Directional Sensor Algorithms

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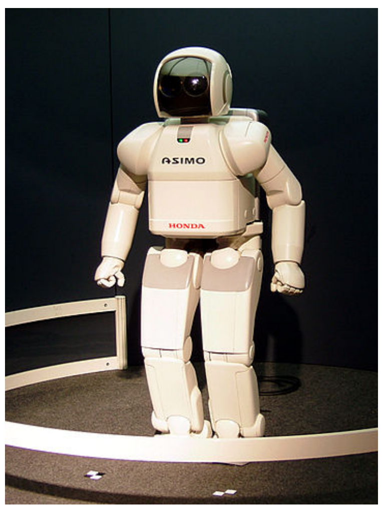
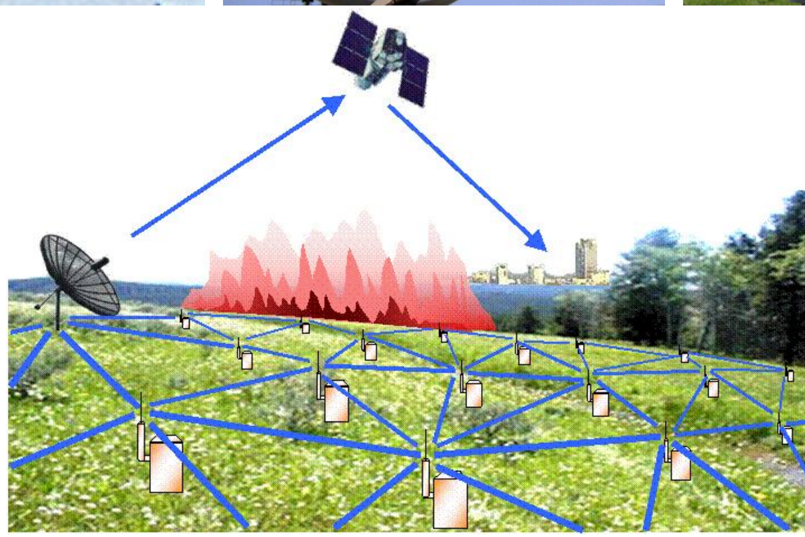
Motivation









Directional Sensors

Directional sensors equipped with ultrasound, infrared, and video are being used in wireless multimedia sensor networks (MSN).

They differ from traditional omni-directional sensors.

Distinct parameters such as angle of view, working direction, and line of sight required careful attention.

Directional antennas affect the network discovery as well as the connectivity and stretch factor of the network.

Why Directional Sensors

Transmitting in a particular direction results in a higher degree of spatial reuse of the shared medium.

Directional transmission uses energy more efficiently.

The transmission range of directional antennas is usually larger than that of omnidirectional antennas, which can reduce hops for routing and make originally unconnected devices connected.

Routing protocols using directional antennas can outperform omnidirectional routing protocols.

Comparison of Omnidirectional & Directional Antennae

	Omnidirectional	Directional
Energy	More	Less
Throughput	Less	More
Collisions	More	Less
Connectivity	Stable	Intermittent
Discovery	Easy	Difficult
Coverage	Stable	Intermittent
Stretch Factor	Less	More
Security	Less	More

Directional Antennas

Antenna Orientation Problem

Question. Given a set of sensors in the plane each equipped with k directional antennas of a given beamwidth ϕ. What is the min range r such that a network with property $\mathcal{P}$ is formed by an appropriate antenna orientation?

- $\mathcal{P}$ can be any or a combination of: k-connectivity, constant stretch factor, min interference, etc.
- Model of Transmission and Reception
  - DD: Directional transmission-directional reception (undirected graphs.)
  - DO: Directional transmission-ominidirectional reception (directed graphs.)
- The antenna model (Single antenna, multiple antennas, dipole antennas, 3D.)
  - Static antennas: Each sensor is free to choose any orientation of its antennas. However, when the choice is made, the antennas remain static.
  - Rotational antennas: A synchronous model where each sensor can activate at most one antenna of an array in each round.

