Polygon reconstruction from local observations

work by
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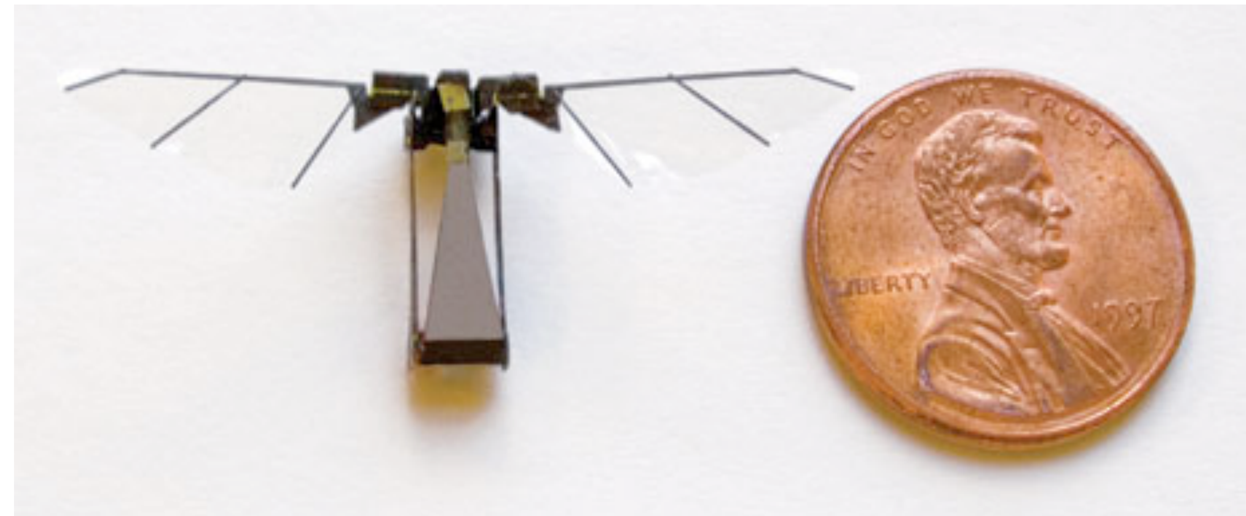
Robots with Little Sensing and Mobility

- due to
  - cost
  - weight
  - form factor
  - environments



For simple sensing model of a vision guided robot:

possibility and **impossibility** results

Microrobots, environments, problems

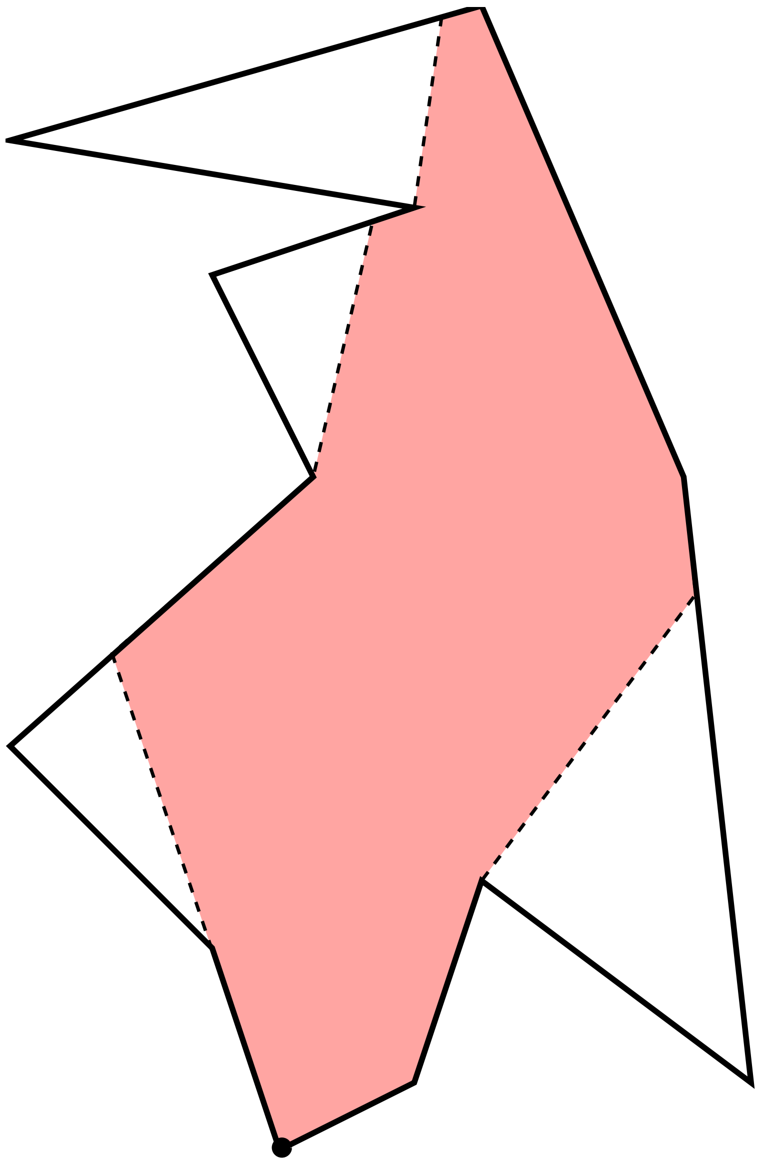
– **Microrobots: autonomous, anonymous, asynchronous,** oblivious, **no coordinates, no compass, ...**

