Minimizing the number of Sensors Moved on Line Segment or Circle Barriers

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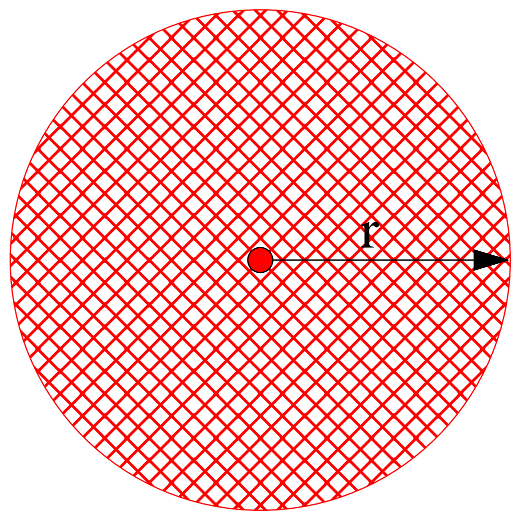
Intrusion Detection by Sensors

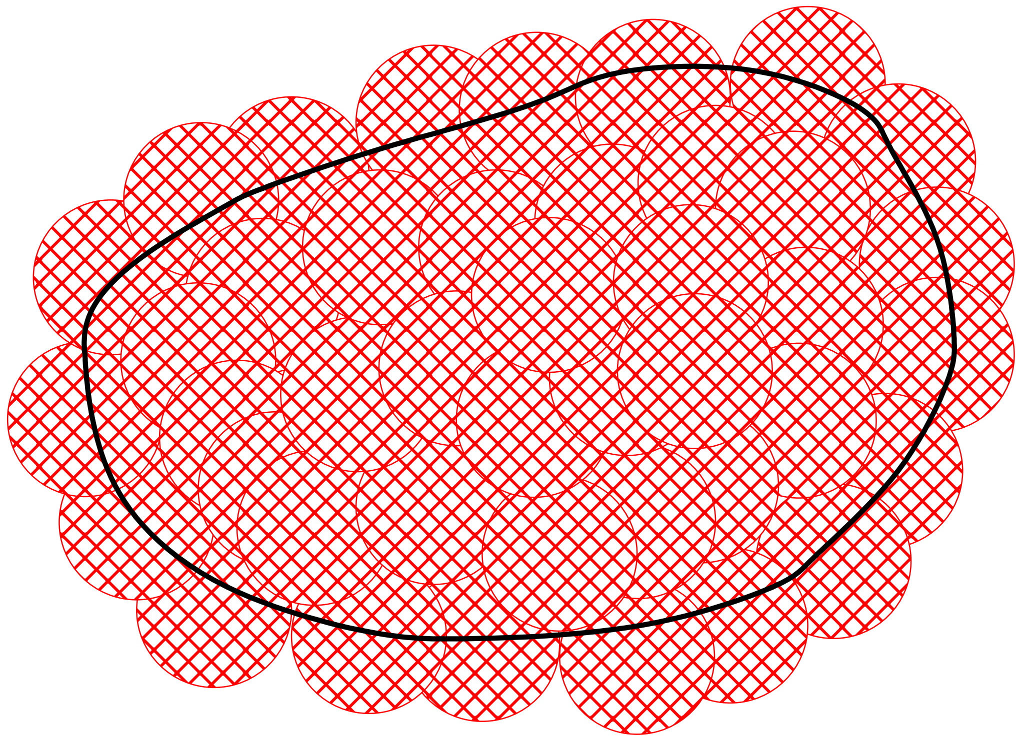
- A region can be protected using a sensor network.
- Each sensor has a sensing range r:



- A sensor detects an object entering its sensing range.