Model of a single antenna

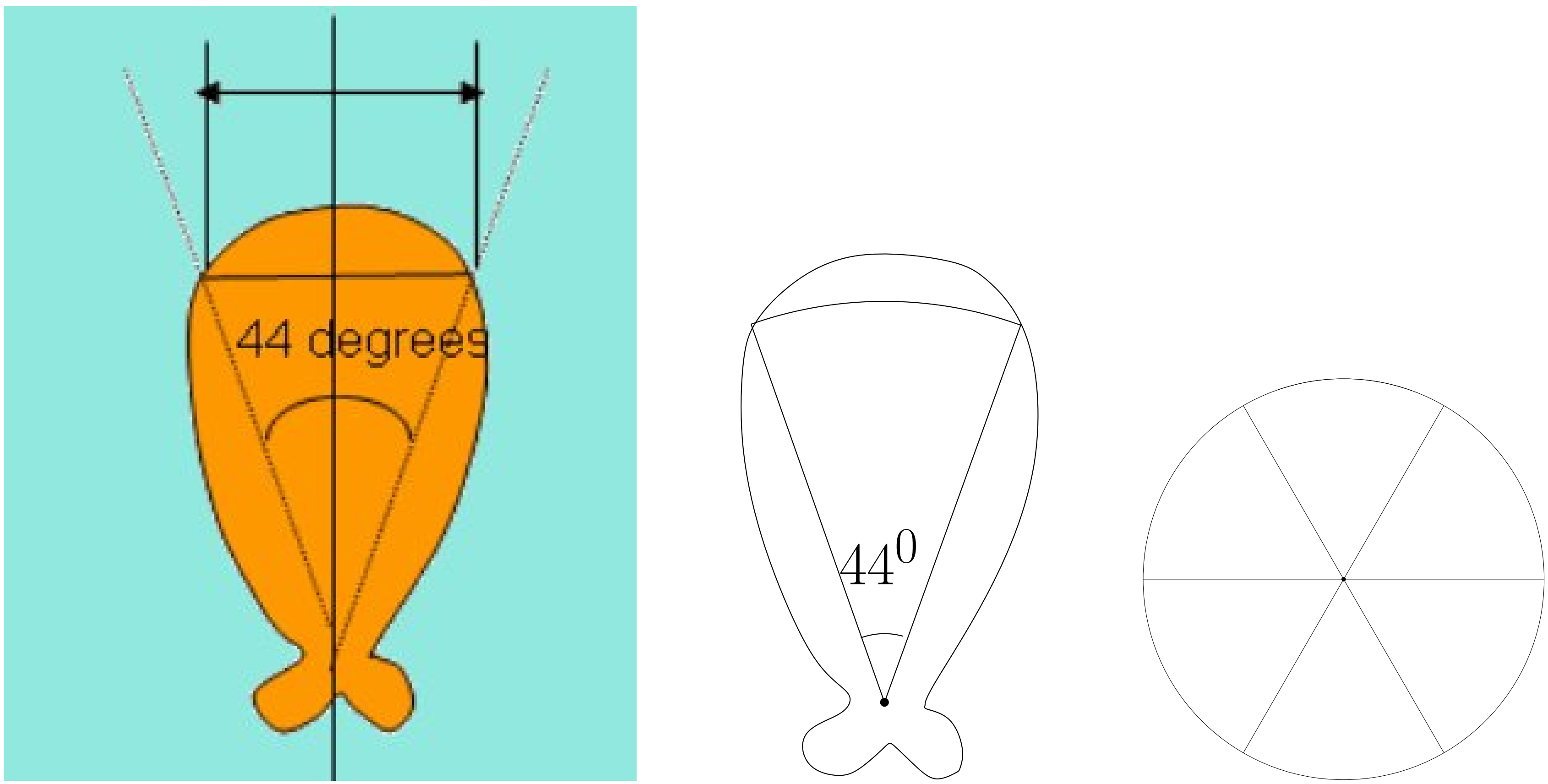


Figure 1: Radiation pattern, single antenna and array antenna

Models of connectivity

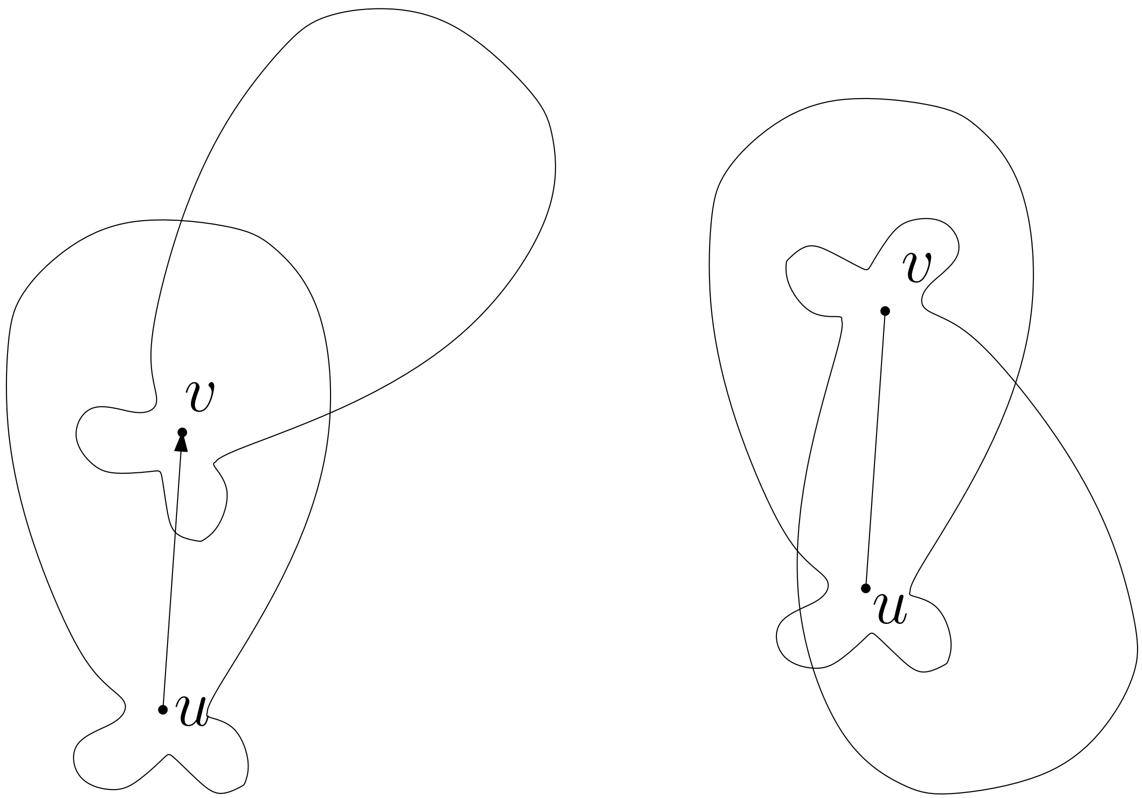


Figure 2: DO vs DD

Connectivity under DO model with single antenna

Problem: Given a set of sensors in the plane with one directional antenna of a given beamwidth ϕ each. Determine the min range r such that a strongly connected network that spans the sensors is formed.

