G_A, such that $G_A \in \mathcal{G}(k,\phi,r)$, for some $r \geq 1$.
- Let $d_A(s,t)$ be the hop-distance between s,t in the graph G_A.

(Hop) stretch factor

- The stretch factor of algorithm A is defined by

$$\sigma_A(\phi) := \max_{s \neq t} \frac{d_A(s,t)}{d_U(s,t)}.$$

- **Problem 2** *We are given a set P of n points in the plane such that the Unit Disk Graph $U := U(P,1)$ is connected. Let ϕ be an angle and $k \geq 1$ an integer.*
  1. *Give an algorithm $A \in \mathcal{A}(k,\phi,P)$ for orienting the antennae and which achieves the optimal stretch factor for antennae of angle ϕ.*
  2. *If there is no algorithm attaining the optimal stretch factor, then give an algorithm $A \in \mathcal{A}(k,\phi,P)$ which attains the best approximation to $\sigma(k,\phi)$.*