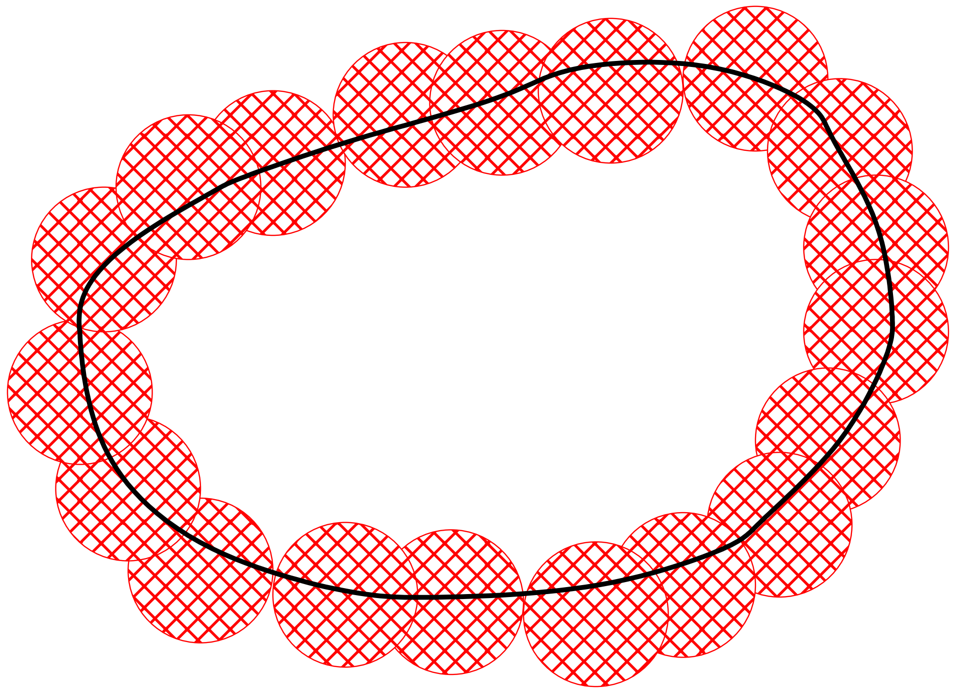
Intrusion Detection by Sensors

Full coverage of a region:



Intrusion Detection by Sensors

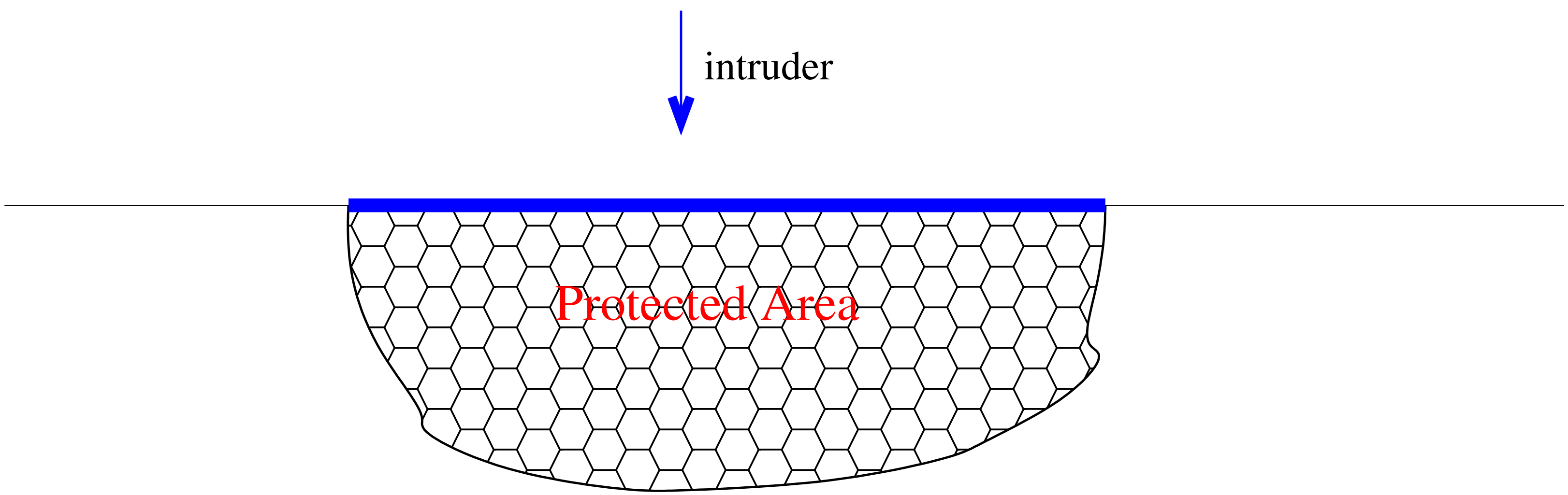
- Barrier coverage of a region: cover only its border.



