Gossiping in Wireless Networks

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Broadcasting vs. gossiping

- **broadcasting** refers to *one-to-all* communication
- used to disseminate a broadcast message from a distinguished source node to all other nodes in the network

- **gossiping** refers to *all-to-all* communication (total information exchange)
- used to exchange messages within all pairs of nodes (points) in the network
Gossiping

• gathering

• broadcasting
Gossiping motivation

• one of the most fundamental communication primitives
• natural extension of broadcasting
• refers to data aggregation (sensor nets)
• distributed coupon collector’s problem
• more complex (graph theory needed)
• ....
Wireless network
Wireless network

(sure)

lunch?

why not

yeah
Wireless network

sure

lunch?

HUH??

coffee?

yeah

why not
Wireless network

Represent as a graph
Wireless network

Topology could be unstable ....
Network parameters

- $n$ number of nodes (devices) in $G$
- $\Delta$ max-degree in $G$
- $D$ diameter of $G$, and
- size of messages (constant ... unbounded)
Wireless network protocol

• (un)directed graph with $n$ nodes
• node ids from set \{1,2,...,$n^c$\}, $c \geq 1$
• full synchronization:
  – discrete time steps
  – shared clock
  – all start at time 0
• communication protocol

transmission

conflict type 1

conflict type 2
Communication algorithms

**input/output:**

\(id, time, history \rightarrow \{receive, transmit(m)\}\)

**algorithm:** sequence of transmission sets, where each set contains a subset of nodes ids

**assumptions:** knowledge of size \(n\) and topology, availability of randomization, capacity of messages

**running time:** \# of steps till all nodes know \(t\) maximized over all networks of size \(n\)

**other complexity measures:** message complexity and energy consumption
Wireless broadcasting (UN)

- **deterministic broadcasting**
  - RoundRobin, folklore
    time: $O(n^2)$
  - Chlebus, Gąsieniec, Gibbons, Pelc, Rytter, 2000
    time: $O(n^{11/6})$
  - Chlebus, Gąsieniec, Ostlin, Robson, 2000
    time: $O(n^{3/2})$
  - Chrobak, Gąsieniec, Rytter, 2000
    time: $O(n \cdot \log^2 n)$
  - Kowalski and Pelc, 2003
    time: $O(n \cdot \log n \cdot \log D)$
  - Czumaj and Rytter, 2003
    time: $O(n \cdot \log^2 D)$
  - De Marco, 2008
    time: $O(n \cdot \log n \cdot \log \log n)$
Wireless broadcasting (UN)

**Deterministic broadcasting:** at time $t$, node $v$ is dormant, $X$ = active neighbors of $v$, nodes in set $S_t$ transmit (if informed)

$v$ will receive $m$ iff $|S_t \cap X| = 1$
Wireless broadcasting (UN)

- \((k,m,n)\)-selector is a family \(F\) of subsets (transmission set) of \(U=\{1,\ldots,n^c\}\), s.t.,
  - for any \(k\)-subset \(X\) of \(U\)
  - there are \(m\) elements \(x_1,\ldots,x_m\) in \(X\), s.t.,
  - for each \(x_i\) there is \(S_j\) in \(F\), s.t., \(X \cap S_j=\{x_i\}\).

- **the size of \((k,m,n)\)-selector** is:
  \[\theta(\frac{k^2 \log n}{(k-m+1)})\]
  - almost linear in \(k\) if \(k-m=\Omega(k)\)
  - quadratic \(m\) is too close to \(k\)
Wireless gossiping (UN)

• **deterministic gossiping**
  – Chrobak, Gąsieniec and Rytter
  in time: $O\left(n^{3/2}\log^2 n\right)$

**Algorithm** `Gossip();`

- perform $n^{1/2}\cdot\log^2 n$ rounds of `RoundRobin;`

**while** max $v \mid K(v) \mid = 0$

  *Find a node $v_{max}$, s.t., $\mid K(v_{max}) \mid = \max v \mid K(v) \mid$;*
  
  *Broadcast from $v_{max}$ message $K(v_{max})$;*

  **for** each node $v$
  
  $K(v) \leftarrow K(v) \setminus K(v_{max});$
Wireless gossiping (UN)

- **deterministic gossiping** follow up
  - Xu, 2003
    time: $O(n^{3/2})$
  - Clementi, Monti and Silvestri, 2001
    time: $O(n \cdot \Delta^2 \cdot \log^c n)$
  - Gaśieniec and Lingas, 2002,
    time: $O(n \cdot D^{1/2} \cdot \log^c n), O(n \cdot \Delta^{3/2} \cdot \log^c n)$
Wireless gossiping (UN)

- **deterministic gossiping** *state of the art*
  - Gąsieniec, Radzik and Xin, 2004
    
    time: $O(n^{4/3} \log^c n)$

  - a *path selector* of length $O(k^2 \log^c n)$ allows a node $v$ to push through its own message $m$ along path $P$ with the neighbourhood $\leq k$.