Angle	Approximation Ratio	Complexity
$\phi < \frac{2\pi}{3}$	$r < \sqrt{3}$	NP-Complete
$\phi \leq \frac{\pi}{3}$	$r \geq 2$	$O(n^2)$
$\frac{\pi}{3} < \phi < \frac{2\pi}{3}$	$r \geq \min(3, 4\sin(\frac{\phi}{2}))$	$O(n^2)$
$\frac{2\pi}{3} \leq \phi < \pi$	$r \geq 2\cos(\frac{\phi}{2}) + 2$	$O(n \log n)$
$\pi \leq \phi < \frac{4\phi}{3}$	$r \geq 2\sin(\frac{5\pi}{6} - \frac{\phi}{2})$	$O(n^2)$
$\frac{4\phi}{3} \leq \phi$	$r = 1$	$O(n^2)$

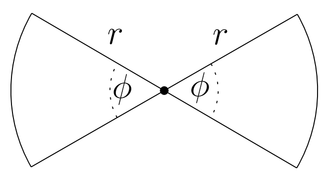
- Can we obtain approximation ratio less than two for $\phi > 0$ and/or optimal approximation ratio when $\frac{2\phi}{3} \leq \phi < \frac{4\phi}{3}$?

Some useful techniques

- Create groups of points within a short distance and orient the antennas to cover their neighbors.
  - The creation of groups of points is often a main issue.
  - The orientation is usually locally done.
- Create strongly connected digraphs such that the longest edge is min and orient the antennas so as they cover the out-going edges.
  - This approach involves global algorithms.

Connectivity under DD model with dipole antennas

Problem: Given a set of sensors in the plane with one dipole antenna each of a given angle ϕ. Determine the min range r such that a connected network is formed by an appropriate antenna orientation.



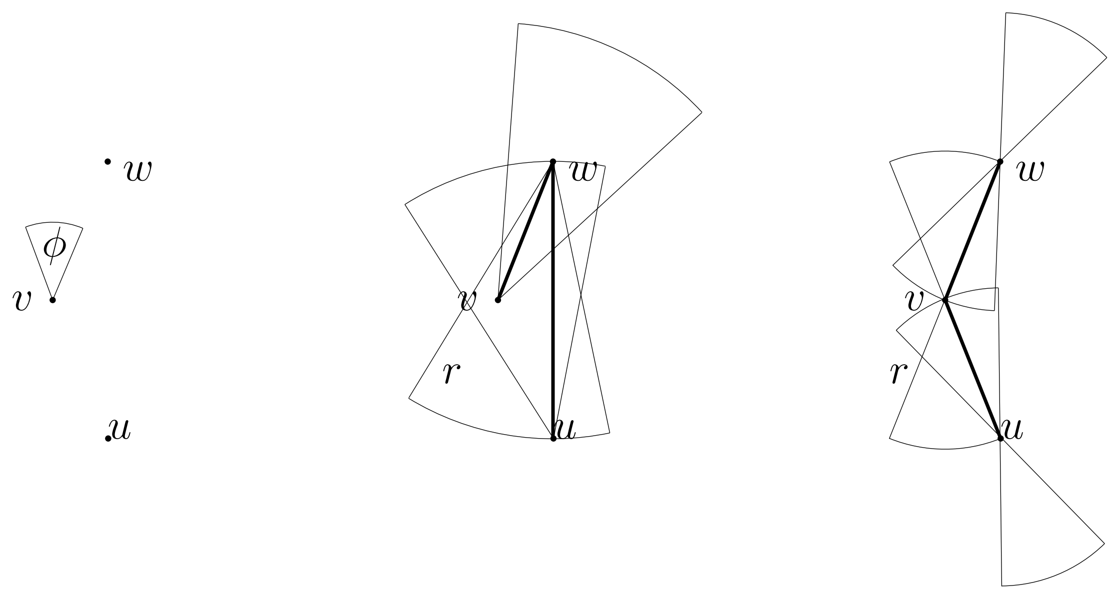
It is known that a sensor can cover all its neighbors in any Euclidean Minimum Spanning Tree with a dipole antenna of angle $\phi \geq 4\pi/5$.