- Barrier coverage is sufficient in many cases,
- and is cheaper.

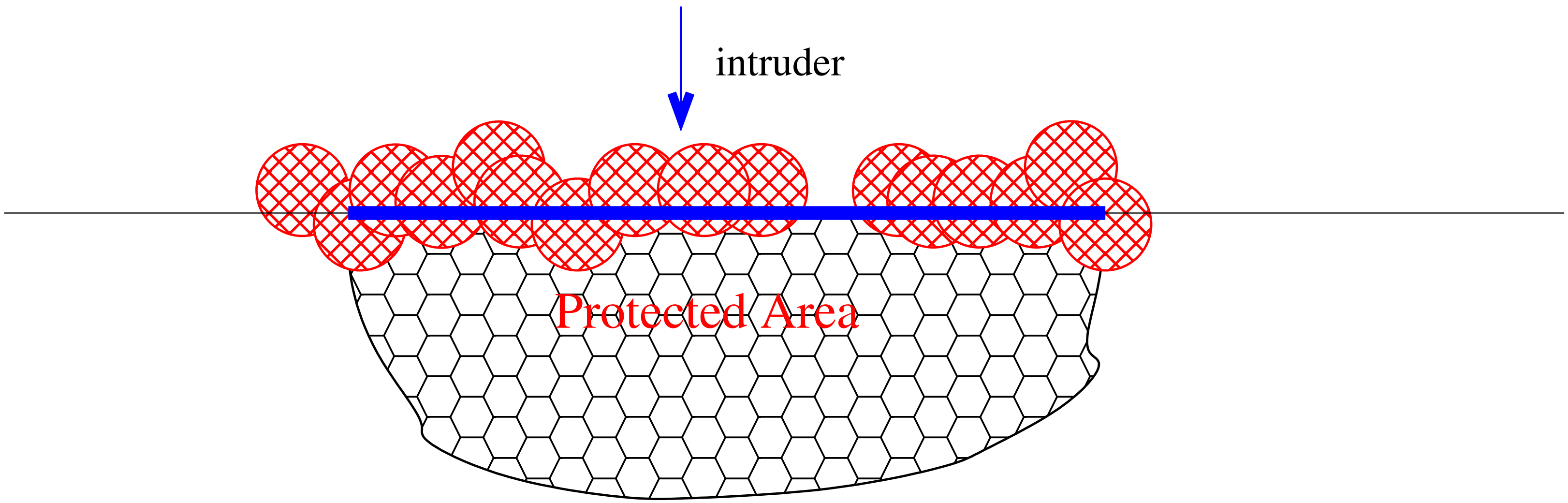
Line Segment Barrier

- We first consider a simplified case of a barrier coverage,
- when we need to cover a line segment (in blue) of the border.



Covering Line Segment Barrier

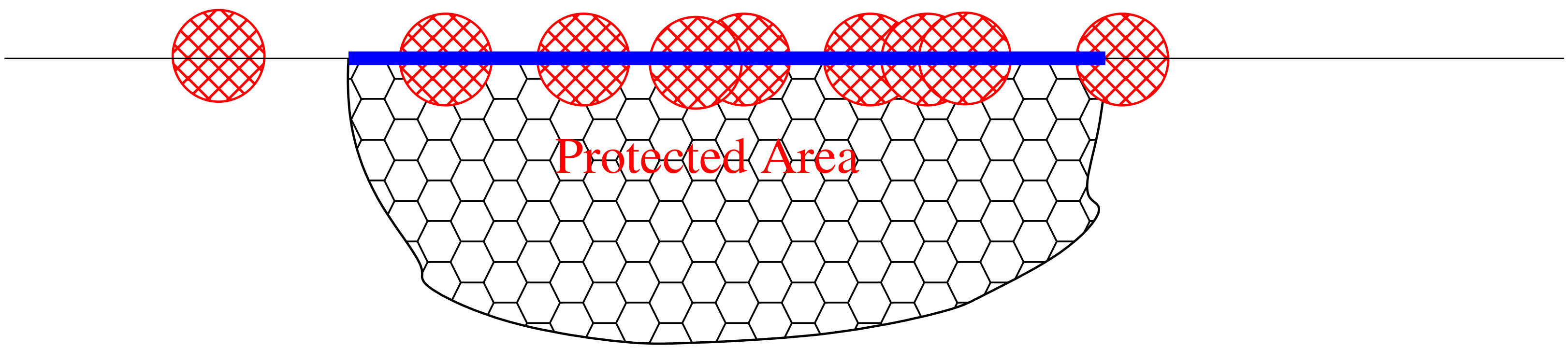
- (1) Using static sensors:
- Sensors are scattered randomly in a band along the barrier



