Capacity of Wireless Networks

Anil Kumar S. Vullikanti

Network Dynamics and Simulation Science Laboratory,
Virginia Bioinformatics Institute
and Department of Computer Science, Virginia Tech

Acknowledgments

Joint work with

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- David Levin (University of Maryland, College Park)
- Madhav Marathe (Virginia Tech)
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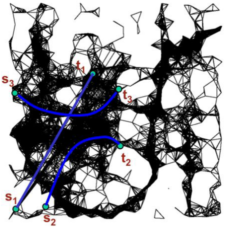
Based on following papers

- V.S. Anil Kumar, M. Marathe, S. Parthasarathy. Cross-layer Capacity Estimation and Throughput Maximization in Wireless Networks, Springer Handbook on *Algorithms for Next Generation Networks*, pp. 67-98, 2009.
- V.S. Anil Kumar, M. V. Marathe, S. Parthasarathy and A. Srinivasan. Algorithmic Aspects of Capacity in Wireless Networks, *ACM SIGMETRICS*, pp. 133-144, 33(1), 2005.
- V.S. Anil Kumar, M. Marathe, S. Parthasarathy and A. Srinivasan. End-to-end packet scheduling in ad hoc networks, *ACM Symposium on Discrete Algorithms (SODA)*, pp. 1021-1030, 2004.
- D. Chafekar, V.S. Anil Kumar, M. Marathe, S. Parthasarathy and A. Srinivasan. Approximating the Capacity of Wireless Networks with SINR constraints, *27th IEEE International Conference on Computer Communications (INFOCOM)*, pp. 1166-1174, 2008.
- D. Chafekar, D. Levin, S. Parthasarathy, V.S. Anil Kumar, M. Marathe and A. Srinivasan. On the capacity of asynchronous random-access wireless networks. *27th IEEE International Conference on Computer Communications (INFOCOM)*, pp. 1148-1156, 2008.
- D. Chafekar, V.S. Anil Kumar, M. Marathe, S. Parthasarathy and A. Srinivasan. Cross-Layer Latency Minimization in Wireless Networks with SINR Constraints, *The ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, pp. 110-119, 2007.
- B. Han, V.S. Anil Kumar, M. Marathe, S. Parthasarathy and A. Srinivasan. Distributed Strategies for Channel Allocation and Scheduling in Software-Defined Radio Networks, *Proc. of the 28th IEEE Conference on Computer Communications (INFOCOM)*, Phoenix, April 21-24, pp. 1521-1529, 2009.
- G. Pei, V.S. Anil Kumar, S. Parthasarathy and A. Srinivasan. Approximation algorithms for throughput maximization in wireless networks with delay constraints, *IEEE Conference on Computer Communications (INFOCOM)*, 2011 (9 pages).



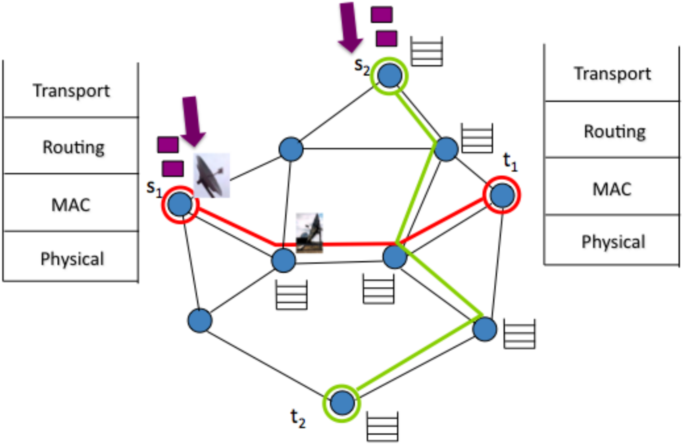
Fundamental Question

Max total rate of communication possible between a set of pairs (s_i, t_i), $i = 1, \ldots, k$, in a given wireless network $G(V, E)$?

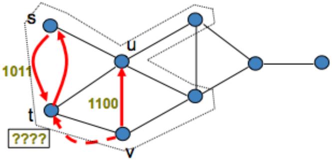
Involves choosing:

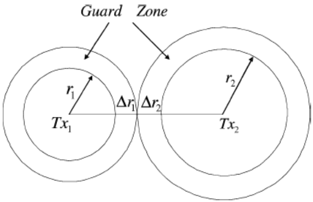
- Route for each connection and rate of arrivals
- Schedule which determines the edges to transmit at each time, and channels and power level
- Objectives: maximize total throughput
- Additional constraints: average delay, total power, fairness

Protocol stack basics



Physical layer abstraction: model broadcast region of a node as a disk (omnidirectional) or sector (directional)





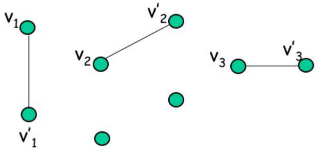
Distance-2 Matching model [Balakrishnan et al., 2004]
$N(e) = \{e' : dist(e, e') \leq 1\}$: interfering edges

Tx-model [Yi et al., 2007]
Transmissions Tx_1 and Tx_2 are simultaneously possible if and only if $d(Tx_1, Tx_2) \geq (1 + \Delta)(r_1 + r_2)$

Other models based on node/edge independent sets

SINR model: Pairs (v_i, v_i') communicate using power level P_i, $i = 1, 2, \ldots$ if and only if:

$$\frac{\frac{P_i}{d(v_i, v_i')^\alpha}}{N + \sum_{j \neq i} \frac{P_j}{d(v_j, v_1')^\alpha}} \geq \beta$$



- β: gain (depends on antenna)
- N: ambient noise
- Joint physical+ MAC abstraction

- Assumption: synchronous time slots of uniform length τ
- Schedule $\mathcal{S}$ specifies the time slots when packets move on links: $X(e,t) = 1$ if packet moves on edge e in time slot t
- $\mathcal{S}$ is feasible if: $\forall t$, $X(e,t) = X(e',t) = 1 \Rightarrow e, e'$ do not interfere
- Link utilization vector, $\bar{x}$, corresponding to $\mathcal{S}$ is defined as

$$\forall e : x(e) = \lim_{T \to \infty} \frac{\sum_{t \le T} X(e,t)}{T}$$

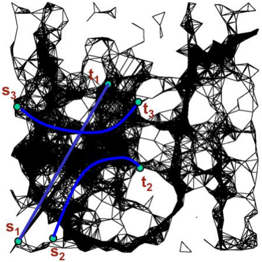
- Flow rate vector, $\bar{f}$, corresponding to $\mathcal{S}$ is defined as

$$\forall e : f(e) = x(e) \cdot cap(e),$$

where $cap(e)$ is the capacity of edge e.

Definition

A rate vector $\bar{f}$ is feasible if it has a corresponding feasible/stable schedule $\mathcal{S}$ that achieves rate $\bar{f}$ and is able to schedule all the packets in bounded time.

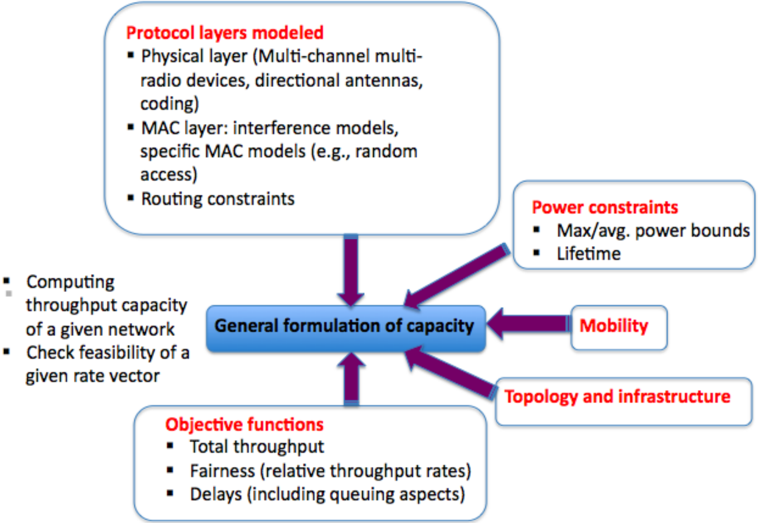


Setting

- Set V of n nodes in the plane
- Radius vector $r = (r(v))$
- Directed graph $G(V, r)$
- k source destination pairs: $(s_1, t_1), \ldots, (s_k, t_k)$

Objective: Find feasible flow vector $\bar{f}$ such that

- There is a feasible schedule $\mathcal{S}$ corresponding to $\bar{f}$
- $\sum_{i=1}^{k} f_i$ is maximized, where f_i is the total flow out of s_i
- Additional QoS constraints: delays/fairness/total power.


