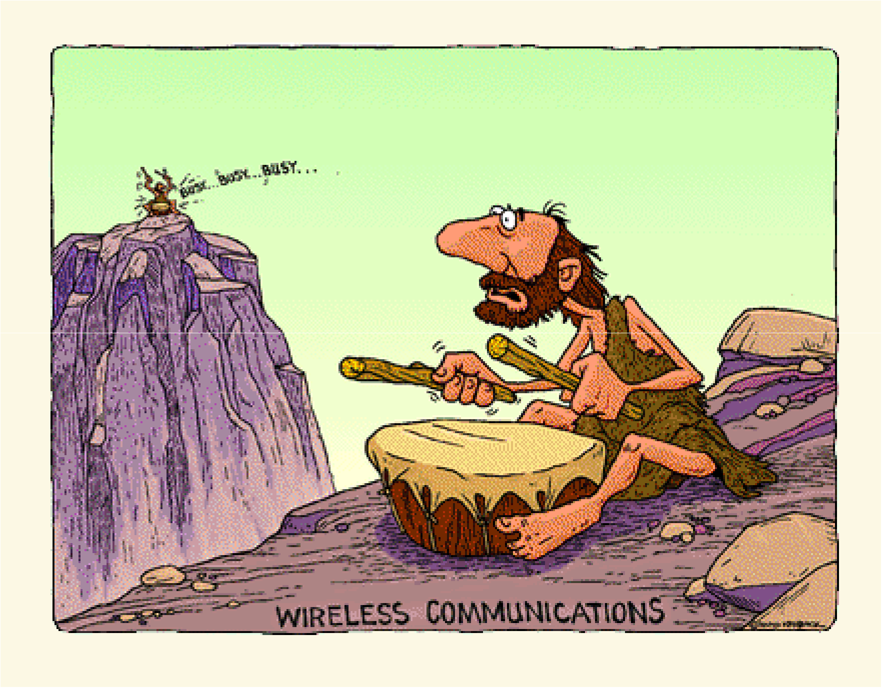
Gossiping in Wireless Networks



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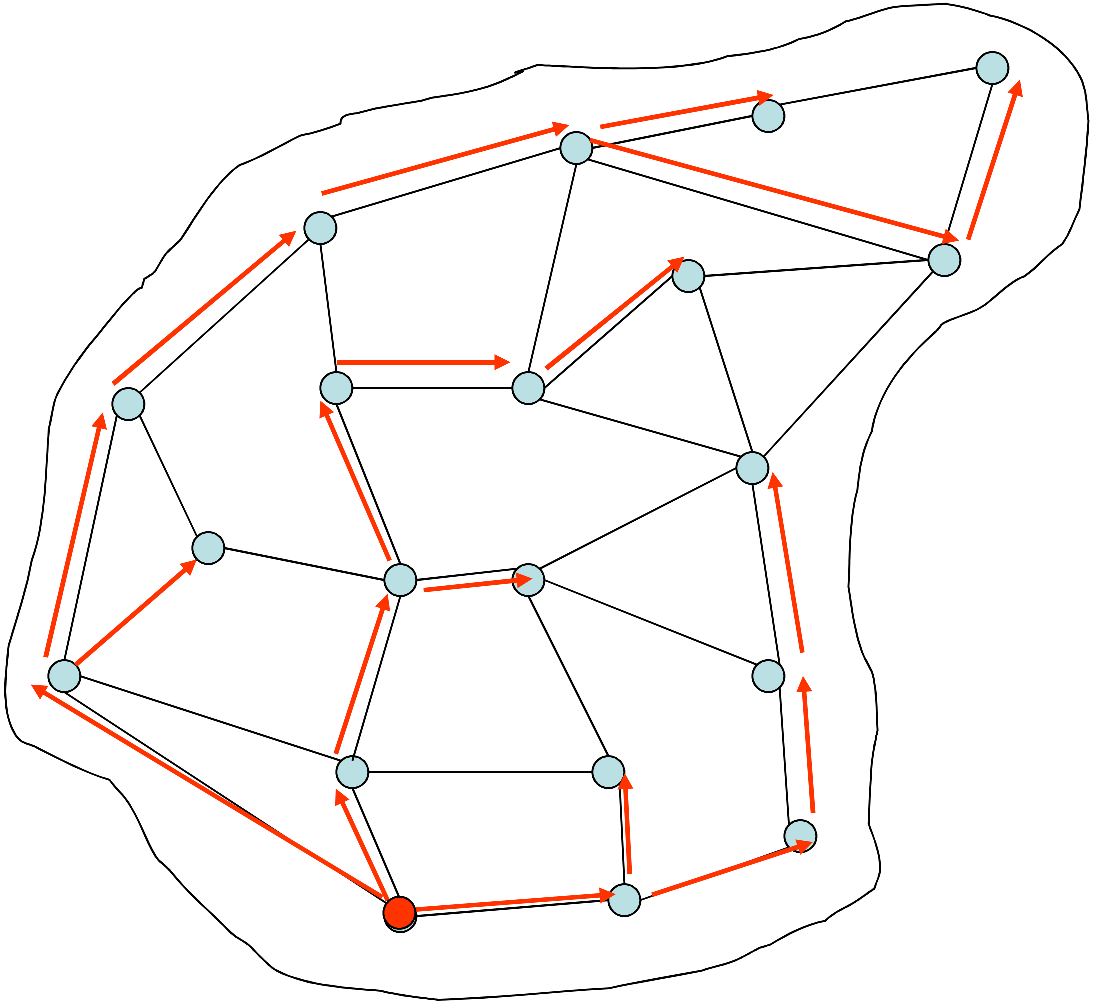
Broadcasting vs. gossiping

- *broadcasting* refers to *one-to-all* communication
- used to disseminate a broadcast message from a distinguished source node to all other nodes in the network

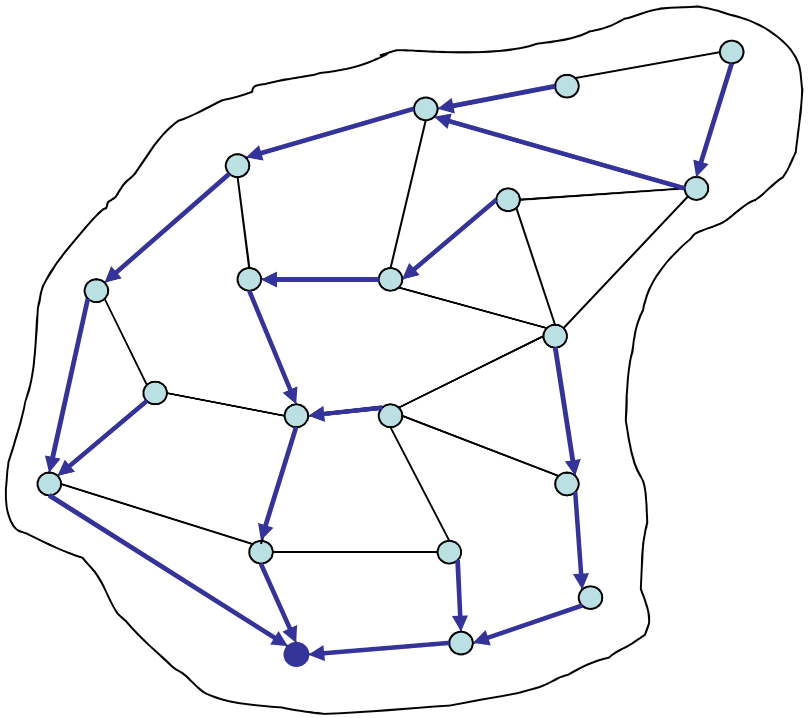
- *gossiping* refers to *all-to-all* communication (total information exchange)
- used to exchange messages within all pairs of nodes (points) in the network