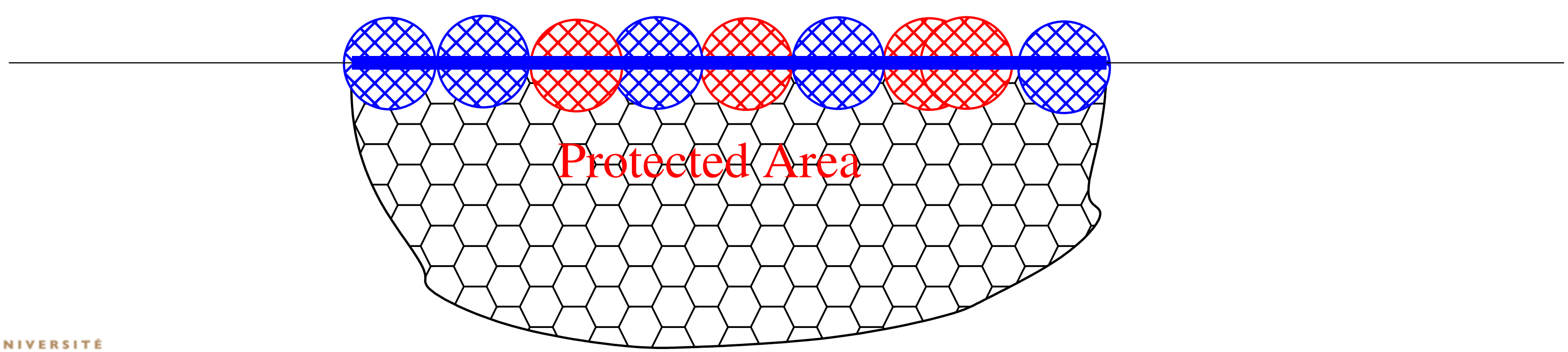
- People often study how many sensors are needed to provide a coverage with high probability.
- Drawback: Large number of sensors is needed.

Covering Line Segment Barrier

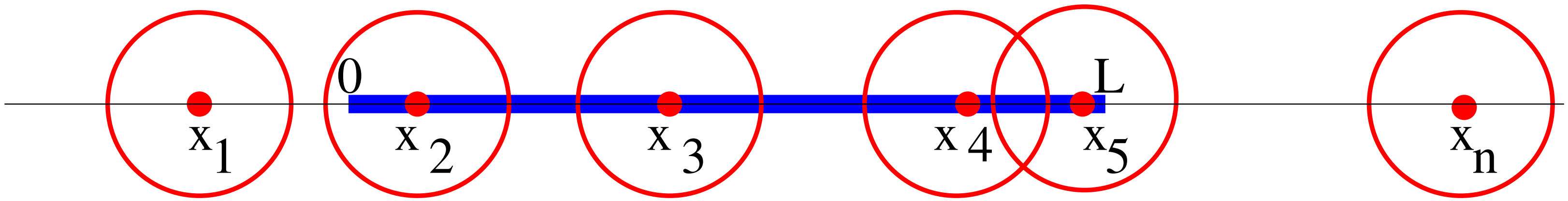
- (2) Using mobile sensors:
- Sensors are scattered on the line along the barrier



- Some sensors move to provide a barrier coverage.



Line Segment Barrier Problem:



- Given a line segment [0,L],
- and n sensors of sensing range $r_1, r_2, \ldots, r_n$
- in initial positions $x_1 \leq x_2 \leq \cdots \leq x_n$ on the line,
- determine the final positions of sensors so that
  1. the line segment is covered, and
  2. a particular aspect of sensors moves is optimized.