Protocol layers modeled
▪ Physical layer (Multi-channel multi-radio devices, directional antennas, coding)
▪ MAC layer: interference models, specific MAC models (e.g., random access)
▪ Routing constraints
Power constraints
▪ Max/avg. power bounds
▪ Lifetime
▪ Computing throughput capacity of a given network
▪ Check feasibility of a given rate vector
General formulation of capacity
Mobility
Topology and infrastructure
Objective functions
▪ Total throughput
▪ Fairness (relative throughput rates)
▪ Delays (including queuing aspects)

Outline for this tutorial

- Part I: Capacity of random networks
- Part II: Arbitrary networks: LP framework
- Part III: Dynamic control for network stability
- Open questions

Outline for Part I

- Basic setting, problem formulation
- Summary of related work
- Upper bound result: $O(\frac{1}{\sqrt{n}})$ scaling
- Lower bound: $\Omega(\frac{1}{\sqrt{n \log n}})$ scaling
- Extensions:
  - Directional antennas
  - Mobility and delays
  - Multi-channel multi-radio networks
  - Hybrid networks

Outline for Part II

- Summary of related work
- LP based cross-layer formulation of the end-to-end capacity of wireless networks
  - Deriving linear necessary and sufficient constraints in a variety of models: $O(1)$ approximation
  - Inductive ordering to deal with non-uniform power levels: $O(1)$ approximation
- $O(\log n)$ approximation for Physical interference model based on SINR constraints
- $O(1)$ approximation for random access networks with uniform power levels
- $O(1)$ approximation for networks with adaptive channel/power allocation
- Logarithmic bounds on average end-to-end delays
- PTAS for computing maximum throughput capacity

Outline for Part III

- Background: arrival processes, queuing
- Backpressure algorithm and its analysis
- Approximate version of backpressure algorithm
- Random access approach
- Summary of related research

Part I: capacity of random networks

Outline for Part I

- Basic setting, problem formulation
- Summary of related work
- Upper bound result: $O(\frac{1}{\sqrt{n}})$ scaling
- Lower bound: $\Omega(\frac{1}{\sqrt{n \log n}})$ scaling
- Extensions:
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  - Multi-channel multi-radio networks
  - Hybrid networks

Basic setting

1. n nodes distributed uniformly at random in the unit square
2. Each node has transmission range $r = \Theta(\sqrt{\frac{\log n}{n}})$.
3. n connections, with each node being a source for a connection, destination chosen randomly (let s_i, t_i denote source and destination for connection i).
4. Each connection has to support rate $\lambda(n)$
5. Each link has capacity W
6. Transport rate of connection i: *connection throughput* $\times$ *distance between* s_i *and* t_i (bit-meters/sec)

Basic Question

How does the expected per-connection throughput which can be supported by a random network evolve as $n \to \infty$?

Related work: capacity of random geometric networks

- *Initial results*: Capacity scaling of $\Theta(\sqrt{n/\log n})$ bit-meters/sec in protocol model of interference [Gupta-Kumar, 2001], simplifications by [Kulkarni-Vishwanatan, 2004], ...
- *Extensions to other interference models*: Capacity of $\Theta(\sqrt{n})$ in SINR/Physical model of interference [Agarwal-Kumar, 2004]
- *Extensions for different physical layer technologies*: improvements using Directional antennas [Peraki, Servetto, 2003], [Yi, et al., 2003], multi-channel and multi-radio (MCMR)/cognitive networks [Kyanasur et al., 2006], [Bhandari et al., 2007]
- *Hybrid networks*: some intermediate nodes with higher bandwidth: improved capacity of $\Omega(\sqrt{n})$ hybrid nodes are added [Liu, Liu, Towsley, 2003], [Negi, Rajeswaran]
- *Impact of mobility* [Grossglauser, Tse], [Bansal, Liu]
- *Impact of delays*: [El Gamal et al., 2004]

[Gupta, Kumar]: tighter upper bound of $\lambda(n) = O(\frac{1}{\sqrt{n \log n}})$ (discussed in Part II)

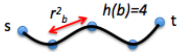
Theorem (Yi et al., 2003)

Expected per-connection throughput is $O(\frac{1}{\sqrt{n}})$.

Proof sketch

- Let $\overline{L}$ denote the average distance between the source and destination of a connection
- Each connection has rate of $\lambda \Rightarrow$ transport capacity of $n\lambda\overline{L}$ per second.
- Consider the b^{th} bit, where $1 \leq b \leq \lambda nT$. Suppose it moves from its source to its destination in a sequence of $h(b)$ hops, where the h^{th} hop covers a distance of r_b^h units. We have:

$$\sum_{b=1}^{\lambda nT} \sum_{h=1}^{h(b)} r_b^h = \lambda nT\overline{L}$$



Proof of upper bound (continued)

- Let indicator $\Gamma(h, b, s)$ be 1 if the h^{th} hop of bit b occurs during slot s. We have

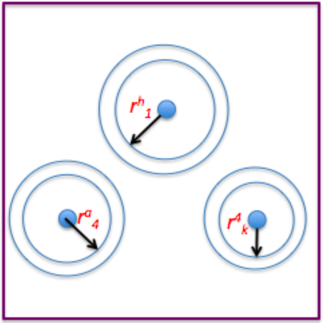
$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \Gamma(h, b, s) \leq \frac{Wn}{2}$$

- Summing over all slots over the T-second period:

$$H \doteq \sum_{b=1}^{\lambda n T} h(b) \leq \frac{WTn}{2}$$

- Because of Tx-model of interference, disks of radius $(1 + \Delta)$ times the lengths of hops centered at the transmitters are disjoint.

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \Gamma(h, b, s) \pi (1+\Delta)^2 (r_b^h)^2 \leq W$$



$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \pi(1+\Delta)^2 (r_b^h)^2 \leq WT$$

$$\Rightarrow \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{WT}{\pi(1+\Delta)^2 H}$$

$$\left(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)\right)^2 \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 (\text{ convexity})$$

$$\Rightarrow \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h) \leq \sqrt{\frac{WT}{\pi(1+\Delta)^2} \cdot H}$$

$$\begin{aligned} \lambda n T\overline{L} &\leq \sqrt{\frac{WTH}{\pi(1+\Delta)^2}} \\ \Rightarrow \lambda n\overline{L} &\leq \frac{1}{\sqrt{2\pi}}\frac{1}{(1+\Delta)}W\sqrt{n} \text{ bit-meters / second} \\ \Rightarrow \lambda &= O(\frac{1}{\sqrt{n}}) \end{aligned}$$

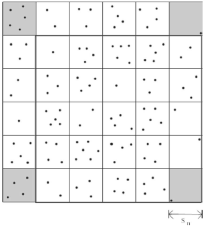
Tighter upper bound using cuts and flows (discussed later)

Lower bound

Theorem (Kulkarni et al., 2004)

Expected per-connection throughput is $\Omega(\frac{1}{\sqrt{n \log n}})$.

Proof strategy: reduction to permutation routing.



1. Grid formed by horizontal and vertical lines uniformly spaced s_n apart: $\frac{1}{s_n^2}$ squarelets of area s_n^2.
2. *Crowding factor*: maximum number of nodes in any squarelet

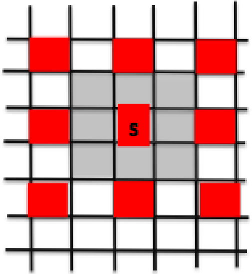
Reduction to permutation routing

1. $\ell \times \ell$ lattice of processors
2. Each processor can communicate with its adjacent vertical and horizontal neighbors in a single slot simultaneously (with one *packet* being a unit of communication with any neighbor during a slot).
3. Each processor is the source and destination of exactly k packets.
4. The $k \times k$ *permutation routing* problem: routing all the $k\ell^2$ packets to their destinations.

Lemma (Kauffman et al., 1994, Kunde, 1993)

$k \times k$ permutation routing in a $\ell \times \ell$ mesh can be performed deterministically in $\frac{k\ell}{2} + o(k\ell)$ steps with maximum queue size at each processor equal to k.