The problem for single antennas of angle $\pi/2$ is studied by Aschne, Katz and Morgenstern [1]

[1] Aschner, Katz and Morgenstern. Symmetric Connectivity with Directional Antennas. 2011

Connectivity under DD model with dipole antennas

Example: Consider three sensors and a given angle ϕ as depicted in the figure on the left. We can reduce the required range with the use of dipole antennas.



Interference with dipole antennas

Let P be a set of sensors in the plane with one double antenna of a given angle ϕ. Let $I(u)$ be the number of antennas that cover u. We denote $I(G) = \max\{I(u) : u \in P\}$ as the interference of G.

Problem: Compute the min range r so as there exists an orientation of the antennas of angle ϕ and range r that forms a (strongly-)connected graph such that $I(G) = O(1)$?

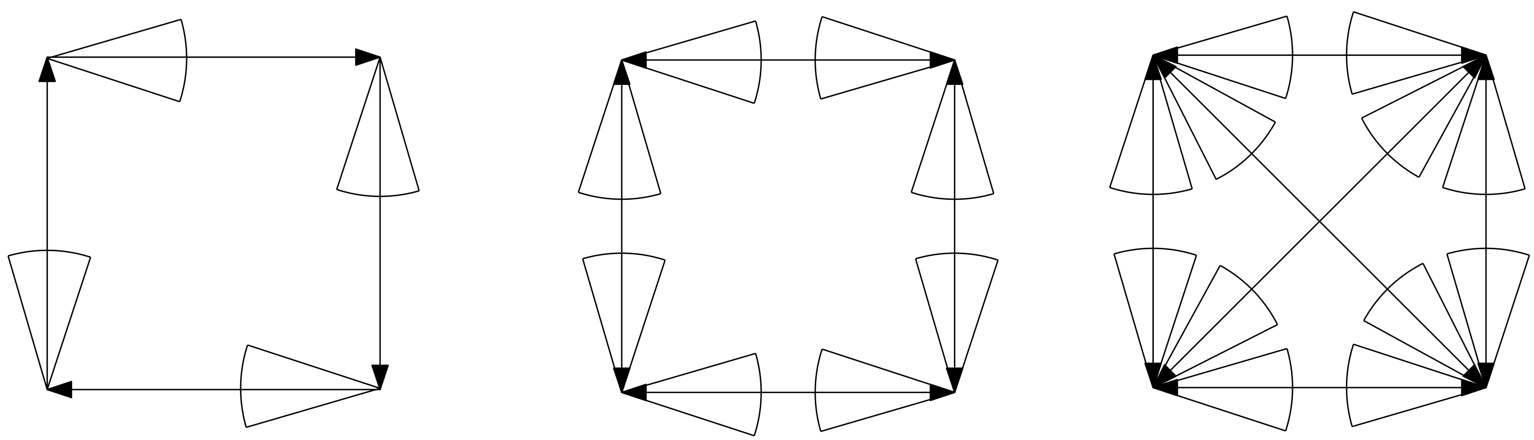
Observe that in the DO model $I(G)$ is given by the max in-degree of G. However, in the DD model $I(G)$ is at least the max degree of G.

Some results are recently presented in [2] for single antennas.

[2] Aschner, Katz and Morgenstern. Do Directional Antennas Facilitate in Reducing Interferences? 2012

