Neighbor Discovery with Directional Antennae

Outline

- Introduction
- Deterministic Algorithms
- Randomized Algorithms
- Conclusions

Introduction

Goals

- Investigate the complexity of discovering neighbors in a setting of rotating antennae:
  - What knowledge is required?
  - How long does it take?
  - What protocols are possible?
  - How does it compare to the omnidirectional setting?

Communication Models with Directional Antennae

- (O, O) model: two sensors can communicate if they are within transmission range of each other,
- (D, O) (respectively, (O, D)) model: the sender (respectively, receiver) must turn its antenna so as to reach its neighbor, and
- (D, D) model: both sender and receiver must direct their antennae towards each other at the same time.
  This is the model we look at!

Neighbor Discovery Process

- Usually entails the exchange of identities (e.g., MAC addresses) between two adjacent nodes.
- It will be sufficient to assume that this is a one step process whereby one sensor sends its identity and the other acknowledges by sending back its own.
- We assume that the sensors have distinct identities but their corresponding locations (i.e., (x, y)-coordinates) in the plane are not known to each other.
- There is a vertex coloring $\chi : V \to \{0, 1, \ldots, c-1\}$

Parameters of the Directional Antennae Model

- For simplicity, for each node u assume an angle (or beam width) $\phi_u = \frac{2\pi}{k_u}$, for some integer k_u.

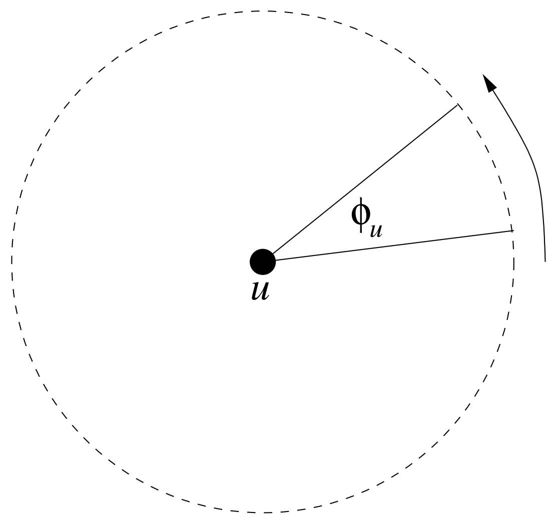


Figure 1: An antenna at u rotating counter-clockwise.

- Sensor network is synchronous

Deterministic Algorithms

Deterministic Algorithms

- Lower Bound: $\Omega(k_u k_v)$ time steps, for two sensors u, v within communication range of each other.
- Upper Bounds

Antenna at u	Knowledge	Running Time	Theorems
$2\pi/k$	Identical	$O(k^{c-1})$	Theorem 1
$2\pi/k$	Identical	$O(k(c \ln c)^3)$	Theorem 2

Table 1: Theorems and running times of deterministic algorithms.

- Recall our basic assumption that there is a coloring $\chi : V \to \{0, 1, \ldots, c-1\}$ of the vertices of the sensor network using c colors.

Lower Bound

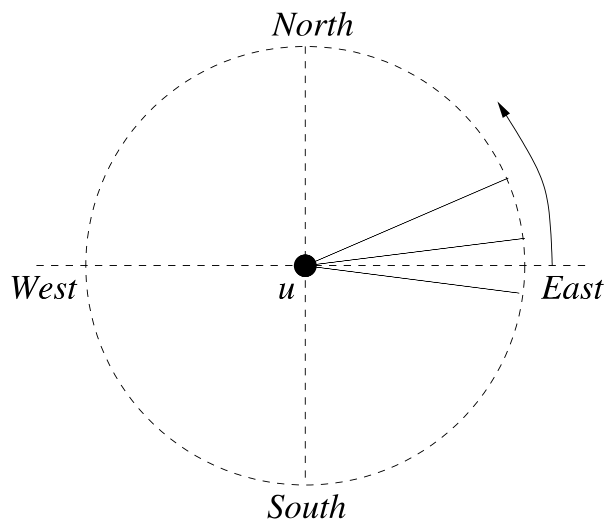
- Consider two sensors u, v within communication range of each other and respective antenna beam widths $\frac{2\pi}{k_u}$ and $\frac{2\pi}{k_v}$, respectively. If the sensors do not know each other's location then any algorithm for solving the neighbor discovery problem in the (D, D) communication model requires at least $\Omega(k_u k_v)$ time steps.
- This is because, for a successful communication to occur each sensor must be within the beam of the other sensor's antenna at the same time. Since the sensors do not know each other's location they must attempt transmissions in all their respective sectors.