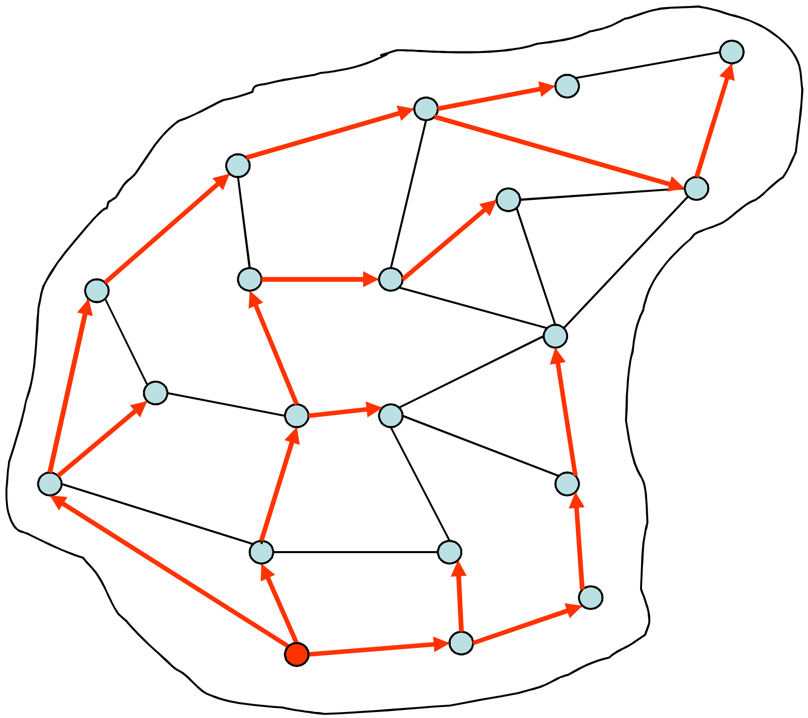
Broadcasting



Gossiping

- gathering
- broadcasting

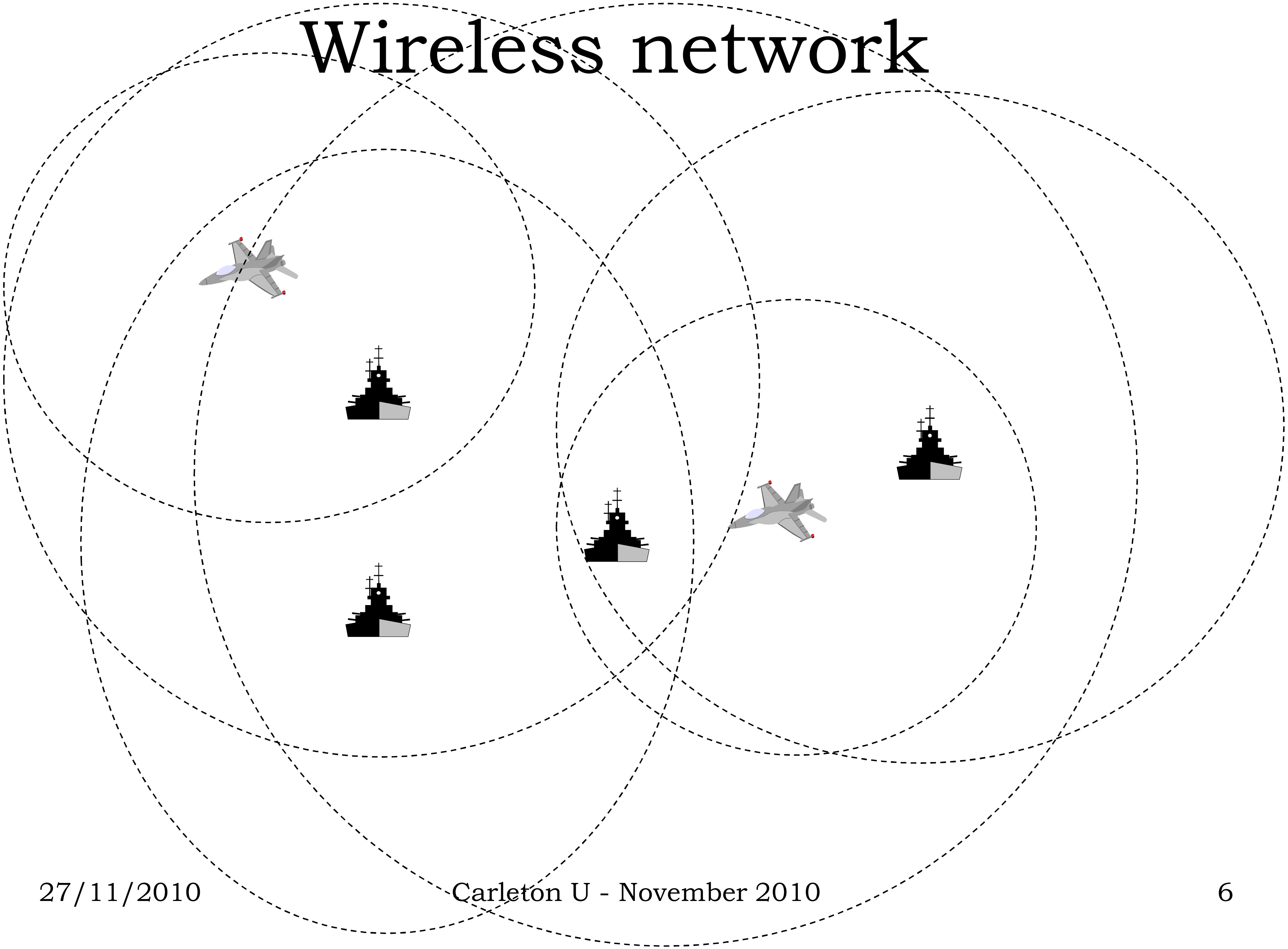




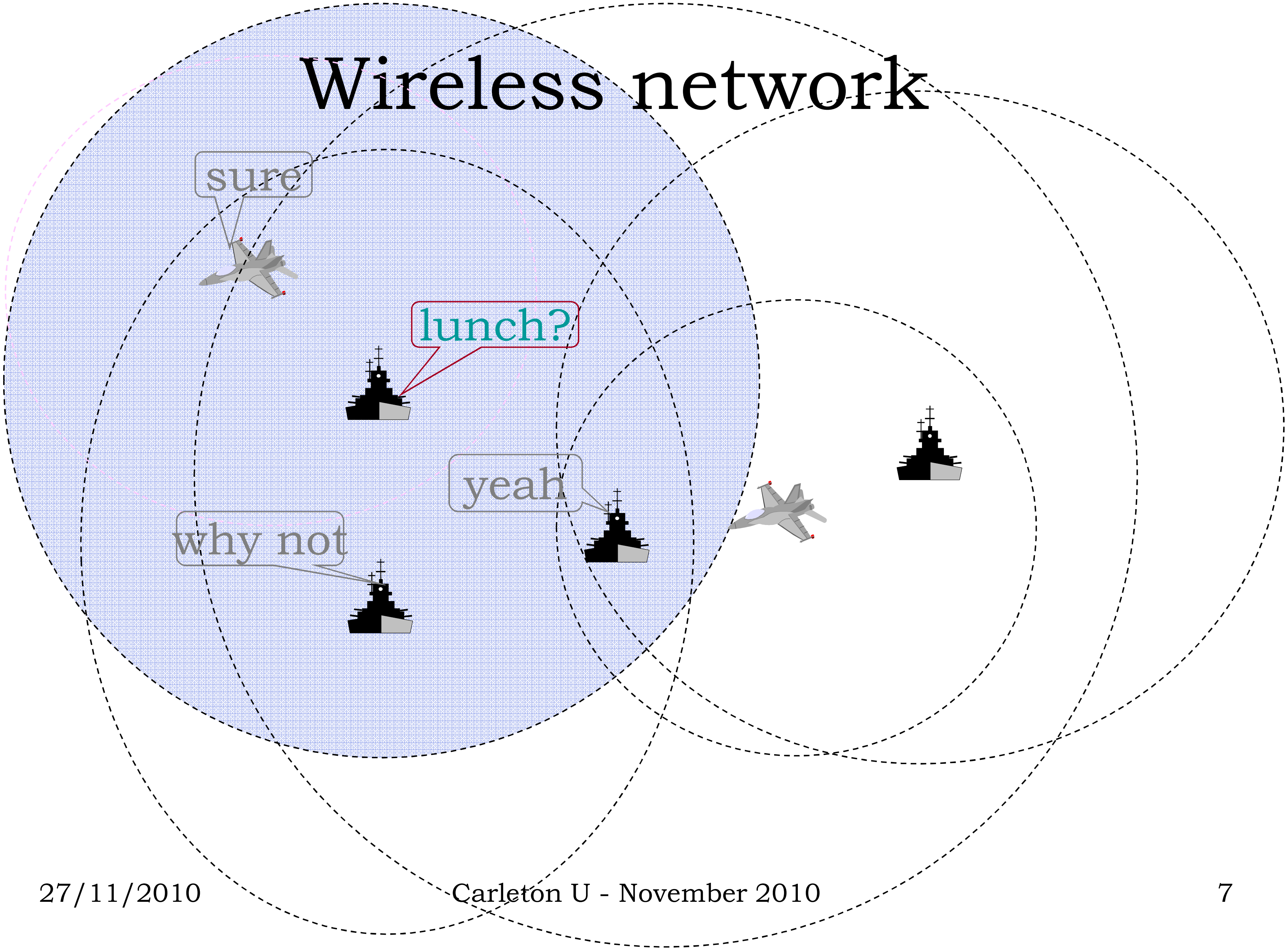
Gossiping motivation

- one of the most fundamental communication primitives
- natural extension of broadcasting
- refers to data aggregation (sensor nets)
- distributed coupon collector's problem
- more complex (graph theory needed)
- ....