1. Map nodes in each specific squarelet onto a particular processor ($\ell = \frac{1}{s_n}$).
2. Each node has m packets and set $k = mc_n$. Map to permutation routing on lattice.
3. Equivalence class for each squarelet s: squarelets whose vertical and horizontal separation from s is an integral multiple of K squarelets:
    1. K depends on Δ.
    2. Transmissions only within squarelet, or to neighboring squarelets $\Rightarrow$ for any transmission on $e = (u, v)$, $d(u, v) \leq \sqrt{5}s_n$.
    3. Minimum distance between two transmitters in the same equivalence class is $(K - 2)s_n$.
    4. By interference condition: $(K - 2)s_n > 2(1 + \Delta)\sqrt{5}s_n$, or $K > 4 + 2\sqrt{5}\Delta$. Thus, we could set $K = 5 + \lceil 2\sqrt{5}\Delta \rceil$.
    5. Number of equivalence classes $= K^2$ (a fixed constant dependent only on Δ).



Step II: Reduction to permutation routing (contd.)

1. Construct schedule for packets on mesh. Each processor in the mesh can transmit and receive up to four packets in the same slot.
2. *Serialize* transmissions of the processors not in the same equivalence class:
   1. Expands the total number of steps in the mesh routing algorithm by a factor of K^2 (# of equivalence classes).
   2. Serialize the transmissions of a single processor: increases the total number of steps in the mesh routing by a further factor of 4.
3. m packets from all nodes reach in time $4K^2\frac{k\ell}{2} = \Theta(\frac{K^2 m c_n}{s_n})$

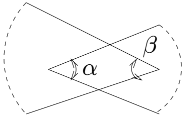
Lemma

Assuming each squarelet has at least one node, the per-connection throughput for a network with squarelet size s_n and crowding factor c_n is $\Omega(\frac{s_n}{c_n})$.

Step II: Reduction to permutation routing (contd.)

1. Set $s_n = \sqrt{\frac{3 \log n}{n}}$
2. With high probability, no squarelet is empty (union bound)
3. $c_n \leq 3e \log n$ (Chernoff bound).

Extensions: directional antennas

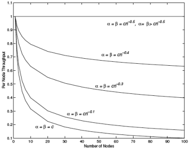


Transmission beamwidth: α
Reception beamwidth: β

Lemma (Yi et al., 2007)

The expected per-connection throughput in random networks with directed antennas with transmission and reception beamwidth α and β, respectively is:

$$\lambda(n) = \begin{cases} \frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Omni Tx, Omni Rv} \\ \frac{2\pi}{\alpha}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Dir Tx, Omni Rv} \\ \frac{2\pi}{\beta}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Omni Tx, Dir Rv} \\ \frac{4\pi^2}{\alpha\beta}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Dir Tx, Dir Rv} \end{cases}$$

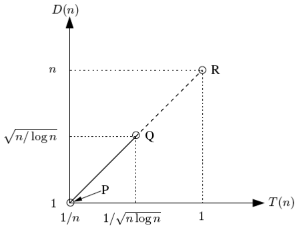


Extensions: delays and mobility

- End-to-end delay $D(n)$: average delay between packet arrival at source and delivery at destination
- $v(n)$: speed of a node
- $T(n)$: expected per-node throughput

Delay-throughput tradeoffs

How does $T(n)$ vary with $D(n)$ and $v(n)$?

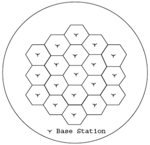


THEOREM (EL GAMAL ET AL., 2004)

In a mobile network with average delay $D(n)$ and per-connection throughput $T(n)$, we have

- $D(n) = \Theta(nT(n))$ *for* $T(n) = O(1/\sqrt{n \log n})$
- $D(n) = O(\sqrt{n}/v(n))$ *when* $T(n) = \Theta(1)$

Several unrealistic assumptions, e.g., arbitrarily large packets and buffering



- n nodes distributed randomly, each choosing a random destination
- m hybrid base stations distributed randomly
- hybrid nodes are all connected by high capacity wired links

THEOREM (LIU ET AL., 2003)

In a hybrid network with n nodes and m base stations, the per-connection throughput $\lambda(m,n)$ satisfies:

$$\lambda(m,n) = \begin{cases} \Theta\left(\sqrt{\frac{1}{n\log n}}W\right) & \text{if } m = O\left(\sqrt{\frac{n}{\log n}}\right) \\ \Theta\left(\frac{mW}{n}\right) & \text{if } m = \omega\left(\sqrt{\frac{n}{\log n}}\right) \end{cases}$$

Part II: approximating the capacity of arbitrary networks

Small sample of results...

- Formulation of rate region using LPs and conflict graphs: [Hajek, Sasaki, 1988], [Jain et al., 2003], [Kodialam and Nandagopal, 2003],...
- Constant factor approximation of the capacity under primary interference [Kodialam and Nandagopal, 2003]
- Constant factor approximation of the capacity for uniform power levels in disk graph models: [Lin, Schroff, 2005], [Kumar et al, 2005], [Kar, Sarkar, Chaporkar, 2005]
- Local multi-commodity flow algorithms [Awerbuch-Leighton, 1993]
- Stability based on Max-weight matching policy [Tassiulas-Ephrimedes, 1993]
- Convex programming methods for capacity [Low et al.]

Feasible Schedules and link rates (recap)

- Assumption: synchronous time slots of uniform length τ
- Schedule $\mathcal{S}$ specifies the time slots when packets move on links: $X(e,t) = 1$ if packet moves on edge e in time slot t
- $\mathcal{S}$ is feasible if: $\forall t,\ X(e,t) = X(e',t) = 1 \Rightarrow e, e'$ do not interfere
- Link utilization vector, $\bar{x}$, corresponding to $\mathcal{S}$ is defined as

$$\forall e : x(e) = \lim_{T\to\infty} \frac{\sum_{t\le T} X(e,t)}{T}$$

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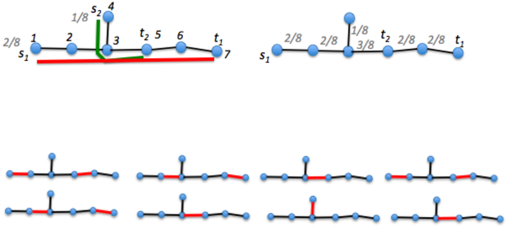
where $cap(e)$ is the capacity of edge e.

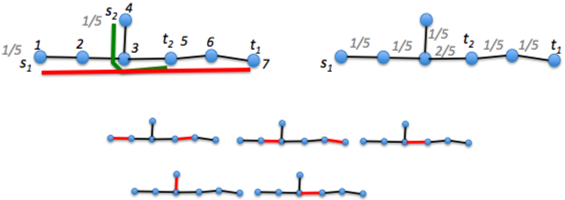
Definition

A rate vector $\bar{f}$ is feasible if it has a corresponding feasible/stable schedule $\mathcal{S}$ that achieves rate $\bar{f}$ and is able to schedule all the packets in bounded time.

Example



- The flow vector $\vec{f}$ with $f_1 = 2/8$, $f_2 = 1/8$ corresponds to periodic schedule $\mathcal{S}$, and is feasible



$f_1 = f_2 = 1/5$ for this schedule

Goal: Given a network, and source-destination pairs, find a feasible flow vector $\vec{f}$ with high total throughput

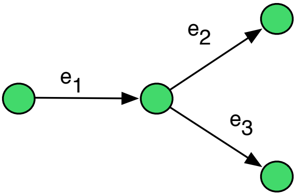
- Define suitable interference set $\hat{N}(e)$ for each link e
- Construct LP $\mathcal{P}(\lambda)$ with flow constraints, and congestion constraints of the form

$$x(e) + \sum_{e' \in \hat{N}(e)} x(e') \leq \lambda,$$

  for each e
- Prove that $\mathcal{P}(c_1)$ gives necessary conditions – any feasible solution $\vec{f}, \vec{x}$ satisfies the constraints of $\mathcal{P}(c_1)$
- Prove that $\mathcal{P}(c_2)$ gives sufficient conditions – corresponding to any feasible solution $\vec{f}, \vec{x}$ of $\mathcal{P}(c_2)$, we can construct a schedule $\mathcal{S}$ that corresponds to $\vec{f}, \vec{x}$

Summary: techniques used

1. Linearization of joint physical and MAC constraints: upper bounds on the rate region expressed by weaker linear constraints
2. Scheduling based on inductive ordering: packets on edge e scheduled after those on edges in $N_{\geq}(e)$ - lower bounds on the optimum



For any edge e:
$x(e) = \lim_{T\to\infty} \sum_{t\leq T} X(e,t)/T$

Capacity Constraint: One packet per edge
$\Rightarrow X(e_i, t) \leq 1$
$\Rightarrow x(e_i) \leq 1$

Primary Interference: For any node, at most one incident edge is used at a time
$\Rightarrow \forall t : X(e_1, t) + X(e_2, t) + X(e_3, t) \leq 1$
$\Rightarrow x(e_1) + x(e_2) + x(e_3) \leq 1$

Objective: $\max \sum_i f_i$ Subject to:

$$
(\mathcal{P}(\lambda)) \quad
\begin{aligned}
\forall i,\quad f_i &= \sum_{e=(s_i,v)} f(e) - \sum_{e=(v,s_i)} f(e)\\
\forall e,\quad x(e) &= f(e)/cap(e)\\
\forall v, \sum_{e\in N(v)} x(e) &\leq \lambda \ (C)\\
\forall e,\quad f(e) &\geq 0
\end{aligned}
$$

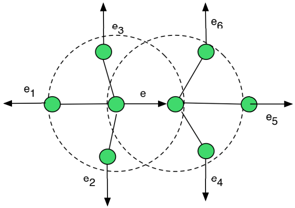
Observation Any feasible link utilization vector $\bar{x}$ is a feasible solution to $\mathcal{P}(1)$.