Communicating Position

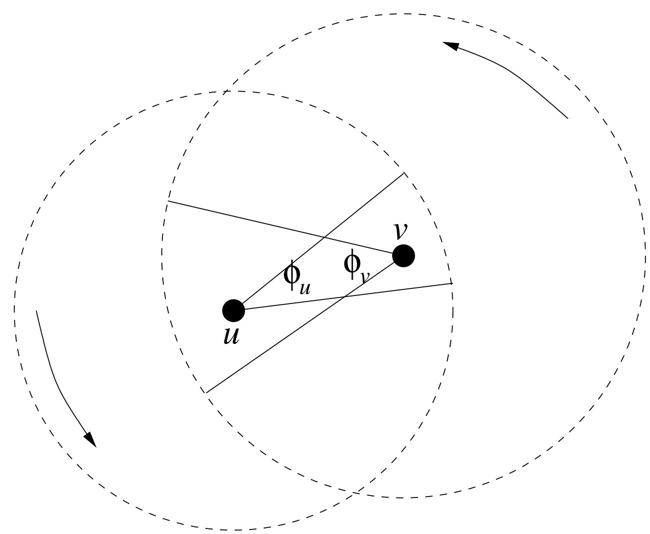
1. Sensors must be within range of each other.

Figure 2: Directional antennae in communicating position.

(a) An antenna at u with sectors counted counter-clockwise.



(b) Neighbor discovery for sensors u, v.



2. Directional antennae must be facing each other.

Communication Failure of Deterministic Algorithms

- Not every deterministic algorithm would work!
- Example: Sensor u employs delay $d_u = 2$ and sensor v delay $d_v = 1$, under which sensors with directional antennae will never be able to communicate as illustrated in Figure 3.

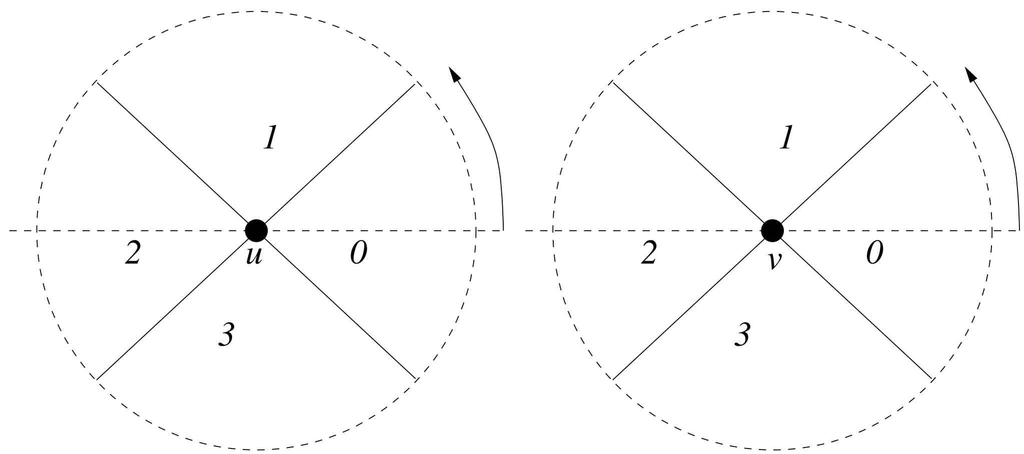


Figure 3: Neighbor discovery for sensors u, v is not possible.