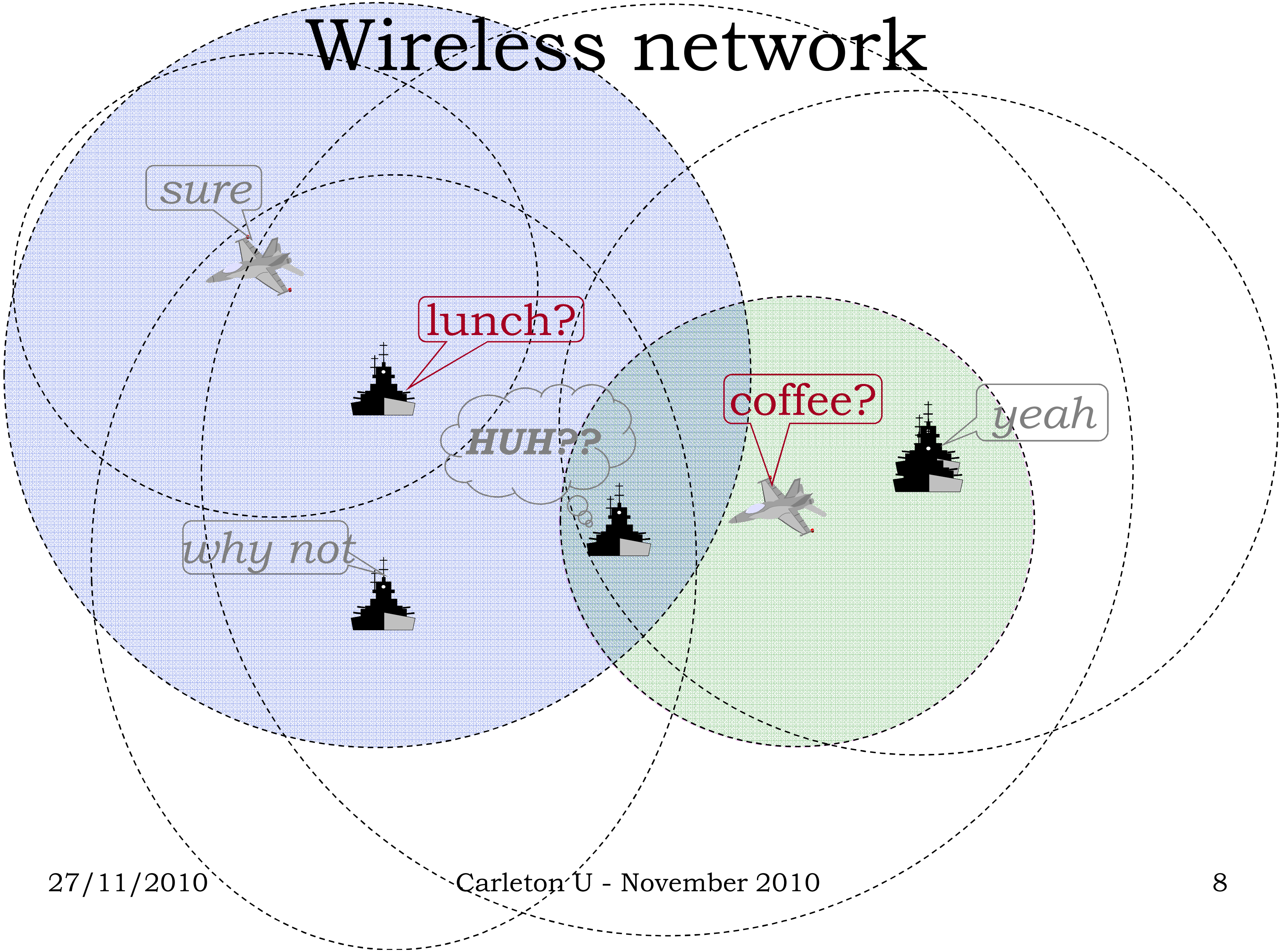
Wireless network



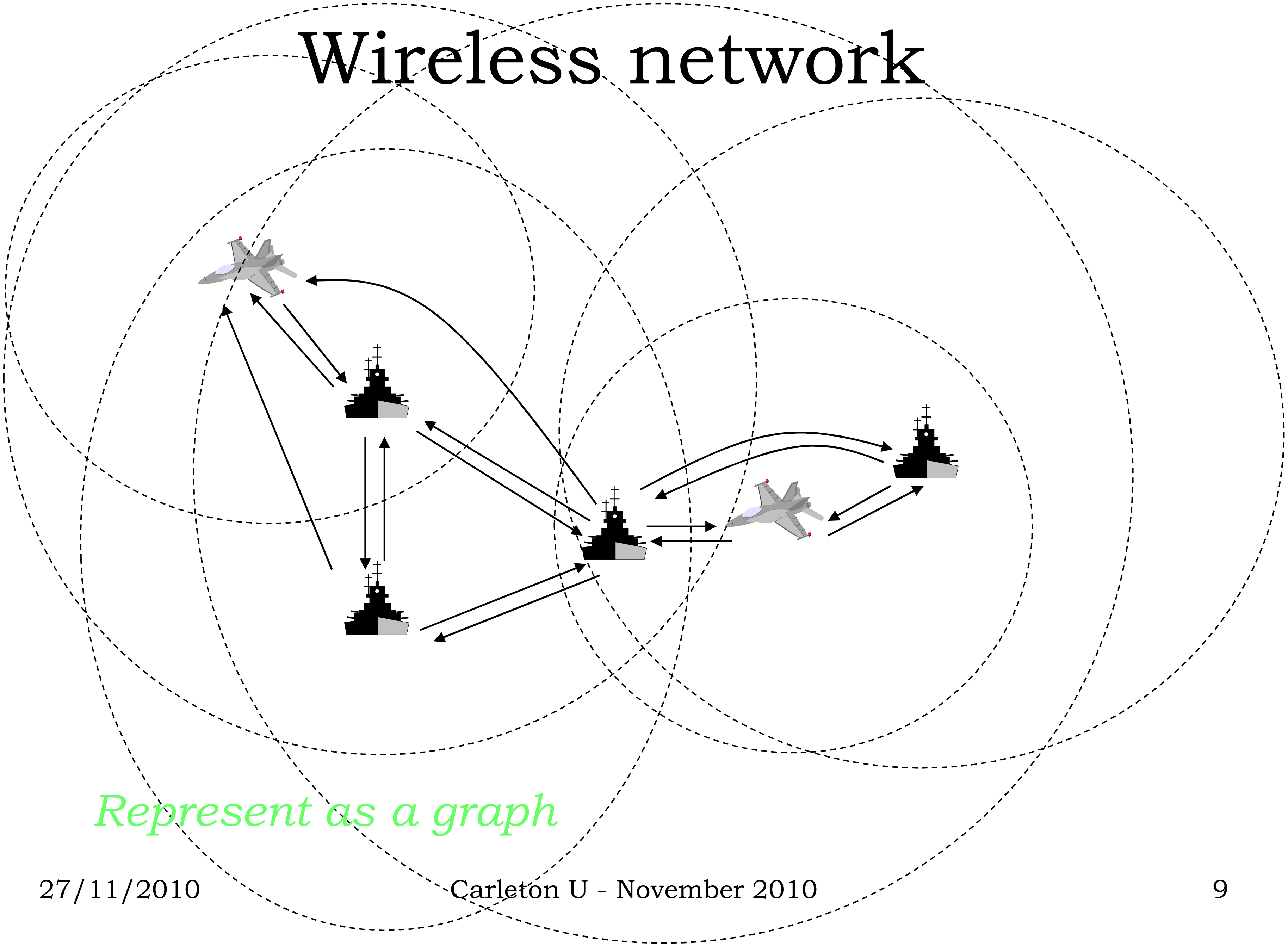
Wireless network



Wireless network

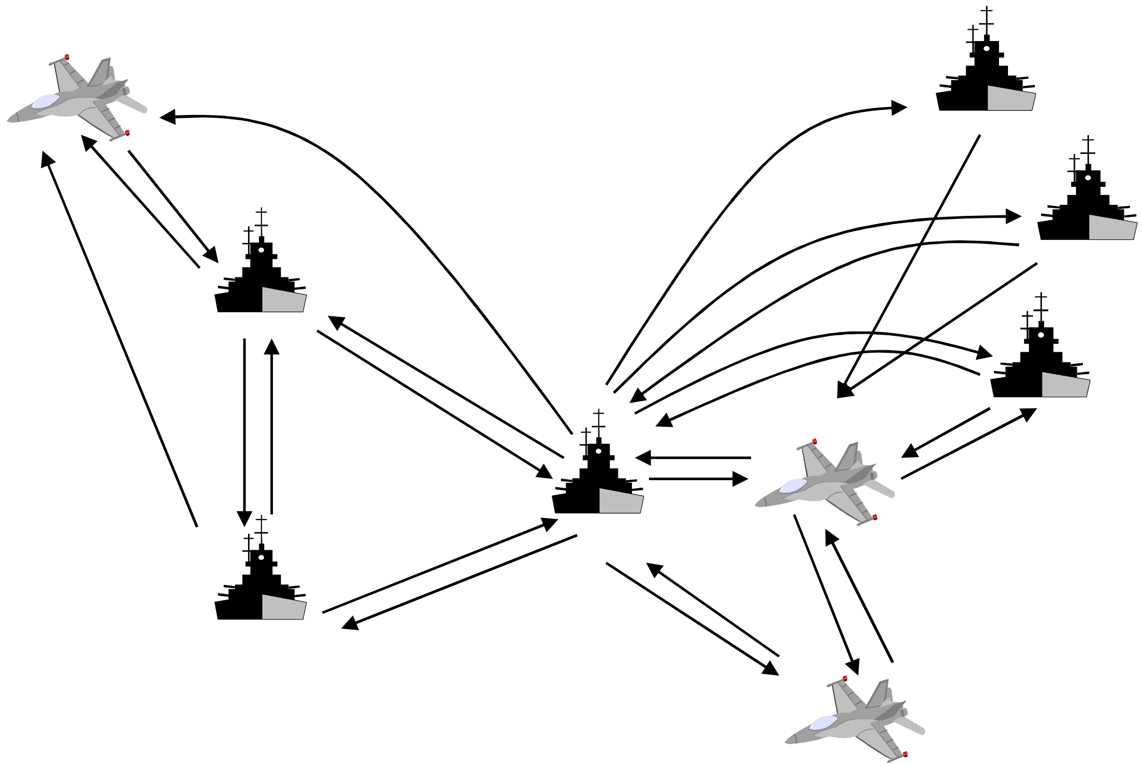


Wireless network



Represent as a graph

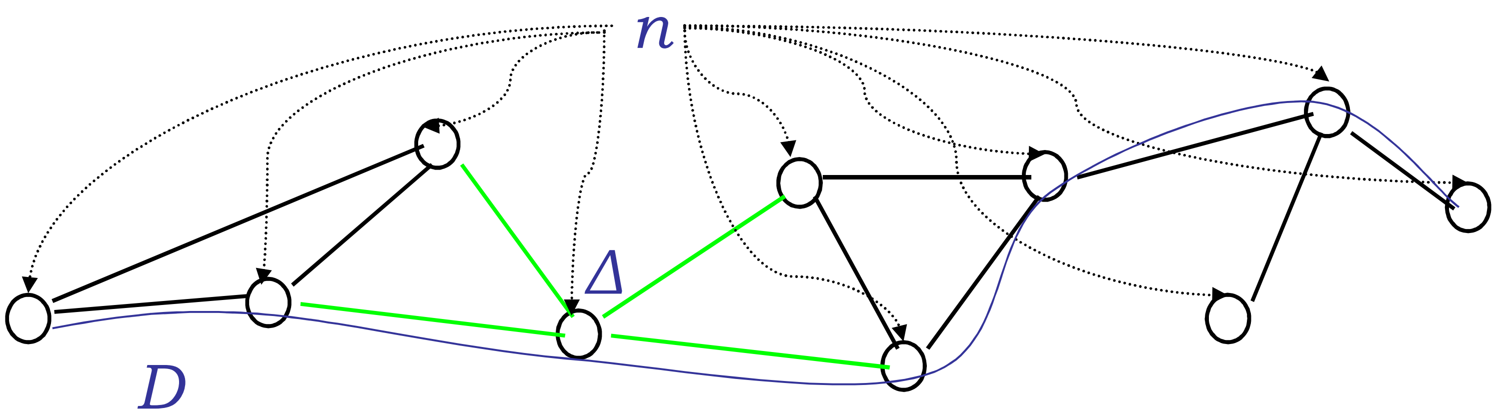
Wireless network



Topology could be unstable ….

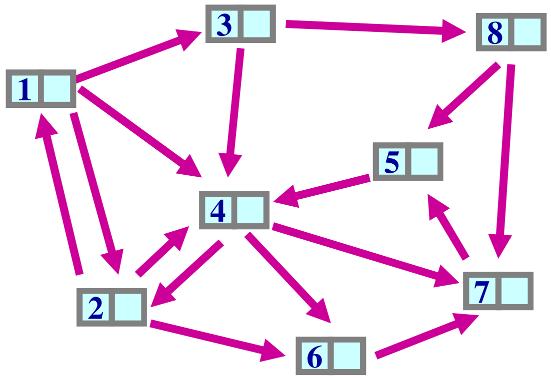
Network parameters

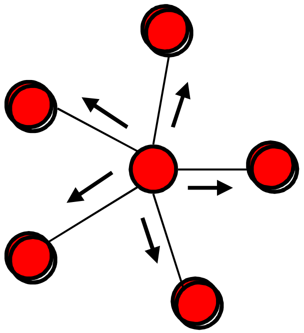
- n number of nodes (devices) in G
- Δ max-degree in G
- D diameter of G, and
- size of messages (constant … unbounded)



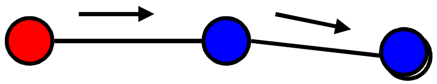
Wireless network protocol