Lemma (Kodialam and Nandagopal, 2003)

Any solution to the program $\mathcal{P}(2/3)$ can be scheduled feasibly.

Theorem (Kodialam and Nandagopal, 2003)

The optimum solution to the program $\mathcal{P}(2/3)$ gives a $2/3$-approximation to the total throughput capacity, under primary interference constraints.



$X(e,t) = 1 \Rightarrow X(e_i,t) = 0, \ \forall e_i$
$X(e,t) = 0 \Rightarrow$ all edges e_i can simultaneously transmit
$\Rightarrow$ non-linear constraints

Linearization
$X(e,t) + \sum_{i=1}^{6} X(e_i,t) \leq 6$
$\Rightarrow x(e) + \sum_{i=1}^{6} x(e_i) \leq 6$

Lemma

Any feasible utilization vector $\bar{x}$ satisfies the congestion constraints:
$\forall e = (u,v), x(e) + \sum_{e' \in N(e)} x(e') \leq \lambda.$

$N(e) = \{e' = (u', v') : u' \in N(u) \cup N(v)\}.$

Objective: $\max \sum_i f_i$ Subject to:

$$\begin{aligned}
\forall i, \quad f_i &= \sum_{e=(s_i,v)} f(e) - \sum_{e=(v,s_i)} f(e) \\
\forall e, \quad x(e) &= f(e)/cap(e) \\
\forall e, x(e) + \sum_{e' \in N(e)} x(e') &\leq \lambda \text{ (Congestion Constraints)} \\
\forall e, \quad f(e) &\geq 0
\end{aligned}$$

Lemma

The constraints of program $\mathcal{P}_{uniform}(\lambda)$ are necessary for some constant λ: every feasible utilization vector $\bar{x}$ is a feasible solution to the program $\mathcal{P}_{uniform}(\lambda)$.

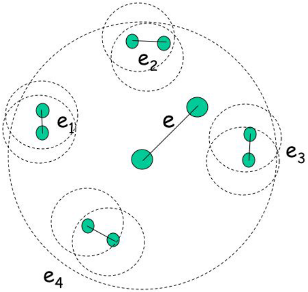
Lemma

The optimum solution to the program $\mathcal{P}_{uniform}(1)$ can be scheduled feasibly.

The solution $\bar{x}$ to $\mathcal{P}_{uniform}(1)$ can be scheduled using a periodic greedy schedule.

Theorem

Program $\mathcal{P}_{uniform}(1)$ gives an $O(1)$-approximation to the total throughput capacity of a wireless network with uniform power levels.



Large number of edges e_i can transmit simultaneously

$X(e,t) + \sum_i X(e_i,t)$ could be large
$\Rightarrow x(e) + \sum_{e' \in N(e)} x(e') \leq 1$ could be highly suboptimal

Non-uniform power levels

Idea: Inductive ordering - ignore "small" edges in the constraint
For $e = (u, v)$, define $r(e) = \max\{r(u), r(v)\}$
$N_{\geq}(e) = \{e' \in N(e) : r(e') \geq r(e)\}$

Lemma

$\forall e, t$, $X(e,t) + \sum_{e' \in N_{\geq}(e)} X(e', t) \leq \lambda$, for a constant λ.

Objective: $\max \sum_i f_i$

Subject to:

$$\begin{aligned}
\forall i, \quad f_i &= \sum_{e=(s_i,v)} f(e) - \sum_{e=(v,s_i)} f(e) \\
\forall e, \quad x(e) &= f(e)/cap(e) \\
\forall e, \ x(e) + \sum_{e' \in N_{\geq}(e)} x(e') &\leq \lambda \text{ (Congestion Constraints)} \\
\forall e, \quad f(e) &\geq 0
\end{aligned}$$

Non-uniform power levels: Formulation $\mathcal{P}_{non-uniform}(\lambda)$

Lemma

There is a constant λ such that the constraints $\forall e,\ x(e) + \sum_{e' \in N_{\geq}(e)} x(e') \leq \lambda$ are necessary: every feasible vector $\bar{x}$ is a feasible solution to program $\mathcal{P}_{non-uniform}(\lambda)$.

Lemma

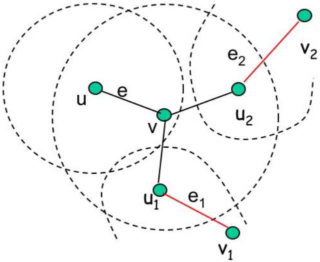
The constraints $\forall e,\ x(e) + \sum_{e' \in N_{\geq}(e)} x(e') \leq 1$ are sufficient: the solution to $\mathcal{P}_{non-uniform}(1)$ can be scheduled feasibly.

Theorem

The program $\mathcal{P}_{non-uniform}(1)$ gives an $O(1)$-approximation to the total throughput capacity under non-uniform power levels.

LEMMA

For any edge e and any D-2 matching E', $|E' \cap N_{\geq}(e)| \leq \lambda$



1. $e \in E' \;\Rightarrow\; |E' \cap N_{\geq}(e)| = 1$
2. Let $e \notin E'$
   - Suppose $e_1 = (u_1, v_1), e_2 = (u_2, v_2) \in E' \cap N_{\geq}(e)$
   - $u_1, v_1 \notin D(u_2) \cup D(v_2)$
   - $D(u) \cup D(v)$ can be partitioned into disjoint regions of area $\pi r(e)^2/\lambda$
3. Let $n(e) = \#$ packets sent on e in time T
4. $\forall e,\; n(e) + \sum_{e' \in N_{\geq}(e)} n(e') \leq \lambda T$
5. Set $x(e) = n(e)/T$

Lemma

The constraints $\forall e,\ x(e) + \sum_{e' \in N_{\geq}(e)} x(e') \leq 1$ *are sufficient: the solution to* $\mathcal{P}_{non-uniform}(1)$ *can be scheduled feasibly.*

Objective: Need to show existence of stable schedule that can send all packets
Different approaches:

1. Periodic scheduling: stable, not necessarily polynomial time, in general
2. Randomized scheme: stable, centralized
3. Random access scheduling: completely local
   1. Lose a factor of $\frac{1}{e}$ for synchronous random access
   2. Lose a factor of $O(\frac{1}{\gamma})$, where γ is the ratio of the maximum transmission duration to the minimum transmission duration
4. Distributed collision free scheduling: based on access hash functions

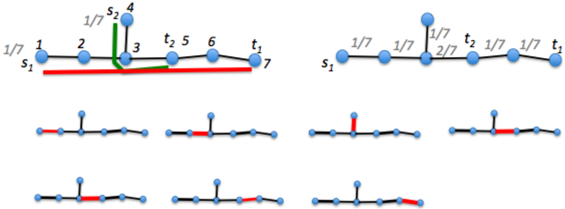
Step I: Choosing time slots

1. Choose W s.t. $S(e) = Wx(e)$ integral for each e
2. Order edges so that $r(e_1) \geq \ldots \geq r(e_m)$
3. (Inductive Scheduling) Choose time slots $S(e)$ for edges in this order:
   - For edge e_i choose any $Wx(e_i)$ slots from the set $\{1, \ldots, W\} \setminus (\cup_{j \leq i-1,\ e_j \in N_{\geq}(e_i)} S(e_j))$

Step II: Periodic scheduling

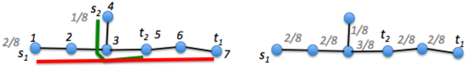
- For each packet, move one edge in W steps

Example



- $W = 7$
- Need $Wx(e) = 1$ slot for all links other than $(3,5)$; $Wx(3,5) = 2$
- Assign slots: $S(1,2) = \{1\}$, $S(2,3) = \{2\}$, $S(3,4) = \{3\}$, $S(3,5) = \{4,5\}$,...