  [selection is done across a number of levels]

  - supplemented by *virtual reduction of degrees* provides the *best currently known solution!*
Wireless gossiping (UN)

• **distributed coupon collection problem**
  - the nodes stand for *n bins* and their messages serve as *n coupons*. Each coupon has *> k copies* in different bins, $M_v$ is the content of bin $v$.
  - at each step, we open bins at random, by choosing each bin, independently, with probability $1/n$.
  - if exactly one bin, say $v$, is opened, all coupons from $M_v$ are collected. Otherwise, a failure occurs and no coupons are collected.
  - for any positive constant $d<1$, repeating the step $(4n/k)\ln(n/d)$ times with probability at least $1-d$, all coupons will be collected.
Wireless gossiping (UN)

• *randomized gossiping*

**Algorithm** RANDGOSSIP

\[ d = \frac{\varepsilon}{\log n}; \]

\[ \text{for } i = 0, 1, \ldots, \log n - 1 \text{ do } \{ \text{round } i \} \]

\[ \text{repeat } (\frac{4n}{2^i}) \ln (n/d) \text{ times} \]

\[ \text{with probability } \frac{1}{n} \text{ do } \]

\[ \text{LTDBROADCAST}_{v}(2^{i+1}); \]

• **INVARIANT**: on the conclusion of round \( i \) each message has \( 2^{i+1} \) copies in different nodes
Wireless gossiping (UN)

- **randomized gossiping**
  - Chrobak, Gaśieniec, and Rytter, 2001
    time: $O(n \cdot \log^4 n)$
  - Liu and Prabhakaran, 2002
    time: $O(n \cdot \log^3 n)$
  - Czumaj and Rytter, 2003
    time: $O(n \cdot \log^2 n)$
Wireless gossiping (UN)

• deterministic *gossiping in symmetric graphs with unbounded messages*
  – Gąsieniec, Pagourtzis, and Potapov 2002
  \[ \text{time: } O(n \log^4 n) \]
Wireless gossiping (UN)

• **gossiping with unit-size messages**
  – Christersson, Gąsieniec, and Lingas, 2002
    \[\text{time: } O(n^{3/2} \log^c n)\]

• gossiping with messages of size \(n^t\)
  – Christersson, Gąsieniec, and Lingas, 2002
    \[\text{time: } O(n^{2-t} \log^c n)\]

• rand. gossiping with unit messages
  – Christersson, Gąsieniec, and Lingas, 2002
    \[\text{time: } O(n \log^c n)\]
Wireless M2M multicast (KN)

• deterministic M2M multicast
  – Gąsieniec, Kranakis, Pelc and Xin, 2004
    time: $O(d \cdot \log^2 n + k \cdot \log^4 n)$.
  – where M2M is the problem of exchanging messages within a fixed group of $k$ nodes at unknown position and the maximum distance between any two participating nodes is $d$
  – an interesting problem of checking whether the whole (sub)network has been discovered is considered
Wireless communication (KN)

- **broadcasting**
  - tree ranks ($< \log n$)
    - Strahler numbers
  - 2-layer broadcast and
    - slow transmissions
  - pipelining
    - wave of fast transmissions

$O(\log^2 n)$
Wireless communication (KN)

Chlamtac and Weinstein’s broadcast procedure for bipartite graphs

$O(\log^2 n)$

$O(\log^2 n)$

$O(\log^2 n)$

$O(\log^2 n)$

$O(\log^2 n)$

$O(\log n)$ slow transmissions
Wireless communication (KN)

• Broadcasting
    time: $\Omega(\log^2 n)$, shallow graphs
  – Chlamtac and Weinstein, 1984
    time: $O(D \log^2 n)$
  – Kowalski and Pelc, 2004, 2005
    time: $O(D \log n + \log n)$, $O(D + \log^2 n)$
  – Gaber and Mansour, 1995
    time: $O(D + O(\log^5 n))$
  – Elkin and Kortsarz, 2005
    time: $O(D + O(\log^4 n))$
  – Gąsieniec, Peleg and Xin, 2005
    time: $D + O(\log^3 n)$, $D + O(\log^2 n)$
Wireless gossiping (KN)

- fast transfer of all messages between two layers of a bipartite graph can be done in time $O(\Delta)$ using a sequence of *minimal covering sets (MCS)*
  - Gąsieniec, Potapov, and Xin, 2004

\[U\]

All nodes in $U$ have neighbours in $MCS(L)$ and neither node in $MCS(L)$ can be removed without violating this condition.
Wireless gossiping (KN)

- gossiping
  - Gąsieniec, Peleg and Xin, 2005

\[ \text{time: } O(D + \Delta \log n) \]
Wireless gossiping (KN)

• there are graphs (e.g., star, line) that require \( n \) steps for radio gossiping and in any graph \( n \) steps suffice.

• **best topology** gives the gossiping time \( \lceil \log(n-1) \rceil + 2 \) for a fraction of integers.
  
  – Gąsieniec, Potapov, and Xin, 2004
Wireless gossiping (KN)

• *gossiping with small messages*
  – Gaśieniec and Potapov, 2002
    time: lines $3n$, ring $2n$, trees $\sim 3.5n$
    time: general graphs $\Omega(n \log n)$ $O(n \log^2 n)$

• *randomised* counterpart
  – Manne and Xin, 2006
    time: $O(n \log n)$
Other problems in WN

- wake-up problem
- broadcasting
- neighbourhood search
- leader election
- consensus
- mutual exclusion
- ...

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Thank you!

A Wireless Gossip

After digging to a depth of 100 meters last year, Japanese scientists found traces of copper wire dating back 1000 years and came to the conclusion that their ancestors already had a telephone network one thousand years ago.

Not to be outdone in the weeks that followed, Chinese scientists dug 200 meters and headlines in the Chinese papers read: "Chinese scientists have found traces of 2000 year old optical fibers and have concluded that their ancestors already had advanced high-tech digital telephone 1000 years earlier than the Japanese."

One week later, the Greek newspapers reported the following: "After digging as deep as 800 meters, Greek scientists have found absolutely nothing." They have concluded that 3000 years ago, their ancestors were already using wireless technology.