- (un)directed graph with n nodes
- node ids from set {1,2,…,n^c}, c ≥ 1
- full synchronization:
  - discrete time steps
  - shared clock
  - all start at time 0
- communication protocol

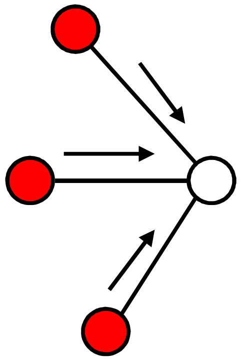




transmission

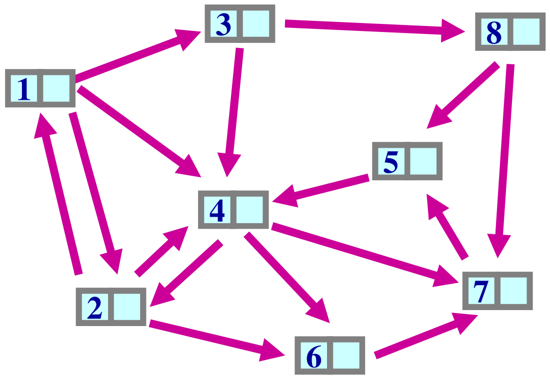


conflict type 1



conflict type 2

Communication algorithms



input/output:

id, *time*, *history* $\rightarrow$ {*receive*, *transmit(m)*}

algorithm: sequence of transmission sets, where each set contains a subset of nodes ids

assumptions: knowledge of size *n* and topology, availability of randomization, capacity of messages