Consider the utilization vector:



- $W = 8$. Assign slots $\{1, \ldots, 8\}$
- Consider an ordering with link $(3, 5)$ in the end
- Suppose greedy assigns: $S(1, 2) = \{1, 2\}$, $S(2, 3) = \{3, 4\}$, $S(3, 4) = \{5\}$, $S(5, 6) = \{1, 2\}$, $S(6, 7) = \{3, 4\}$
- Not enough free slots for $(3, 5)$

Lemma

For each edge e_i,

$$|\{1,\ldots,W\} \setminus (\cup_{j\le i-1, e_j\in N_{\ge}(e_i)} S(e_j))| \ge Wx(e_i)$$

Proof.

If not,

$$Wx(e_i) + \sum_{j\le i-1, e_j\in N_{\ge}(e_i)} Wx(e_j) > W$$

which violates the congestion constraint in $\mathcal{P}_{non-uniform}(1)$. □

$\Rightarrow S(e) = W \cdot x(e)$ slots can be allocated for each edge e

Sufficient condition: proof (continued)

- Schedule is valid since $N()$ is symmetric:
  - Suppose $e \in N(e'), e' \in N(e)$, $r(e') \geq r(e) \Rightarrow e' \in N_{\geq}(e)$
  - Suppose e' is scheduled at time t. Then, $t \in S(e')$. Since $e' \in N_{\geq}(e)$, slot t is not assigned to edge e
- Schedule is stable (constant bit rate): in a frame of length W, number of packets required to flow through e is $x(e)W$, and exactly this many slots are assigned for this edge.
- Lyapunov technique for proving stability for stochastic arrivals

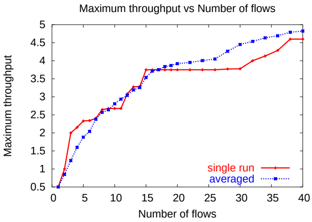
Objective: $\max \sum_i f_i$
Subject to:

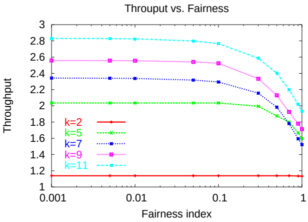
$$
\begin{aligned}
\forall i, \quad f_i &= \sum_{e=(s_i,v)} f(e) - \sum_{e=(v,s_i)} f(e) \\
\forall e, \quad x(e) &= f(e)/cap(e) \\
\forall v, x(e) + \sum_{e' \in N_{\geq}(e)} x(e') &\leq 1 \\
\forall e, \quad f(e) &\geq 0 \\
\forall i,j, \quad f_i &\leq f_j/\gamma \qquad \text{Fairness constraints}
\end{aligned}
$$

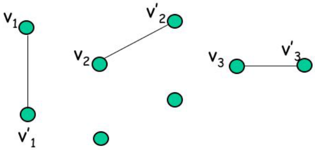
Fairness:

- $\gamma = 1 \Rightarrow$ completely fair
- $\gamma = 0 \Rightarrow$ throughput maximization

- Same approximation ratio holds
- Can quantify the relationship between fairness and capacity







SINR model: If pairs (v_1, v_1'), (v_2, v_2'), ... communicate

$$\frac{\frac{P_1}{d(v_1, v_1')^\alpha}}{N + \sum_{i>1} \frac{P_i}{d(v_i, v_1')^\alpha}} \geq \beta$$

- $\forall e : N(e) = E$
- $\forall e = (u, v) : N_{\geq}(e) = \{e' = (u', v') : \ell(e') \geq \max\{\ell(e), a \cdot d(u, u')\}$
- Assumptions: Power levels for all links are fixed, For each edge e, $cap(e)$ is fixed under an additive white Gaussian noise assumption

$\Delta = \max_e\{\ell(e)\}/\min_{e'}\{\ell(e')\}$

Lemma

The program $\mathcal{P}_{non-uniform}(\lambda)$ gives necessary conditions for a constant λ, while the program $\mathcal{P}_{non-uniform}(1/\log\Delta)$ gives sufficient conditions.

Extensions: power constraints

- Setting: $\mathcal{S}$ has to determine which edges e to use at time t, *and what power level to use*
- Capacity of link e at power level p

$$cap(e, p) = W \log_2(1 + \frac{p}{d(u, v)^{\alpha} N_0 W})$$

- Setting: $\mathcal{S}$ has to determine which edges e to use at time t, *and what power level to use*
- Capacity of link e at power level p

$$cap(e, p) = W \log_2(1 + \frac{p}{d(u, v)^{\alpha} N_0 W})$$

- J= set of possible choices of power levels; need not be finite
- Define $\mathcal{T}(J) = \{(e, p) \in E \times J\}$
- Define $N(e, p) = \{(e' = (u', v'), p') : e' \in V^2,\ p' \in J, d(u, u') \leq (1 + \Delta)(range(p) + range(p'))\}$
- Define $N_{\geq}(e, p) = \{(e' = (u', v'), p') \in N(e, p) : p' \geq p\}$

Program $\mathcal{P}_{pctm}(\lambda)$

$$\max \sum_i f_i \qquad \text{s.t.:}$$

$$\forall i, \quad f_i = \sum_{(e=(s_i,v),p)\in\mathcal{T}} f(e,p) - \sum_{(e=(v,s_i),p)\in\mathcal{T}} f(e,p)$$

$$\forall (e,p) \in \mathcal{T}, x(e,p) = f(e,p)/cap(e,p)$$

$$\forall (e,p) \in \mathcal{T}, x(e,p) + \sum_{(e',p')\in N_{\geq}(e,p)} x(e,p) \leq \lambda$$

$$\forall i, \forall u \neq s_i, t_i \sum_{e\in N_{out}(u)} f(e,p) = \sum_{e\in N_{in}(u)} f(e,p)$$

$$\sum_{(e,p)\in\mathcal{T}} x(e,p)\cdot p \leq B$$

B= total bound on power usage

Joint power and throughput capacity optimization: special case

Lemma

Any feasible rate vector and power assignment must satisfy the constraints of $\mathcal{P}(c)$ for a constant c. Further, any solution to $\mathcal{P}(1)$ is feasible.

Joint power and throughput capacity optimization: special case

Lemma

Any feasible rate vector and power assignment must satisfy the constraints of $\mathcal{P}(c)$ for a constant c. Further, any solution to $\mathcal{P}(1)$ is feasible.

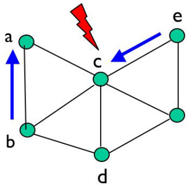
Assumption: $|J| \leq poly(n) \Rightarrow |\mathcal{P}_{pctm}|$ is polynomial sized.

Joint power and throughput capacity optimization: general case

- Let $p_{max} = \max\{p \in J\}$ and $p_{min} = \min\{p' \in J\}$
- Assumption: $p_{max}/p_{min} \leq poly(n)$
- $J' = \{p_{min}, (1+\epsilon)p_{min}, \ldots, p_{max}\}$

Lemma

The program $\mathcal{P}_{pctm}(1)$ defined using set J' (instead of set J) gives a constant factor approximation to throughput capacity under a given bound on total power consumption.



- Node v attempts to transmit on link $e = (v, w)$ only if no neighbor of v is currently transmitting
- If channel free, v transmits on e with probability $\tau(e)$

- T_{id}: idle slot length
- $T_{xmit}(\ell)$: length of transmission on link ℓ
- $N_{pri}(\ell)$: links within primary interference of ℓ
- $N_{sec}(\ell) = N(\ell) \setminus N_{pri}(\ell)$
- Probability of accessing the link ℓ: $\tau(\ell) = 1 - e^{-x(\ell)}$

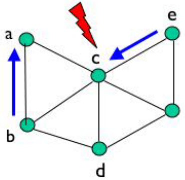
Lemma

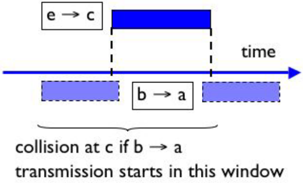
Let $\bar{x}$ be a feasible solution to the program $\mathcal{P}(1)$. Then, $\frac{1}{e}\bar{x}$ can be achieved by synchronous random access scheduling.