Optimizations Studied Previously

- Minimize the maximal movement of sensors (MinMax).

  (A centralized algorithm is given in J. Czyzowicz et al., LNCS v. 5793, 2009)

- Minimize the sum of movement of sensors (MinSum).

  (A centralized algorithm is given in J. Czyzowicz et al., LNCS v. 6288, pp. 29-42, 2010)

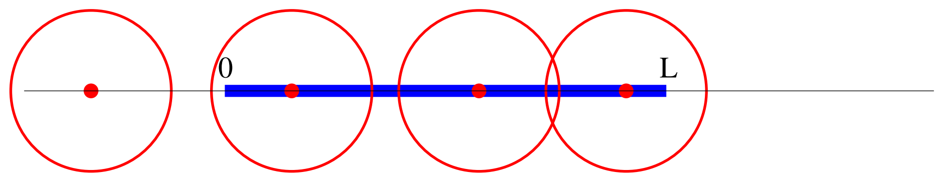
- Algorithms for the two problem are different.

- Both motivated by saving sensor's energy.

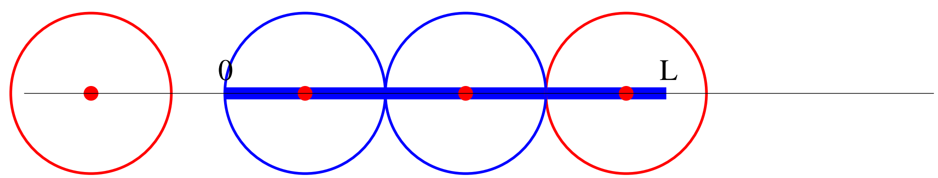
Our Optimization Problem: MinNum

- Minimize the number of sensors that must move.
  We call it MinNum.
- Why MinNum:
  - The energy cost of the movement start-up of a sensor can be more important than the eventual size of the move.
  - It would be easier to organize a move of a smaller number of sensors.

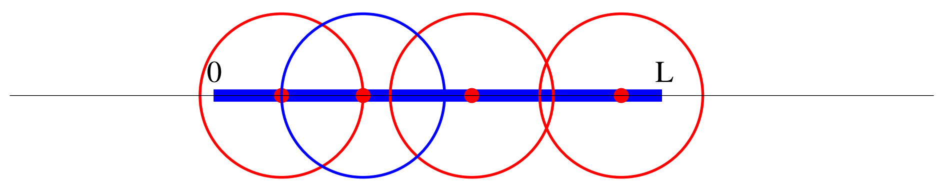
- Given an instance of the barrier coverage problem, MinMax, MinSum, MinNum optimization problem typically give a different solution.



- MinMax solution:



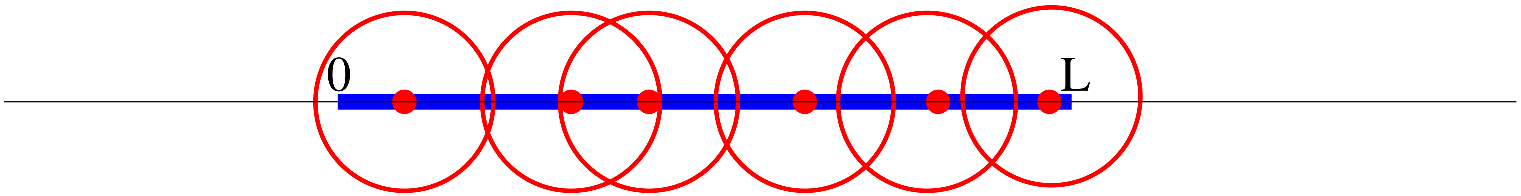
- MinNum solution:



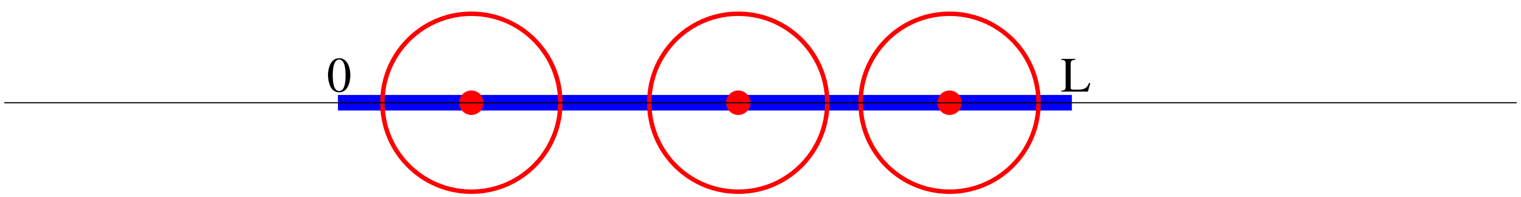
Sub-problems of MinNum

- Let R be the sum of the sensing diameters of the sensors.
- The coverage of the barrier segment is possible only when $R \geq L$.
- We consider several sub-problems of MinNum:
  1. $R \geq L$, full coverage,
  2. $R < L$ and the coverage is maximized,
  3. $R < L$ and the coverage is maximized and contiguous.

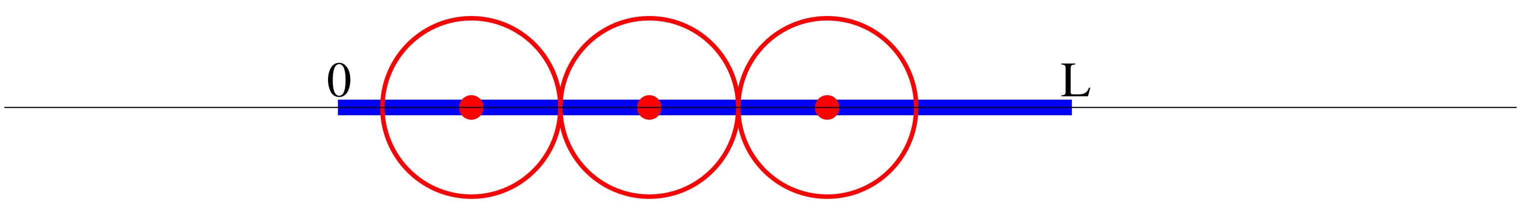
1. $R \geq L$, full coverage:



2. $R < L$, maximal coverage:



3. $R < L$, maximal coverage, contiguous:



Our Results

- The MinNum problem on a line segment $[0; L]$ is NP-hard,
  when sensors have unequal sensing ranges.
- The proof is done by reducing the partition problem to the MinNum problem.
- It remains NP-hard even on the infinite line in the contiguous case.
- Thus we now consider the case of homogeneous sensors with the identical sensor ranges.

Identical Sensor Ranges

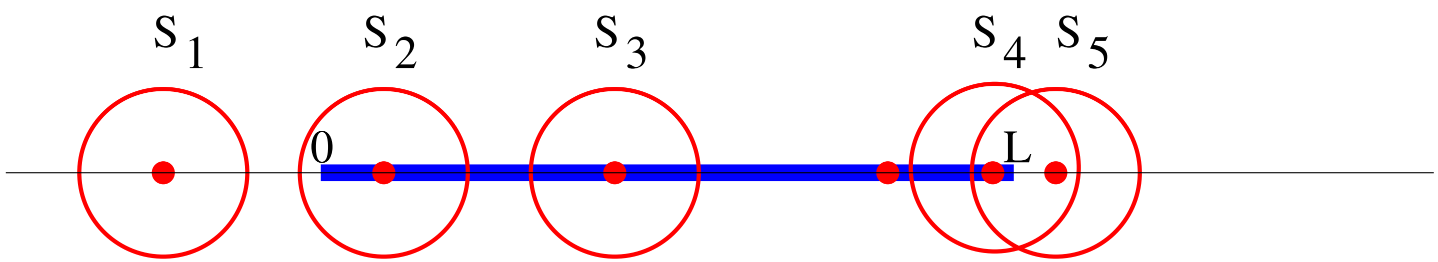
- We have low-degree centralized algorithm for each case:

	$Contiguous$	$non-contiguous$
$R = L$	$O(n)$	$n.a.$
$R > L$	$O(n^3)$	$n.a.$
$R < L$	$O(n^2)$	$O(n^3)$