(Basic) Antenna Rotation Algorithm (with Delay)

- For each sensor u, let d_u be an integer delay parameter and k be defined so that $\phi = \frac{2\pi}{k}$

Algorithm 1: Antenna Rotation Algorithm $ARA(d_u, k_u)$

Start at a given orientation;
while *true* **do**
    for $i \leftarrow 0$ **to** $d_u - 1$ **do**
        `//For` d_u `steps stay in chosen sector`
        Send message to neighbor(s);
        Listen for messages from neighbor(s) (if any);
    Rotate antenna beam one sector counter-clockwise;
    `//rotate by an angle equal to` ϕ

A Simple Choice of Delays

- A simple theorem is the following:
- **Theorem 1** *Consider a set of sensors in the plane with identical antenna beam widths equal to* $\phi = \frac{2\pi}{k}$. *For each sensor* u *let the delay be defined by* $d_u := k^{\chi(u)}$. *If each sensor* u *executes algorithm* $ARA(d_u, k)$ *then every sensor in the network will discover all its neighbors in at most* k^{c-1} *time steps.*
- Running time can be improved by choosing delays appropriately!

Improving on Delay

Theorem 2 *Consider a set of sensors in the plane such that the antenna beam width of sensor u is equal to $\phi = \frac{2\pi}{k}$. Assume the sensor network is synchronous. Suppose that the delays d_u at the nodes are chosen so that*

1. $\gcd(k, d_u) = 1$, *for all u, and*
2. *if u, v are adjacent then* $\gcd(d_u, d_v) = 1$.

If each sensor u executes algorithm $ARA(d_u, k)$ then every sensor in the network will discover all its neighbors in at most $O\left((k(\max_u d_u)^3\right)$ time steps.

In addition, the delays d_u can be chosen so that every sensor in the network will discover all its neighbors in at most $O(k(c \log c)^3$ time steps.

In particular, this is at most $O\left((c \ln c)^3\right)$ time steps, if $k \in O(1)$.

Proof (1/2)

- Without loss of generality assume that
  1. u and v are in horizontal position and sensor u is to the left of sensor v, and
  2. that both antennae orientations are initially set to *East*.
- u, v can communicate when v's antenna is facing *West* which is sector $\lfloor \frac{k}{2} \rfloor$.
- Since $\gcd(d_u, d_v) = 1$, by Euclid's algorithm there exist integers $0 < a_u < d_u, 0 < a_v < d_v$ such that

$$a_u d_u = a_v d_v + 1. \tag{1}$$

- Lets look at sensor u first. After $d_u k$ steps sensor u will be in its starting position and, clearly, the same applies for any time duration that is a multiple of $d_u k$. Thus sensor u is in its initial position (facing East) at time $j a_u d_u k$, for any $j > 0$.

Proof (2/2)

- Multiply both sides of Equation $a_u d_u = a_v d_v + 1$ by jk to obtain $ja_u d_u k = ja_v d_v k + jk$
- So at time $t = ja_u d_u k$ sensor u is facing *East*. If there is a j such that $jk = \lfloor \frac{k}{2} \rfloor d_v + r$ for $0 \leq r < d_v$, then sensor v is facing West and therefore the sensors u, v can discover each other.
- Find a j such that,

$$jk \leq \left\lfloor \frac{k}{2} \right\rfloor d_v < jk + k \tag{2}$$

  which means that $jk + k = \lfloor \frac{k}{2} \rfloor d_v + r$, with $r \leq k < d_v$.
- A simple modification of the proof will prove the result when the two sensors are not necessarily on a horizontal line.
- # of rotations required is $ja_u d_u k$, where j satisfies Inequality (2).

Randomized Algorithms

Randomized Neighbor Discovery Algorithms

- **Upper Bounds**

Antenna at u	Knowledge	Running Time	Theorems
$2\pi/k$	Identical	$kn^{O(1)}$	Theorem 3
$2\pi/k$	Identical	$O(k^2 \log n)$	Theorem 4
$2\pi/k_u$	$\max_u k_u \le k$	$O(k^4 \log n)$	Theorem 5

Table 2: Theorems and running times of randomized algorithms.