High Connectivity under DO model

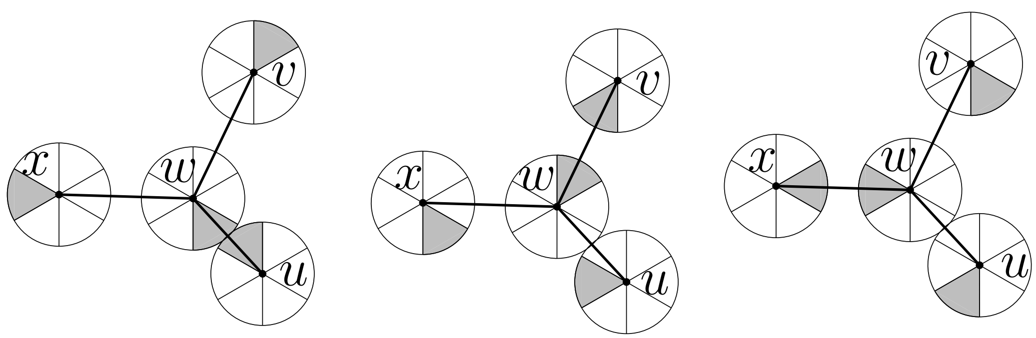
Problem: Given a set of sensors in the plane equipped with k directional antennas of a given beamwidth $\phi \leq 2\pi/k$, determine the min range r such that a k-connected network (k-edge connected network) is formed by an appropriate antenna rotation.



Min Delay Problem under DD Model

Let G be a connected UDG on a set of points P. Assume that each sensor has a k-antenna array. Consider a synchronous model. In each round a sensor can activate at most one antenna of the array. A message can be transmitted in each round at any distance providing that the links are active.

Problem: Let $t(G)$ be the max number of rounds needed to transmit a message for any pair of sensors u and v. Determine a schedule that minimizes $t(G)$.



Rotational Sensors

Rotational Sensor Problem

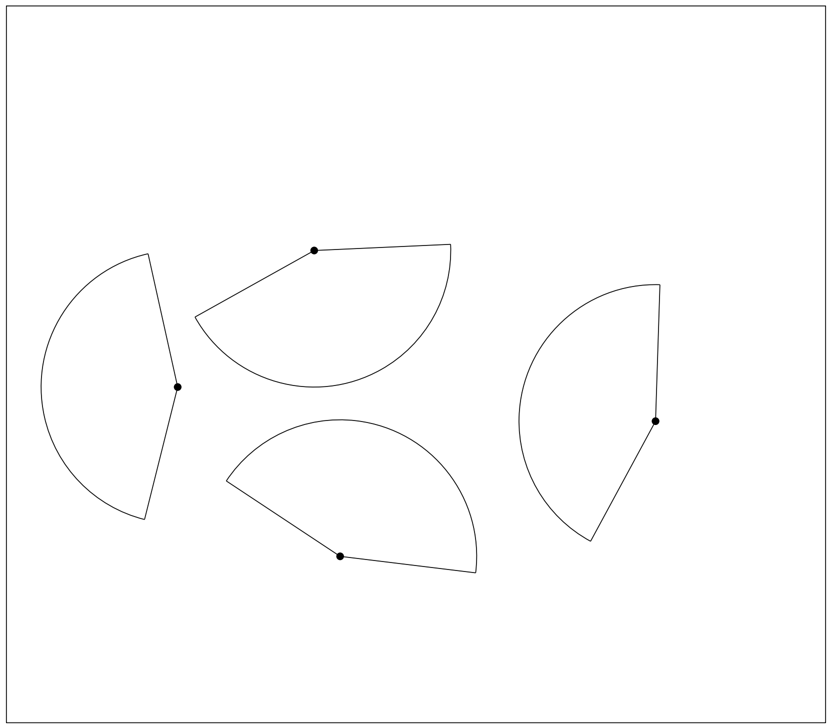
Consider a set $\mathcal{P}$ of directional sensors (e.g. radars) in the plane that rotate at constant and equal speed and a domain $\mathcal{D}$ in the plane.

Question. Given angle $\phi > 0$, is there an initial orientation of the sensors so that $\mathcal{D}$ is fully covered without interruption?

