Social exchange networks with distant bargaining

Context and motivation

The study of bargaining has been a central theme in economics and sociology, since it constitutes a basic activity in any human society. The most basic bargaining model is that of two agents A and B that negotiate how to divide a good of a certain value (say, 1) amongst themselves, while at the same time each has a fallback alternative of value α and β respectively. The famous *Nash bargaining solution* tries to give each the corresponding alternative value, and then split equally the surplus $s = 1 - \alpha - \beta$ (if $s < 0$ then obviously there can be no bargaining). This basic model can be extended to an (exchange) network of agents, where two agents in this network connected by an edge e of value w_e can bargain over this edge (which abstracts the notion of a good of value w_e). Then the obvious question is: how should we divide the value of the edges (under certain constraints) amongst agents, so that none of them will want to change their minds on which edge(s) to bargain for? In a seminal work, Kleinberg and Tardos [2] solve the bargaining problem in general exchange networks, under the constraint that any agent can only bargain with at most one of her neighbors. Their results are based on the structural properties of matchings in general graphs, since the final bargaining solution defines a matching on the exchange network.

Problem description

In this project, we propose to study extensions and generalizations of network bargaining. In this short proposal, we mention two specific problems for concreteness.

Distant bargaining. The original model of Kleinberg and Tardos allows for a player to interact only with direct neighbors. We propose to study the generalization where a pair of agents (not necessarily network neighbors) can bargain over the edges/goods of the *path* that connects them. Of course, in this case the intermediate nodes of the path need to be compensated for not participating (and winning) in the bargaining, since if an intermediate pair acquires an edge, then the path will be useless to the two agents it connects. This model tries to capture the nature of goods that are in fact composites of other goods that may be individually coveted by other agents; e.g., two national ISPs may want to bid for an East coast - West coast channel, but smaller (provincial) ISPs can also bid for segments of the channel. We aim at (i) Characterize the combinatorial properties of solutions, and (ii) Solve the bargaining problem in as a general setting as possible.

Stability in bargaining. The work of Kleinberg and Tardos yields a prediction of the distribution of wealth over the nodes of a network through bargaining only if the underlying graph admits a stable solution. As Bateni et al. [1] note, this is the case if and only if the *core* of the underlying instance of the matching game is non-empty. It is unfortunately known that matching game instances often have empty cores. We propose to investigate more forgiving models of network bargaining, that allow for equilibrium solutions in larger classes of graphs. For example, one may allow each player to engage in deals with more than one of its neighbours simultaneously. Alternatively, given an unstable instance, one could attempt to remove the smallest number of edges so that the resulting graph is stable.

References

[1] M. Bateni, M. Hajiaghayi, N. Immorlica, and H. Mahini. The cooperative game theory foundations of network bargaining games. In *Proceedings of International Colloquium on Automata, Languages and Processing*, pages 67–78, 2010.

[2] J. Kleinberg and E. Tardos. Balanced outcomes in social exchange networks. *In Proceedings STOC'08*, 2008.