	$Contiguous$	$non-contiguous$
$infinite\ line$	$O(n^2)$	$O(n)$

Algorithm for R >L

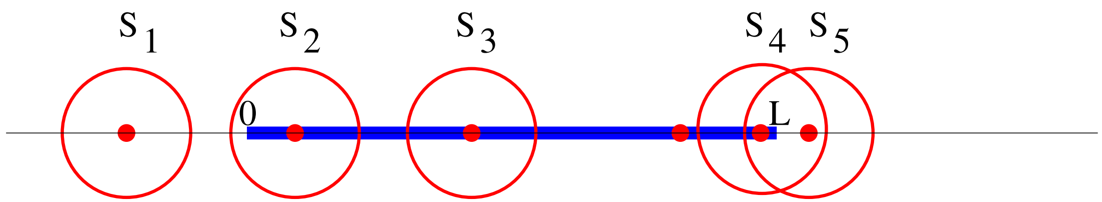
- Given an instance of the problem with n sensors,
- Find the largest number j such that:
  1. j sensors don't move and
  2. the gaps left on the line segment can be covered with at most n-j sensors.
- Given and instance



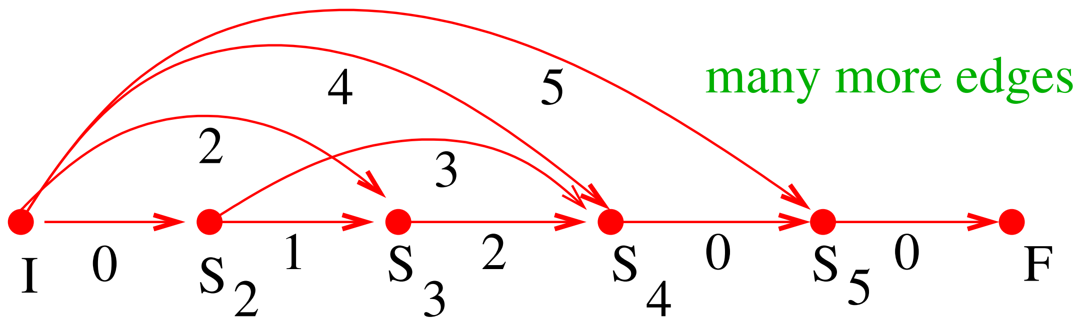
we can represent it using a directed graph:

Algorithm for R >L

- Instance:



Its representation: edge cost = # of sensors needed to cover the remaining gap between these two sensors.