- The solution depends on:
  - The domain
  - The deployment of the sensors.
  - The number of sensors

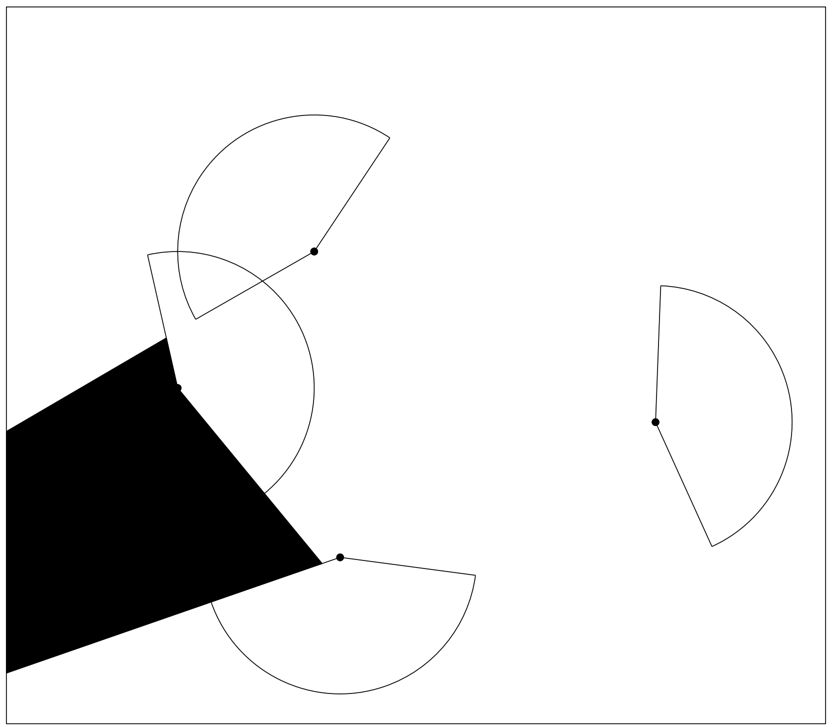
Covering the Plane with Rotational Sensors

Consider for example four sensors at time 0 and angle ϕ.



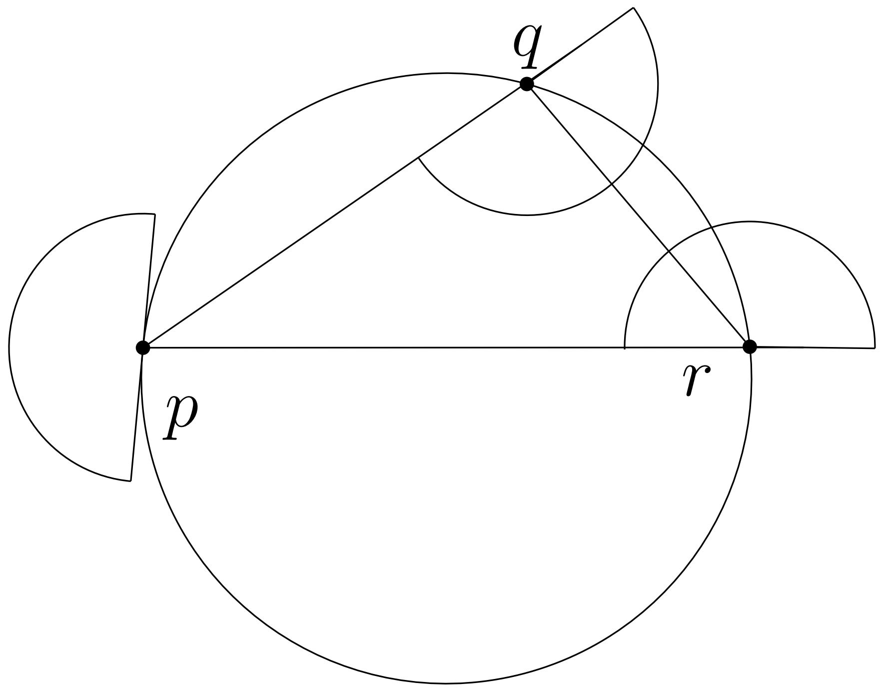
Covering the Plane with Rotational Sensors

At time ϕ the four sensors leave uncovered area.