Randomized Neighbor Discovery Algorithm (1/4)

Algorithm 2: Randomized Antenna Rotation Algorithm $RARA(d_u, k)$

1 Select $d_u \leftarrow RANDOMPRIME(k..R)$;

2 Execute $ARA(d_u, k)$;

Theorem 3 *Consider a set of sensors in the plane such that the antenna beam width of sensor* u *is equal to* $\phi = \frac{2\pi}{k}$. *Assume the sensor network is synchronous. If each sensor* u *executes algorithm* $RARA(k; R)$, *where* $R = n^{O(1)}$ *and* n *is an upper bound on the number of sensors, then every sensor in the network will discover all its neighbors in at most* $kn^{O(1)}$ *expected time steps, with high probability.*

Randomized Neighbor Discovery Algorithm (2/4)

- For every node u, let $N(u)$ denote the neighborhood of u and $\deg(u)$ the degree of u.
- Let $D = \max_u \deg(u)$ denote the maximum degree of a node of the sensor network.
- By the prime number theorem, the number of primes $\leq R$ and $> k$ is approximately equal to $\frac{R}{\ln R} - \frac{k}{\ln k}$ and therefore the probability that the primes chosen by two adjacent nodes, say u and v, are different is $1 - \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}}$.
- Let E_u be the event that the prime chosen at u is different from all the primes chosen by its neighbors.

Randomized Neighbor Discovery Algorithm (3/4)

- It is easily seen that

$$\begin{aligned}
\Pr[E_u] &= 1 - \Pr[\neg E_u] \\
&= 1 - \Pr\left[\exists v \in N(u)(d_u = d_v)\right] \\
&\geq 1 - \sum_{v \in N(u)} \Pr\left[d_u = d_v\right] \\
&\approx 1 - \deg(u) \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}} \\
&\geq 1 - D \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}}.
\end{aligned}$$

Randomized Neighbor Discovery Algorithm (4/4)

- Similarly, we can prove that

$$
\begin{aligned}
\Pr\left[\bigcap_u E_u\right] &= 1 - \Pr\left[\bigcup_u \neg E_u\right] \\
&\geq 1 - \sum_u \Pr[\neg E_u] \\
&\geq 1 - nD \frac{1}{\frac{R}{\ln R} - \frac{k}{\ln k}} \\
&\geq 1 - \frac{1}{n}.
\end{aligned}
$$

- By choosing R in $n^{O(1)}$ and recalling that $D \leq n$ we see that all the primes chosen by all the nodes in the network are pairwise distinct, with high probability.

Additional Algorithms

- **Theorem 4** *Consider a set of n sensors in the plane with identical antenna beam width equal to $\phi = \frac{2\pi}{k}$. Assume the sensor network is synchronous. There is an algorithm so that every sensor in the network will discover all its neighbors in at most $O(k^2 \log n)$ expected time steps, with high probability.*
- **Theorem 5** *Consider a set of n sensors in the plane such that sensor u has antenna beam width equal to $\phi_u = \frac{2\pi}{k_u}$. Assume the sensor network is synchronous and that an upper bound k is known to all sensors so that $\max_u k_u \leq k$. There is an algorithm so that every sensor in the network will discover all its neighbors in at most $O(k^4 \log n)$ expected time steps, with high probability.*

Conclusions

- **Interesting Problem:** Efficiency of broadcasting
  1. in the single channel UDG model, i.e., there is a single send/receive channel and multiple transmissions on the same node produce packet collisions, and
  2. a link between two sensors u, v exists if and only if $d(u, v) \leq 1$.
- If broadcasting time with omnidirectional antennae without collisions is B then the result of Theorem 3 indicates that broadcasting in the directional antennae model can be accomplished in time $O(B(c \ln c)^3)$, where c is the number of colors of a vertex coloring of the sensor network. The main question arising is whether we can improve on this time bound when using directional antennae.

Additional Work

- J. Du, E. Kranakis, O. Morales Ponce, S. Rajsbaum, Neighbor Discovery in a Sensor Network with Directional Antennae. In proceedings of Algosensors 2011, Saarbruecken, Germany, September 08-09, 2011.