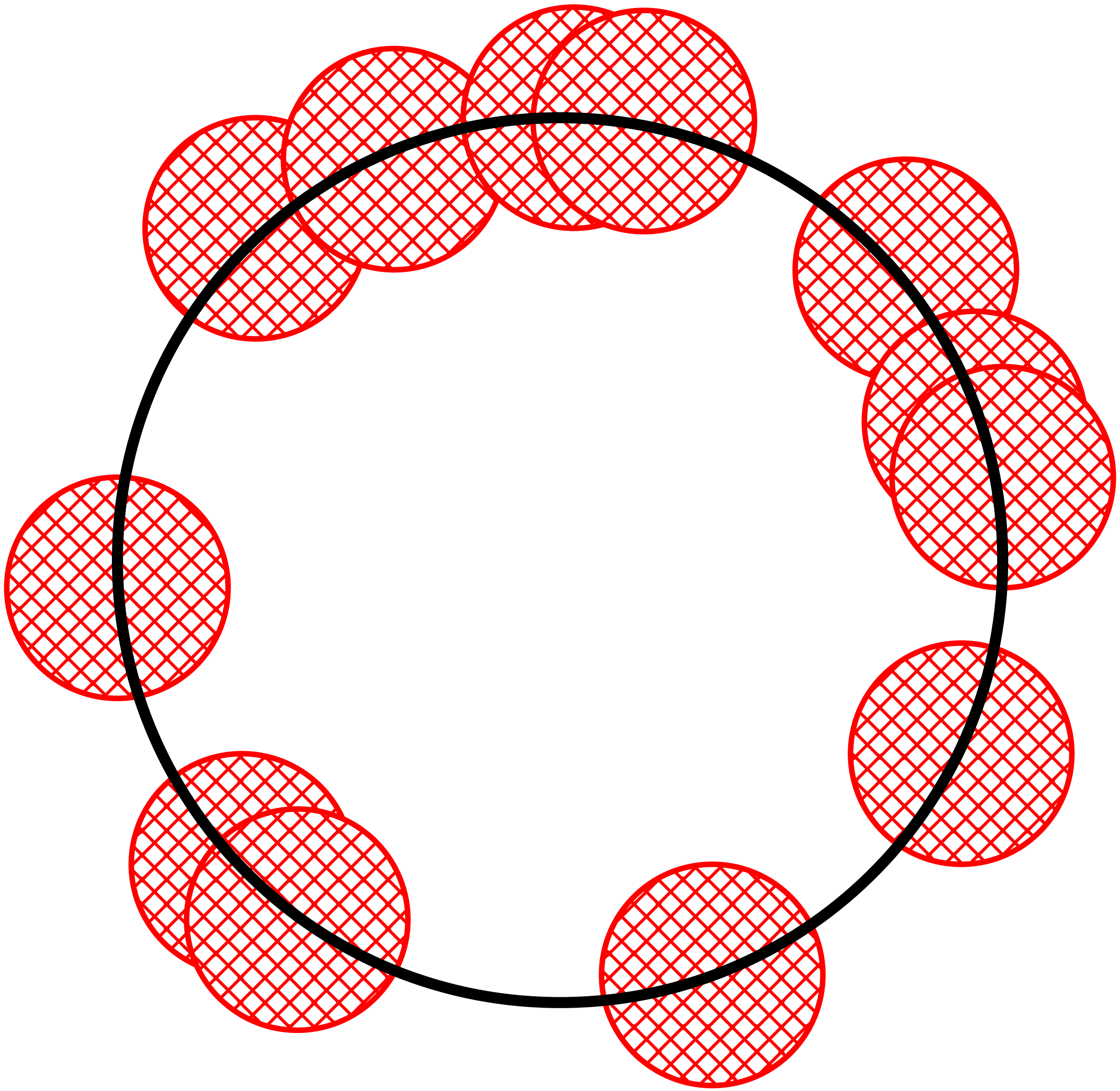


Find a longest directed path from I to F such that

$$length + cost \leq n - 1.$$

- Can be done by dynamic programming in $O(n^3)$

MinNum on a Circular Barrier



MinNum on a Circle Barrier

- Barrier to cover is a circle,
- we have n sensors of sensing range $r_1, r_2, \ldots, r_n$,
- in initial positions $x_1 \leq x_2 \leq \ldots \leq x_n$ on the circle (angles w.r.t. to the center of the circle).
- Determine the final positions of sensors on the circle so that
  1. the circle is covered (if possible), and
  2. the number of sensors moved in minimal.

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Our Results

- The MinNum problem on a circle barrier $C = (0, d/2)$ of diameter d is NP-hard, when sensors have unequal sensing ranges.
- We consider in the rest the case of homogeneous sensors which have identical sensing range c_r on the circle.
- We can consider several situations depending on the total length of the circle that can be covered.

Our Results

- Length of the circle is πd
  Total potential coverage of sensors is of length nc_r.

- Centralized algorithms:

	$Contiguous$	$non - contiguous$
$nc_r = \pi d$	$O(n^2)$	$n.a.$
$nc_r > \pi d$	$O(n^4)$	$n.a.$
$nc_r < \pi d < 2nc_r$	$O(n^2)$	$O(n^4)$
$2nc_r \leq \pi d$	$O(n^2)$	$O(n)$

Open Problems

- Can the complexity of algorithms be improved?
- Consider the barrier coverage problem when we have a fixed number of sensing ranges.
- Consider other shapes of barriers, e.g., a regular polygon
- Distributed algorithms for the problem.
- References:
M. Mehrandish, L. Narayanan, J. Opatrny, *Minimizing the Number of Sensors Moved on Line Barriers*, Proc. of IEEE WCNC 2011, pp. 1464-1469, 2011.

M. Mehrandish, Ph.D. Thesis, Concordia U., 2011

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