Covering the Plane with Rotational Sensors

For three sensors, π is always sufficient to cover the plane



Covering the Plane with Rotational Sensors

Some Results and open problems:

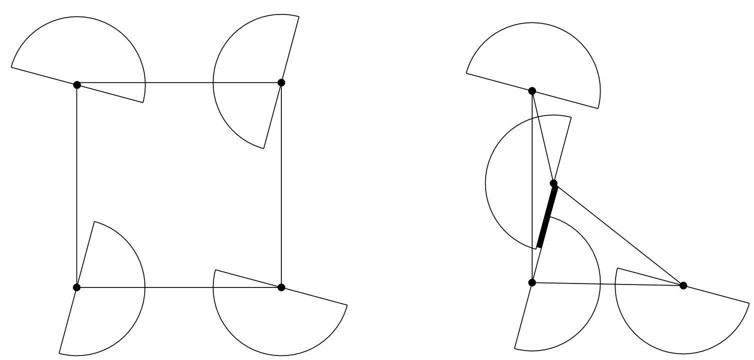
Number of points	Configuration	Lower bound	Upper Bound
2	General Position	2π	2π
3	General Position	π	π
4	General Position	π	π
n	Points on a line	Open	$6\pi/n$
n	Regular polygon	Open	$4\pi/n$
n	Convex Position	Open	Open
n	General Position	Open	$O(1/\sqrt{n})$

- Is there any other configuration of points to be considered?

Art gallery problem with rotational sensors

Consider a simple polygon $\mathcal{P}$ with n vertices. Assume that each vertex of $\mathcal{P}$ has a rotational directional sensor of angle ϕ rotating at constant and equal speed.

Problem: Determine the min angle ϕ such that there exists an initial orientation of the sensors of angle ϕ that covers the interior of $\mathcal{P}$ without interruption.

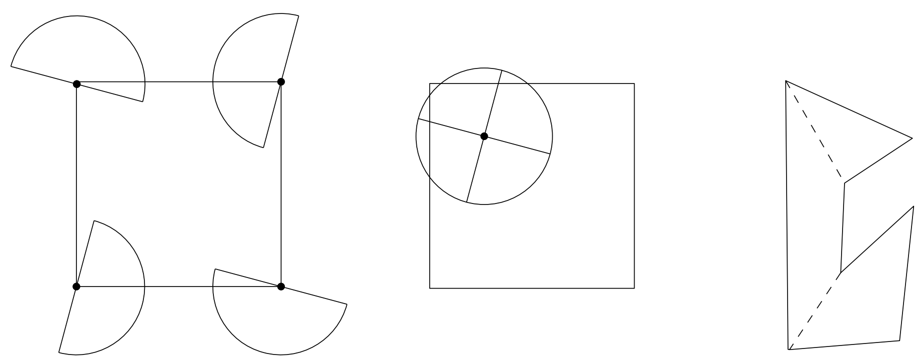


Art gallery problem with mobile rotational sensors

Consider a set of n points with rotational directional sensors of angle ϕ rotating at constant and equal speed and let $\mathcal{P}$ be a single polygon with n vertices.

In this variant, we are allowed to place the sensors in any place inside the polygon.

Problem: Determine the min angle ϕ such that there exists a position of the sensors that minimizes ϕ and the interior of $\mathcal{P}$ is covered without interruption.

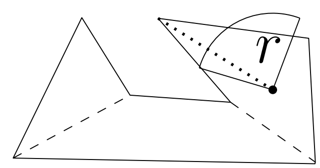


Art gallery problem with mobile rotational sensors and limited visibility

Consider a set of n points with rotational directional sensors of angle ϕ and range r rotating at constant and equal speed and let $\mathcal{P}$ be a single polygon with n vertices.

In this variant, we fix the range r and allow to move the sensors in any place inside the polygon.

Problem: Given a range r, determine the min angle ϕ such that there exists a position of the sensors that covers the interior of $\mathcal{P}$ without interruption.



Thanks