running time: # of steps till all nodes know *t* maximized over all networks of size *n*

other complexity measures: *message complexity* and *energy consumption*

Wireless broadcasting (UN)

- ***deterministic broadcasting***
  - RoundRobin, folklore

    time: $O(n^2)$
  - Chlebus, Gąsieniec, Gibbons, Pelc, Rytter, 2000

    time: $O(n^{11/6})$
  - Chlebus, Gąsieniec, Ostlin, Robson, 2000

    time: $O(n^{3/2})$
  - Chrobak, Gąsieniec, Rytter, 2000

    time: $O(n \cdot \log^2 n)$
  - Kowalski and Pelc, 2003

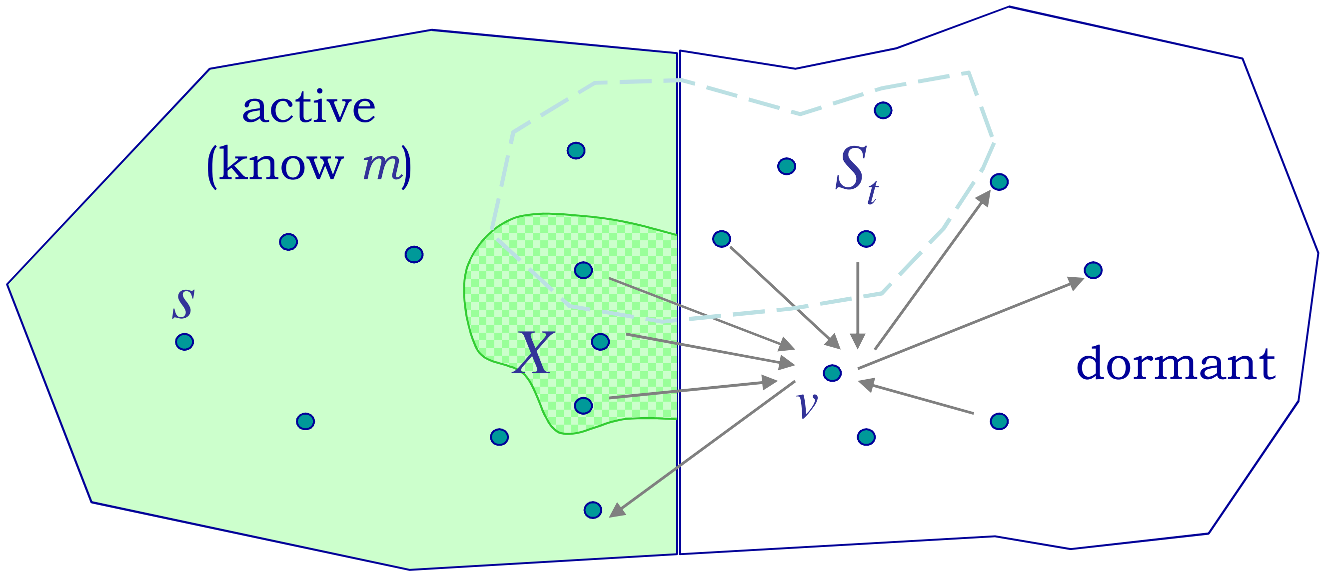
    time: $O(n \cdot \log n \cdot \log D)$
  - Czumaj and Rytter, 2003