Proof:
Choose $\tau(\ell) = 1 - e^{-x(\ell)/\lambda}$, for each ℓ.
Probability of collision free transmission on edge ℓ:

$$\begin{aligned}\eta(\ell) &= \Pi_{\ell' \in I(\ell)}(1 - \tau(\ell')) \\ &= e^{\sum_{\ell' \in I(\ell)} -x(\ell')} \\ &\geq e^{x(\ell)-1}\end{aligned}$$

$$\begin{aligned}
\text{Successful flow through } \ell &= cap(\ell) \cdot \tau(\ell) \cdot \eta(\ell) \\
&\geq cap(\ell) \cdot (1 - e^{-x(\ell)}) \cdot e^{x(\ell)-1} \\
&= cap(\ell) \cdot (e^{x(\ell)-1} - e^{-1}) \\
&\geq cap(\ell) \cdot \left(\frac{1 + x(\ell)}{e} - \frac{1}{e} \right) \\
&= \frac{f(\ell)}{e} \\
&\Rightarrow \tfrac{1}{e}\bar{f} \text{ is stable}
\end{aligned}$$





- T_{id}: idle slot length
- $T_{xmit}(\ell)$: transmission duration on ℓ
- $\gamma = \frac{\max_{\ell} T_{xmit}(\ell)}{\min_{\ell'} T_{xmit}(\ell')}$
- Δ: max #simultaneous transmissions possible in $N(\ell)$ (interference degree)

Theorem

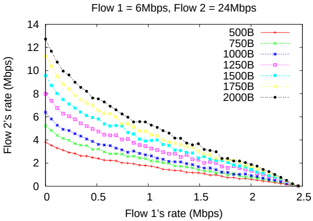
Let $\vec{x}$ be a feasible solution to $\mathcal{P}(1)$. The random access protocol with channel access probability

$$\tau(\ell) = 1 - e^{-\frac{x(\ell)}{\Delta(\ell)} \cdot \frac{T_{id}}{T_{xmit}(\ell)(1+\gamma)}},$$

achieves a link utilization of $\vec{h} \geq \frac{1}{e(\gamma+1)\Delta}\vec{x}$.

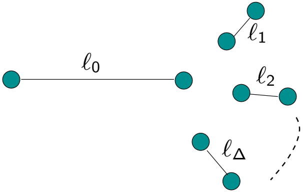
Asynchronous Random Access: effect of packet sizing policies

Random access is more competitive when the packet sizes on links are non-uniform, and are proportional to the link capacity



ℓ_1 and ℓ_2: hidden interfering links
$c(\ell_1)=$ 6Mbps, $c(\ell_2)$=24Mbps
packet size on ℓ_1: 500 Bytes
packet size on ℓ_2 varied from 500 Bytes to 2000 Bytes

Limits on the competitive ratio of asynchronous random access scheduling



$\forall i \geq 1,\ \ell_i \in \mathit{hidden}(\ell_0)$
$\forall i \geq 1,\ \ell_0 \in \mathit{hidden}(\ell_i)$
Assume $T_{xmit}(\ell_i) = T_{xmit} = a_1 T_{id}$
and $T_{xmit}(\ell_0) = \gamma T_{xmit}$

$\vec{f} = \langle 1/2, \ldots, 1/2 \rangle$ is feasible for greedy scheduling

Lemma

$\lambda \vec{f}$ is feasible for random access scheduling only if $\lambda \leq c \frac{\log \Delta\gamma}{\Delta\gamma}$

Characterizing the capacity region for random access protocols

New formulation to approximate the throughput capacity of an asynchronous random access network within an $O(\Delta)$-factor:

Theorem (Necessary Conditions)

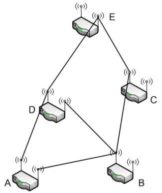
$\vec{x}$ is feasible for asynchronous random access protocol only if:

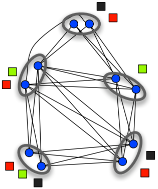
$$\forall \ell : x(\ell) + \sum_{\ell' \in exposed(\ell)} x(\ell') + \sum_{\ell' \in hidden(\ell)} x(\ell') \cdot (1 + \frac{T_{xmit}(\ell) - T_{id}}{T_{xmit}(\ell')}) \leq \Delta$$

Theorem (Sufficient Conditions)

$\vec{x}$ is feasible for asynchronous random access protocol if:

$$\forall \ell : x(\ell) + \sum_{\ell' \in exposed(\ell)} x(\ell') + \sum_{\ell' \in hidden(\ell)} x(\ell') \cdot (1 + \frac{T_{xmit}(\ell) - T_{id}}{T_{xmit}(\ell')}) \leq \frac{1}{e}$$





- Graph $G = (V, E)$
- For each node $u \in V$, $Radios(u)$: set of wireless interfaces associated with it.
- Set Ψ of channels available
- Schedule + channel assignment: at each time t, choose links $e = (u, v)$ which will transmit, which radio interfaces to use at u, v and which channel to use

- Induced Radio Network $\mathcal{G} = (\mathcal{V}, \mathcal{L})$: $\mathcal{V}$ is the set $\cup_u Radios(u)$ and $\mathcal{L} = \cup_{e=(u,v)\in E} Radios(u) \times Radios(v)$
- For link $\ell = (\rho, \rho')$, $parent(\ell) = (u, v)$ if $\rho \in Radios(u)$ and $\rho' \in Radios(v)$
- Consider set $\mathcal{T} = \{(\ell, \psi) : \ell \in \mathcal{L}, \psi \in \Psi\}$

For link $\ell = (\rho, \rho')$ in induced radio network $\mathcal{G} = (\mathcal{V}, \mathcal{L})$:

- $Pri(\ell) = \{\ell' \text{ sharing a radio with } \ell\}$
- $Pri_{\succ}(\ell) = \{\ell' \in Pri(\ell) : parent(\ell') \succ parent(\ell)\}$
- $Sec(\ell) = \{\ell' : parent(\ell') \in Pri(parent(\ell))\} \cup \{\ell' : parent(\ell') \in Sec(parent(\ell))\}$
- $Sec_{\succ}(\ell) = \{\ell' \in Succ(\ell) : parent(\ell') \succ parent(\ell)\}$

Theorem

Flow constraints with the following congestion constraints are necessary for any feasible flow+utilization vector:

$$x(\ell, \psi) + \sum_{\rho \in \Psi \setminus \{\psi\}} x(\ell, \rho) + \sum_{\chi \in \Psi} \sum_{f \in Pri_{\succ}(\ell)} x(f, \chi) + \sum_{g \in Sec_{\succ}(\ell)} x(g, \psi) \leq \lambda + 2$$

Sufficient conditions

Theorem

The rate vector satisfying the following conditions can be scheduled feasibly:

$$\forall(\ell,\psi),\ x(\ell,\psi) + \sum_{\rho\in\Psi\setminus\{\psi\}} x(\ell,\rho) + \sum_{\chi\in\Psi}\sum_{f\in Pri(\ell)} x(f,\chi) + \sum_{g\in Sec(\ell)} x(g,\psi) \leq \frac{1}{e} - \epsilon$$

- Need access-hash function $H(\ell, \psi, t)$ such that:

$$H(\ell,\psi,t) = \begin{cases} 1 & \text{with probability } 1 - e^{-e\cdot x(\ell,\psi)} \\ 0 & \text{with probability } e^{-e\cdot x(\ell,\psi)} \end{cases}$$

- *Key Property*: Value of $H(.,.,)$ fixed no matter who invokes it with the same arguments
- Also known as random oracles in Cryptography
- SHA-1 works well in practice

Executed by each radio ρ:

1. $\forall \ell$ incident on ρ: compute $H(\ell, \psi, t)$, for each ψ, t.
2. Randomly pick a pair (ℓ, ψ) s.t. $H(\ell, \psi, t) = 1$
   - if no such pair exists, sleep during time t
3. If selected link $\ell \in \mathcal{L}_{out}(\rho)$, then schedule an outgoing transmission across ℓ on channel ψ at time t
4. if selected link $\ell \in \mathcal{L}_{in}(\rho)$, then tune to channel ψ and await an incoming transmission across ℓ on channel ψ at time t

Goal: choose flow vector $\vec{f}$ so that:

- $\sum_i f_i$ is maximized
- For each session i such that $f_i > 0$, average delay for each packet is at most D

Our Result

Careful choice of paths plus random access scheduling to get joint bounds on throughput and delays.