    time: $O(n \cdot \log^2 D)$
  - De Marco, 2008

    time: $O(n \cdot \log n \cdot \log\log n)$

Wireless broadcasting (UN)

deterministic broadcasting: at time t node v is dormant, X = active neighbors of v, nodes in set S_t transmit (if informed)



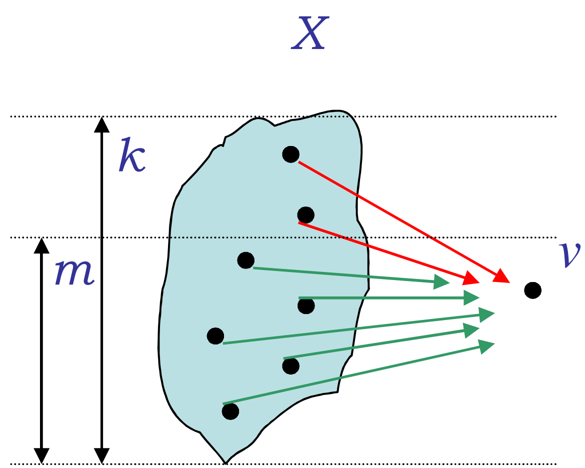
v will receive m *iff* $|S_t \cap X| = 1$

Wireless broadcasting (UN)

- *(k,m,n)-selector* is a family $\boldsymbol{F}$ of subsets (transmission set) of U=\{1,..,n^c\}, s.t.,
  - for any k-subset X of U
  - there are m elements x_1,..,x_m in X, s.t.,
  - for each x_i there is S_j in $\boldsymbol{F}$, s.t., $X \cap S_j = \{x_i\}$.
- *the size of (k,m,n)-selector* is:

$$\theta(k^2 \log n/(k-m+1))$$

  - *almost linear* in k if k-m=$\Omega(k)$
  - *quadratic* m is too close to k



Wireless gossiping (UN)

- ***deterministic gossiping***
  - Chrobak, Gąsieniec and Rytter

in time: $O(n^{3/2}\log^2 n)$

Algorithm *Gossip();*

perform $n^{1/2}\cdot\log^2 n$ rounds of *RoundRobin*;

while $\max_v |K(v)| = 0$

Find a node v_{max}, s.t., $|K(v_{max})| = \max_v |K(v)|$;

Broadcast from v_{max} message $K(v_{max})$;

for each node v

$K(v) \leftarrow K(v) \setminus K(v_{max})$;

Wireless gossiping (UN)

- ***deterministic gossiping*** *follow up*
  - Xu, 2003

    time: $O(n^{3/2})$
  - Clementi, Monti and Silvestri, 2001

    time: $O(n \cdot \Delta^2 \cdot \log^c n)$
  - Gąsieniec and Lingas, 2002,

    time: $O(n \cdot D^{1/2} \cdot \log^c n)$, $O(n \cdot \Delta^{3/2} \cdot \log^c n)$

Wireless gossiping (UN)

- ***deterministic gossiping*** *state of the art*
  - Gąsieniec, Radzik and Xin, 2004

time: $O(n^{4/3}\log^c n)$

  - a *path selector* of length $O(k^2\log^c n)$ allows a node v to push through its own message m along path P with the neighbourhood $\leq k$.

[selection is done across a number of levels]

  - supplemented by *virtual reduction of degrees* provides the *best currently known solution!*

Wireless gossiping (UN)

- ***distributed coupon collection problem***
  - the nodes stand for n *bins* and their messages serve as n *coupons*. Each coupon has > k *copies* in different bins, M_v is the content of bin v.
  - at each step, we open bins at random, by choosing each bin, independently, with probability $1/n$.
  - if exactly one bin, say v, is opened, all coupons from M_v are collected. Otherwise, a failure occurs and no coupons are collected.
  - for any positive constant d<1, repeating the step $(4n/k)\ln(n/d)$ times with probability at least 1-d, all coupons will be collected.

Wireless gossiping (UN)

- ***randomized gossiping***

algorithm RANDGOSSIP$_v$(ε).

$d = \varepsilon/\log n$;

for $i = 0, 1, \ldots, \log n - 1$ ***do*** {*round i*}

repeat $(4n/2^i)\ln(n/d)$ ***times***

with probability $1/n$ ***do***

LTDBROADCAST$_v(2^{i+1})$;

- *INVARIANT:* on the conclusion of round i each message has 2^{i+1} copies in different nodes

Wireless gossiping (UN)

- ***randomized gossiping***
  - Chrobak, Gąsieniec, and Rytter, 2001

    time: $O(n \cdot \log^{4} n)$
  - Liu and Prabhakaran, 2002

    time: $O(n \cdot \log^{3} n)$
  - Czumaj and Rytter, 2003

    time: $O(n \cdot \log^{2} n)$

Wireless gossiping (UN)

- deterministic ***gossiping in symmetric graphs with unbounded messages***
  - Gąsieniec, Pagourtzis, and Potapov 2002

time: $O(n \cdot \log^4 n)$

Wireless gossiping (UN)