- Choose flow $\vec{f}$ that maximizes $\sum_i f_i$ subject to:

$$\begin{aligned}
\forall i, \sum_{p \in P(i)} f(p) cost(p) &\leq Df_i \\
\forall (e,i),\ x(e,i) &= \sum_{p \in P(i):\ e \in p} f(p)/cap(e) \\
\forall e, \sum_i x(e,i) + \sum_{e' \in N(e)} \sum_i x(e',i) &\leq 1
\end{aligned}$$

- (Filter) Drop flows on paths longer than $2D$ for each i
- (Round) Choose a subset S of sessions and a path p_i for each $i \in S$ by iterative rounding
- (Choose flows) Choose flow $f(p_i) = K \log\log D / \log D$

Theorem

The flow vector $\vec{f}$ along with random access scheduling ensures that $\sum_i f_i = \Omega(OPT \cdot \log\log D / \log D)$, and at least $(1 - 1/n)$-fraction of the packets for each session i are delivered within a delay of $O(D \cdot (\log D / \log\log D) \cdot \log n)$.

- Adaptive channel switching delays can be incorporated into the framework in terms of *cost*(p) to quantify the throughput gains of adaptive channel switching
- Similar tradeoffs for adaptive power switching

- Define suitable interference set $\hat{N}(e)$ for each link e
- Construct LP $\mathcal{P}(\lambda)$ with flow constraints, and congestion constraints of the form

$$x(e) + \sum_{e' \in \hat{N}(e)} x(e') \leq \lambda,$$

  for each e
- Prove that $\mathcal{P}(c_1)$ gives necessary conditions – any feasible solution $\vec{f}, \vec{x}$ satisfies the constraints of $\mathcal{P}(c_1)$
- Prove that $\mathcal{P}(c_2)$ gives sufficient conditions – corresponding to any feasible solution $\vec{f}, \vec{x}$ of $\mathcal{P}(c_2)$, we can construct a schedule $\mathcal{S}$ that corresponds to $\vec{f}, \vec{x}$

Summary

- Two techniques for cross-layer formulation of the end-to-end capacity of wireless networks
    - Linearization of interference constraints
    - Inductive ordering to deal with non-uniform power levels
- Framework extends to a number of models, constraints and objective functions

Part III: Dynamic control for network stability

Outline for Part III

- Background: arrival processes, queuing
- Backpressure algorithm and its analysis
- Approximate version of backpressure algorithm
- Random access approach
- Summary of related research

"Arrivals at all sources are well-behaved"

"Arrivals at all sources are well-behaved"

1. Let $A^i(t)$ be the exogenous arrival process for connection i with rate λ_i
2. An arrival process $A^i(t)$ is admissible with rate λ_i if
    1. The time averaged expected arrival rate satisfies:

$$\lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[A^i(\tau)] = \lambda$$

    2. Let $H(t)$ represent the history until time t There exists $A_{\max}$ such that $E[(A^i(t))^2 \mid H(t)] \leq A_{max}^2$ for t.
    3. For any $\delta > 0$, there exists an interval size T, possibly dependent on δ, such that for any initial time t_0:

$$E\left[\frac{1}{T}\sum_{k=0}^{T-1} A^i(t_0+k)|H(t_0)\right] \leq \lambda + \delta$$

Other models: adversarial arrivals

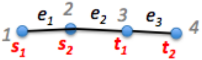
Background (continued)

- Each node v maintains queues for each link (v, w) and each connection i
- Assume unbounded buffer sizes – no packet drops because of buffer overflows
- Let $U_v^i(t)$ denote the queue at node v for connection i at time t; let $\mathbf{U}(t) = \langle U_v^i(t) \rangle$
- $\mu_{(u,v)}^i(t) \leq c(u, v)$: data rate allocated to commodity i during slot t across the link (u, v) by the network controller.

- $I \subset E$ is a conflict free subset if for every $e, e' \in I$, e and e' are conflict-free.
- Let $\mathcal{I}$ denote the set of all possible conflict-free subsets $I \subset E$
- Let $\mu(I)$ denote the vector of transmission rates for each $e \in I$.
- Let

$$\Gamma \doteq Conv(\{\vec{\mu}(I) \mid I \in \mathcal{I}\})$$

  denote the convex hull of all transmission-rate matrices
- Let $\text{inflow}^i_{v,\mu}(t) = \sum_{(w,v)\in E} \mu^i_{(w,v)}(t)$ denote the flow of commodity i into node v for policy μ at time t
- Let $\text{outflow}^i_{v,\mu}(t) = \sum_{(v,w)\in E} \mu^i_{(v,w)}(t)$ denote the flow of commodity i out of node v for policy μ at time t
- Let $\text{netflow}^i_{v,\mu}(t) = \text{outflow}^i_{v,\mu}(t) - \text{inflow}^i_{v,\mu}(t)$ denote the total flow of commodity i out of node v for policy μ at time t



$$I_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Assume primary interference: edges with common end-point conflict
- Two connections (s_1, t_1) and (s_2, t_2)
- $\Gamma = \{\alpha I_1 + \beta I_2 : \alpha + \beta \leq 1\}$

- Traffic matrix corresponding to $\mu = \frac{2}{3} I_1 + \frac{1}{3} I_2$
- $\text{inflow}^1_{2,\mu}(t) = \mu^1_{(1,2)} = 2/3$

Theorem (Grigoriadis et al., 2006)

The connection rate vector $\langle \lambda_i \rangle$ is within the network-layer capacity region Λ if and only if there exists a randomized network control algorithm that makes valid $\mu^i_{(u,v)}(t)$ decisions, and yields:

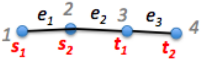
$$\forall i,\ \mathbf{E}[netflow^i_{s_i,\mu}(t)] = \lambda_i$$

$$\forall i,\ \forall w \notin \{s_i, t_i\},\ \mathbf{E}[netflow^i_{w,\mu}(t)] = 0$$

At each time t

- For each link (v, w): let $i = i^*$ be the commodity with maximum differential backlog $\Delta U_v^i - U_w^i$
- For each link (v, w), define $weight(v, w)$ to be the maximum differential backlog
- Choose independent set I with maximum weight $wt(I) = \sum_{e \in I} wt(e)$
- Schedule all links in I simultaneously, and send as much as possible

Example



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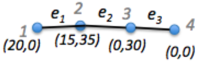

- $\Delta U^1_{(1,2)} = 5$, $\Delta U^2_{(1,2)} = -35$ $\Rightarrow i^*_{(1,2)} = 1$, $W^*_{(1,2)} = 5$
- $\Delta U^1_{(2,3)} = 15$, $\Delta U^2_{(2,3)} = 5$ $\Rightarrow i^*_{(2,3)} = 1$, $W^*_{(2,3)} = 15$
- $\Delta U^1_{(3,4)} = 0$, $\Delta U^2_{(3,4)} = 30$ $\Rightarrow i^*_{(3,4)} = 2$, $W^*_{(3,4)} = 30$
- $wt(I_1) = 5 + 30 = 35$, $wt(I_2) = 15$

At each time t

- For each link (v, w): let $i^*_{(v,w)}(t)$ denote the connection which maximizes the differential backlog $W^*_{(v,w)}(t) = U_v^{i^*_{(v,w)}(t)}(t) - U_w^{i^*_{(v,w)}(t)}(t)$.
- Choose conflict-free link set $I^* \in \mathcal{I}$ which maximizes $\sum_{(u,v)\in I^*} W^*_{(u,v)}(t) \cdot c(u, v)$
- The network controlled chooses links $e = (u, v) \in I^*$ and connection $i^*_{(u,v)}(t)$ if $W^*_{(u,v)}(t) > 0$ (if there is not enough backlogged data, i.e., $U_{(u,v)}^{i^*_{(u,v)}(t)}(t) < c(u, v)$ use dummy bits)

- Consider any valid resource allocation policy that assigns a rate of $\tilde{\mu}^i_{(u,v)}(t)$ to commodity i across link (u,v) at time t.
- Let $\mu^i_{(u,v)}(t)$ denote the corresponding values for the dynamic backpressure algorithm.
- By construction:

$$\begin{aligned}
\sum_{(u,v)} \sum_i \tilde{\mu}^i_{(u,v)}(t)[U^i_u(t) - U^i_v(t)] &\leq \sum_{(u,v)} \sum_i \tilde{\mu}^i_{(u,v)}(t) W^*_{(u,v)}(t) \\
&\leq \sum_{(u,v)} W^*_{(u,v)}(t) \cdot \mu(u,v)
\end{aligned}$$

Analysis (continued)

Rearranging the terms:
"$\sum_v$ of queue-size at $v\cdot$ netflow(v) $= \sum_e$ flow(e)$\cdot$backlog(e)"

Analysis (continued)