- ***gossiping with unit-size messages***
  - Christersson, Gąsieniec, and Lingas, 2002

    time: $O(n^{3/2}\log^c n)$
- gossiping with messages of size n^t
  - Christersson, Gąsieniec, and Lingas, 2002

    time: $O(n^{2-t}\log^c n)$
- rand. gossiping with unit messages
  - Christersson, Gąsieniec, and Lingas, 2002

    time: $O(n \cdot \log^c n)$

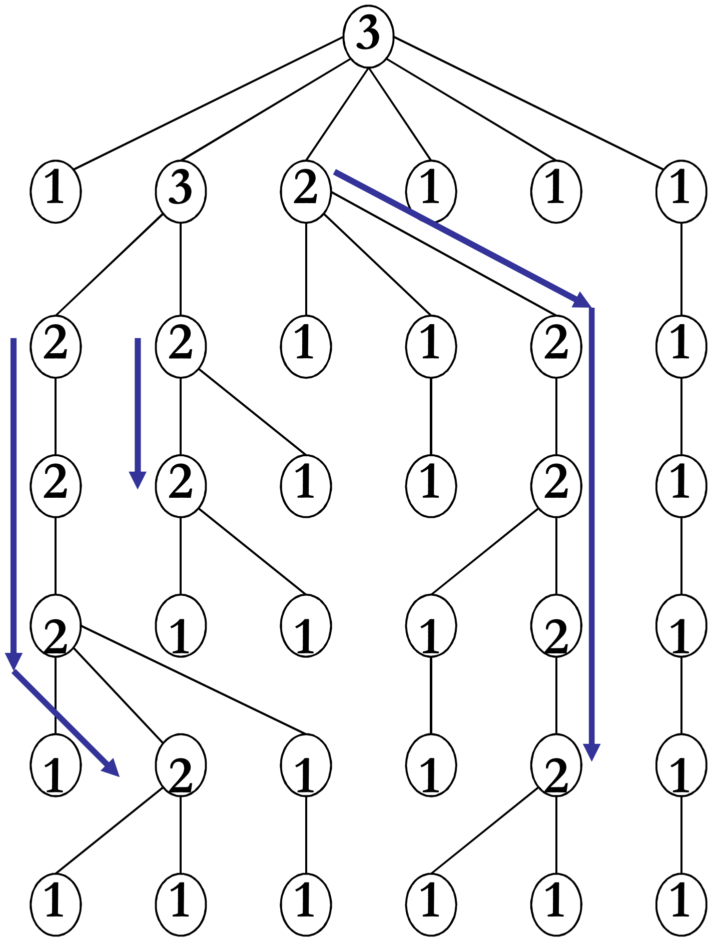
Wireless M2M multicast (KN)

- ***deterministic M2M multicast***
  - Gąsieniec, Kranakis, Pelc and Xin, 2004

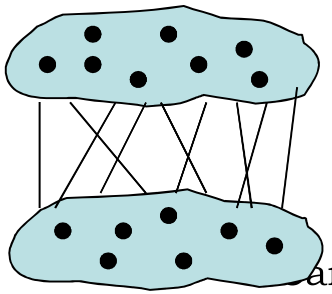
    time: $O(d \cdot \log^2 n + k \cdot \log^4 n)$.
  - where M2M is the problem of exchanging messages within a fixed group of k nodes at unknown position and the maximum distance between any two participating nodes is d
  - an interesting problem of checking whether the whole (sub)network has been discovered is considered

Wireless communication (KN)

- ***broadcasting***
  - tree ranks (< log n)
    - Strahler numbers
  - 2-layer broadcast and
    - slow transmissions
  - pipelining
    - wave of fast transmissions

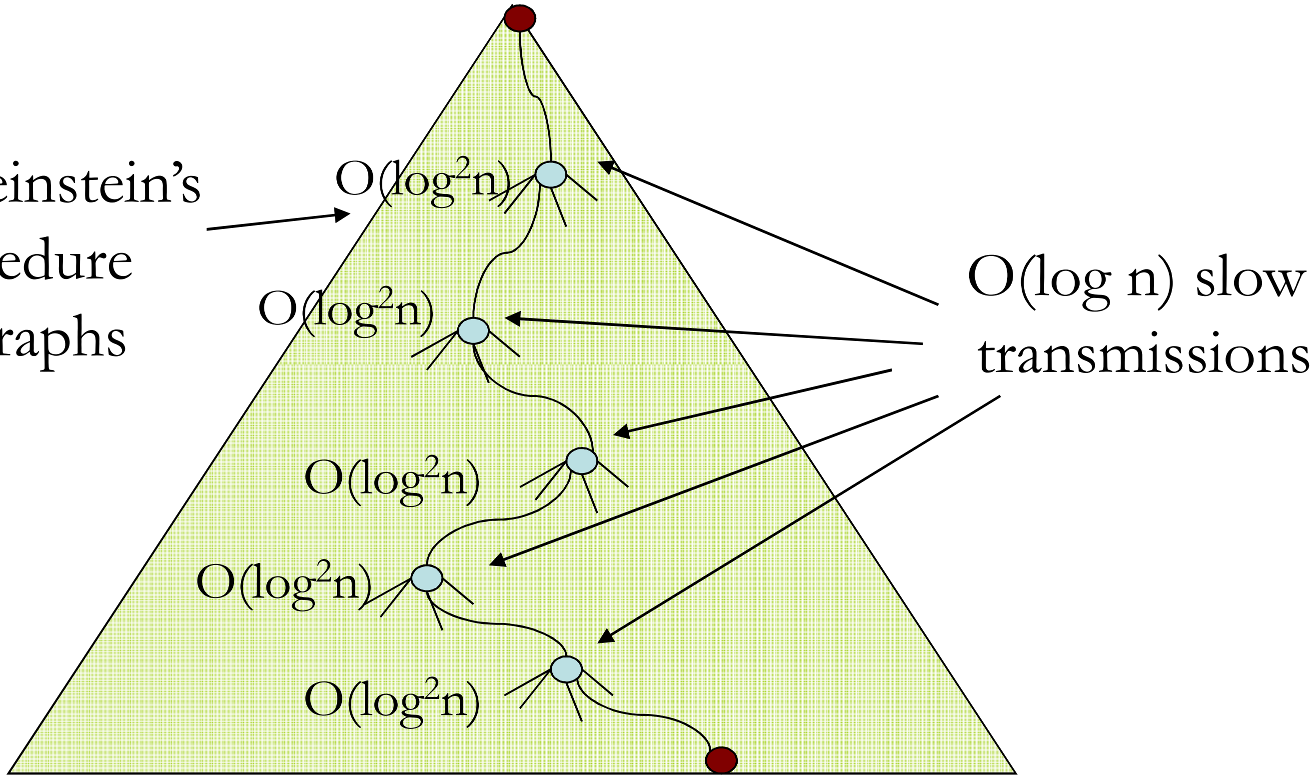


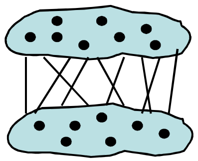
O($\log^2 n$)



Wireless communication (KN)

Chlamtac and Weinstein's broadcast procedure for bipartite graphs



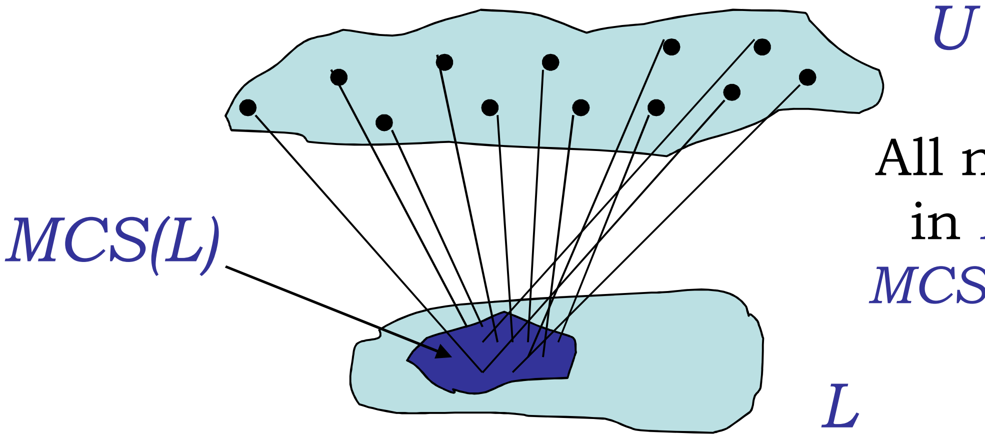


Wireless communication (KN)