Rearranging the terms:
"$\sum_v$ of queue-size at $v\cdot$ netflow$(v) = \sum_e$ flow$(e)\cdot$backlog(e)"

$$\sum_i \sum_v U_v^i(t) \cdot [\sum_w \mu^i_{(v,w)}(t) \quad - \quad \sum_u \mu^i_{(u,v)}(t)] = \sum_{(u,v)} \sum_i \mu^i_{(u,v)}(t)[U_u^i(t) - U_v^i(t)]$$

Lemma (Property)

If $\tilde{\mu}^i_{(u,v)}(t)$ denotes any resource allocation policy, and $\mu^i_{(u,v)}(t)$ denotes the resource allocation for the Backpressure scheme, we have:

$$\sum_v \sum_i U_v^i(t)[\sum_w \tilde{\mu}^i_{(v,w)}(t) \quad - \quad \sum_u \tilde{\mu}^i_{(u,v)}(t)] \leq \sum_v \sum_i U_v^i(t) \left[\sum_w \mu^i_{(v,w)}(t) - \sum_u \mu^i_{(u,v)}(t)\right]$$

Define:

$$L(U(t)) = \sum_i \sum_v (U_v^i(t))^2$$

THEOREM (GRIGORIADIS ET AL., 2006)

If there exist constants $B > 0$ and $\epsilon > 0$ such that for all slots t:

$$\mathbf{E}[L(U(t+1)) - L(U(t)) \mid U(t)] \leq B - \epsilon \sum_v \sum_i U_v^i(t) \quad (1)$$

then, the network is strongly stable.

Theorem

Let $\vec{\lambda}$ denote the vector of arrival rates; if there exists an $\epsilon > 0$ such that $\vec{\lambda} + \vec{\epsilon} \in \Lambda$ (where $\vec{\epsilon}$ is the vector such that $\epsilon_i = 0$ if $\lambda_i = 0$, and $\epsilon_i = \epsilon$ otherwise), then the dynamic backpressure algorithm stably services the arrivals.

- If $V, U, \mu, A \geq 0$ and $V \leq \max\{U - \mu, 0\} + A$, then,

$$V^2 \leq U^2 + \mu^2 + A^2 - 2U(\mu - A)$$

- Since $U_v^i(t+1) \leq \max\{U_v^i(t) - \sum_{e=(v,w)} \mu_e^i(t), 0\} + \sum_i A^i(t) + \sum_{e=(u,v)} \mu_e^i(t)$, we have:

$$U_v^i(t+1)^2 \leq U_v^i(t)^2 + \left(\sum_w \mu_{(v,w)}^i(t)\right)^2 + \left(A_v^i(t) + \sum_u \mu_{(u,v)}^i(t)\right)^2 - 2U_v^i(t) \cdot \left(\sum_w \mu_{(v,w)}^i(t) - A_v^i(t) - \sum_u \mu_{(u,v)}^i(t)\right)$$

Summing over all indices (v, i) and since $\sum_j z_j^2 \leq (\sum_j z_j)^2$, if $z_j \geq 0$,

$$L(U(t+1)) - L(U(t)) \leq 2BN - 2\sum_v \sum_i U_v^i(t) \cdot \left(\sum_w \mu^i_{(v,w)}(t) - A_v^i(t) - \sum_u \mu^i_{(u,v)}(t) \right),$$

where $B \doteq \frac{1}{2N} \cdot \sum_v [(\max_w \mu(v,w))^2 + (\max_i A^i + \max_u \mu(u,v))^2]$.

$$\Rightarrow \mathbf{E}[L(U(t+1)) - L(U(t)) \quad | \quad U(t)] \leq 2BN + 2 \cdot \sum_i U^i_{s_i}(t) \cdot \mathbf{E}[A^i_{s_i}(t) \mid U(t)] - 2\mathbf{E}[\sum_v \sum_i U_v^i(t) \cdot \left(\sum_w \mu^i_{(v,w)}(t) - \sum_{(u,v)} \mu_{(u,v)}(t) \right) \mid U(t)]$$

Simple algebra: "expected change in potential $\leq$ constant $+2 \cdot \sum_i U^i_{s_i}(t)$ expected-arrival at $s_i - 2\sum_v E[U^i_v(t)$ netflow$(v)]$"

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$+2 \cdot \sum_i U^i_{s_i}(t)$ expected-arrival at $s_i - 2\sum_v E[U^i_v(t)$ netflow$(v)]$"

$$\Rightarrow \mathbf{E}[L(U(t+1)) - L(U(t)) \quad | \quad U(t)] \leq 2BN + 2 \cdot \sum_i U^i_{s_i}(t) \cdot \mathbf{E}[A^i_{s_i}(t) \mid U(t)] -$$

$$2\mathbf{E}[\sum_v \sum_i U^i_v(t) \cdot (\sum_w \mu^i_{(v,w)}(t) - \sum_{(u,v)} \mu_{(u,v)}(t)) \mid U(t)]$$

- By definition of arrival process: $\mathbf{E}[A^i_{s_i}(t) \mid U(t)] = \lambda_i$ for all commodities i.
- For optimal allocation vector $\tilde{\mu}$:
  - $\forall i$, $\mathbf{E}$[total flow out of s_i for $\tilde{\mu}$] $= \lambda_i + \epsilon_i$
  - $\forall i$, $\mathbf{E}$[total flow out of v for $\tilde{\mu}$] $= 0$, for all $v \neq s_i, t_i$
- Backpressure algorithm maximizes
  $\mathbf{E}[\sum_v \sum_i U^i_v(t) \cdot \left(\sum_w \mu^i_{(v,w)}(t) - \sum_{(u,v)} \mu_{(u,v)}(t)\right) \mid U(t)]$ at each step t

$$\Rightarrow \mathbf{E}[\sum_v \sum_i U^i_v(t) \cdot (\sum_w \mu^i_{(v,w)}(t) - \sum_{(u,v)} \mu_{(u,v)}(t)) \mid U(t)] \quad \geq \quad \sum_i U^i_{s_i}(t)(\lambda_i + \epsilon_i)$$

$$\Rightarrow \mathbf{E}[L(U(t+1)) - L(U(t)) \mid U(t)] \quad \leq \quad 2BN - 2\sum_i U^i_{s_i}(t)\epsilon_i,$$

which implies stability of backpressure algorithm with arrival rates $\vec{\lambda}$ if $\vec{\lambda} + \vec{\epsilon}$ is stable.

- Finding max-weight independent set is NP-complete in most interference models
- Approximating the max-weight independent set within a γ-factor implies γ-factor approximation of the rate region, $\gamma > 1$:
  - Suppose $\gamma\vec{\lambda} \in \Gamma$, and λ_i is the arrival rate for connection i
  - In earlier analysis: $\sum_i U^i_{s_i}(t) \cdot \mathbf{E}[A^i_{s_i}(t) \mid U(t)] = \sum_i \lambda_i U^i_{s_i}(t)$
  - For any policy $\tilde{\mu}$, approximate backpressure implies:

$$\begin{aligned}
\sum_{(u,v)} \sum_i \tilde{\mu}^i_{(u,v)}(t)[U^i_u(t) - U^i_v(t)] &\leq \sum_{(u,v)} \sum_i \tilde{\mu}^i_{(u,v)}(t) W^*_{(u,v)}(t) \\
&\leq \gamma \sum_{(u,v)} W^*_{(u,v)}(t) \cdot \mu(u,v)
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  - Rearranging terms:

$$\begin{aligned}\frac{1}{\gamma} \sum_v \sum_i U^i_v(t)[\sum_w \tilde{\mu}^i_{(v,w)}(t) &- \sum_u \tilde{\mu}^i_{(u,v)}(t)] \\ &\leq \sum_v \sum_i U^i_v(t) \left[\sum_w \mu^i_{(v,w)}(t) - \sum_u \mu^i_{(u,v)}(t)\right]\end{aligned}$$

- Implies stability condition for approximate backpressure

Summary

Approximation algorithm for one-hop weighted link scheduling problem $\Rightarrow$ approximation algorithm for end-to-end throughput capacity in general interference models.

- Greedy scheduling gives $O(1)$-factor approximation to max-weight scheduling in many models
- Limitations:
  - Does not immediately give us a way to compute the approximate rate vector $\vec{\lambda}$ – need additional characterization
  - Convergence time not necessarily polynomial time

Open problems

- SINR models
- Distributed algorithms
- Delay-throughput tradeoffs
- Incorporating specific protocols for different layers
- Power constraints
- Adaptive channel switching, cognitive networks
- New paradigms: Cooperative networking, Physical layer advances, information theoretic bounds

Thank You