- ***broadcasting***
  - Alon, Bar-Noy,Linial, and Peleg, 1991
    time: $\Omega(\log^2 n)$, shallow graphs
  - Chlamtac and Weinstein, 1984
    time: $O(D\cdot\log^2 n)$
  - Kowalski and Pelc, 2004, 2005
    time: $O(D\cdot\log n+\log n)$, $O(D+\log^2 n)$
  - Gaber and Mansour, 1995
    time: $O(D+O(\log^5 n))$
  - Elkin and Kortsarz, 2005
    time: $O(D+O(\log^4 n))$
  - Gąsieniec, Peleg and Xin, 2005
    time: $D+O(\log^3 n)$, $D+O(\log^2 n)$

Wireless gossiping (KN)

- fast transfer of all messages between two layers of a bipartite graph can be done in time O(Δ) using a sequence of *minimal covering sets (MCS)*
  - Gąsieniec, Potapov, and Xin, 2004

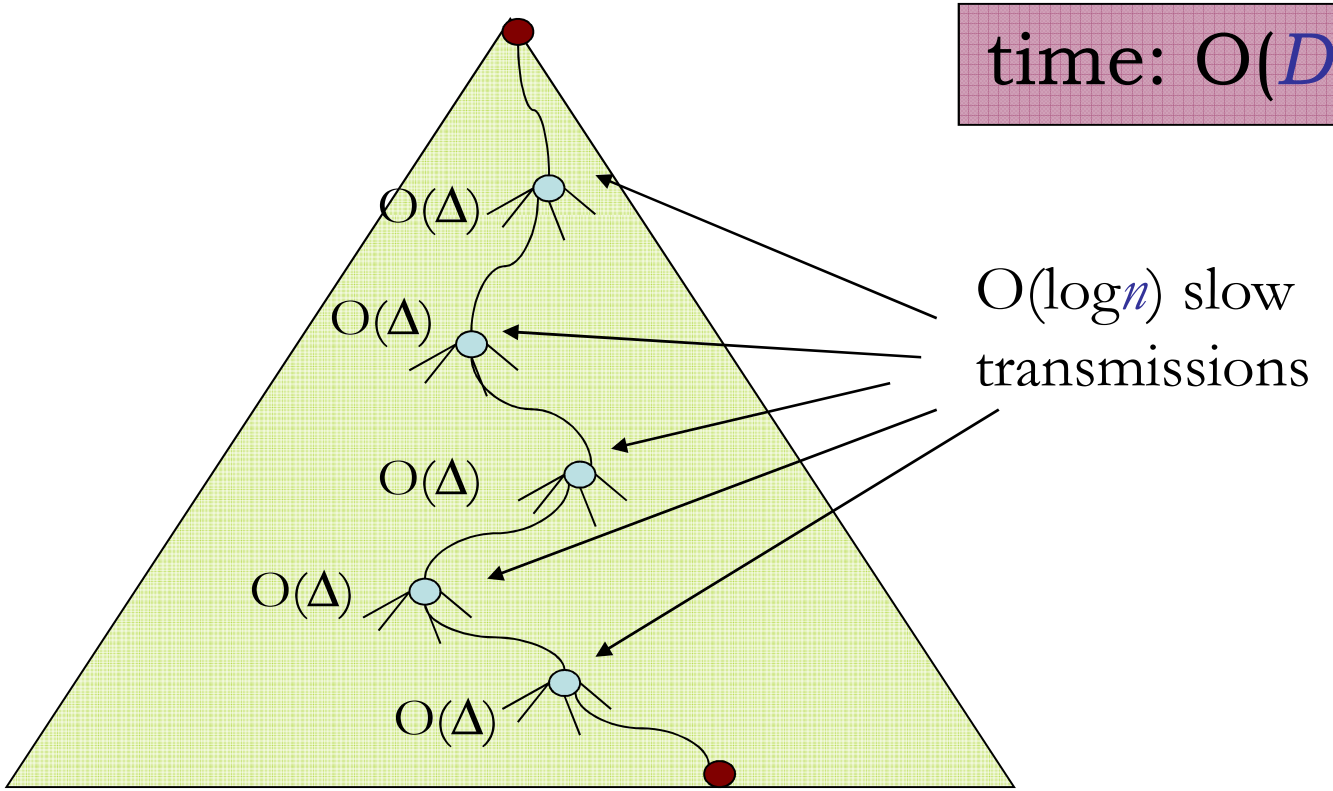


All nodes in *U* have neighbours in *MCS(L)* and neither node in *MCS(L)* can be removed without violating this condition

Wireless gossiping (KN)

- gossiping
  - Gąsieniec, Peleg and Xin, 2005

time: $O(D+\Delta \cdot \log n)$



Wireless gossiping (KN)

- there are graphs (e.g., *star*, *line*) that require n steps for radio gossiping and in any graph n steps suffice.
- ***best topology*** gives the gossiping time $\lfloor \log(n\text{-}1) \rfloor +2$ for a fraction of integers.
  - Gąsieniec, Potapov, and Xin, 2004

Wireless gossiping (KN)

- ***gossiping with small messages***
  - Gąsieniec and Potapov, 2002

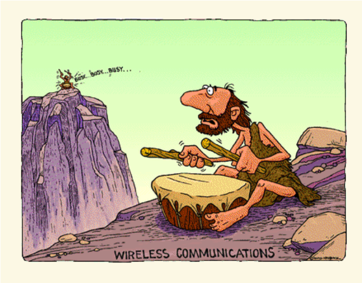
    time: lines $3n$, ring $2n$, trees ~$3.5n$

    time: general graphs $\Omega(n \cdot \log n)$ $O(n \cdot \log^2 n)$
- ***randomised*** counterpart
  - Manne and Xin, 2006

    time: $O(n \cdot \log n)$

Other problems in WN

- wake-up problem
- broadcasting
- neighbourhood search
- leader election
- consensus
- mutual exclusion
- …



Thank you!

A Wireless Gossip

After digging to a depth of 100 meters last year, Japanese scientists found traces of copper wire dating back 1000 years and came to the conclusion that their ancestors already had a telephone network one thousand years ago.

Not to be outdone in the weeks that followed, Chinese scientists dug 200 meters and headlines in the Chinese papers read: "Chinese scientists have found traces of 2000 year old optical fibers and have concluded that their ancestors already had advanced high-tech digital telephone 1000 years earlier than the Japanese."

One week later, the Greek newspapers reported the following: "After digging as deep as 800 meters, Greek scientists have found absolutely nothing." They have concluded that 3000 years ago, their ancestors were already using wireless technology.