– **Unknown environment: graph, local orientation, the plane, a polygon,** obstacles, terrain, **no landmarks, ...**

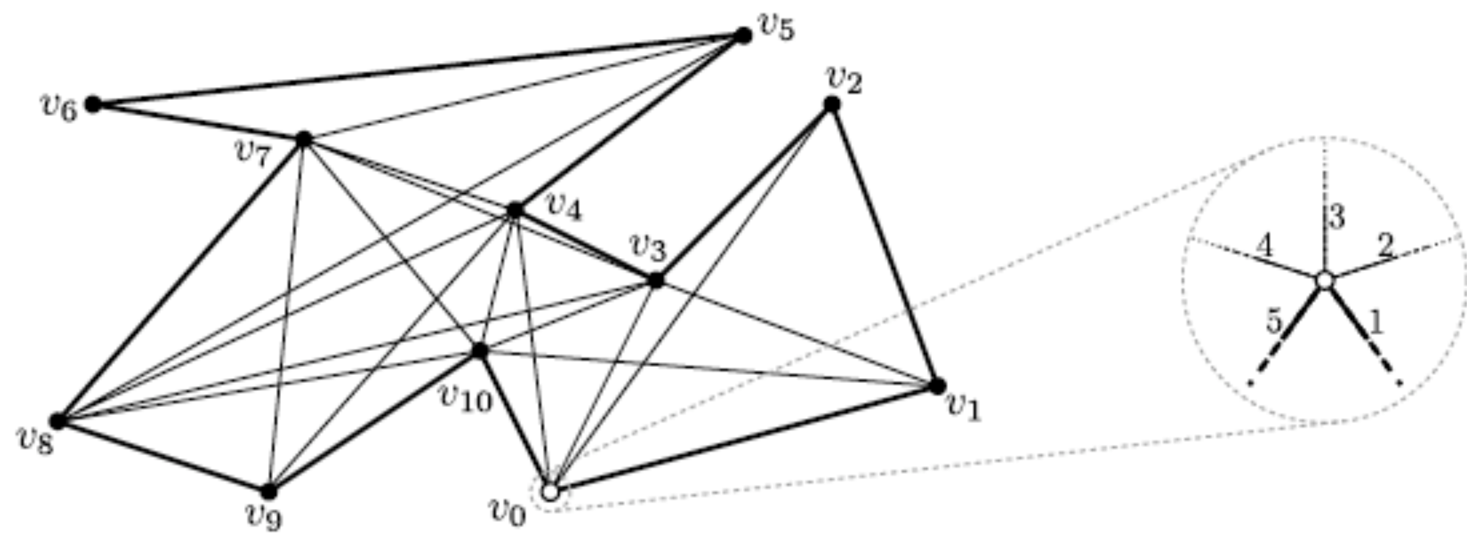
– **Problems:** guard a polygon, **build a map,** form a pattern, count targets, **rendezvous, ...**

Basic Robot Model

- Robot is point on a vertex
- Robot can look at polygon while sitting at a vertex
  → sees all visible vertices (ccw)
- Robot can move to visible vertices
  → cannot sense while moving



Local View at a Vertex

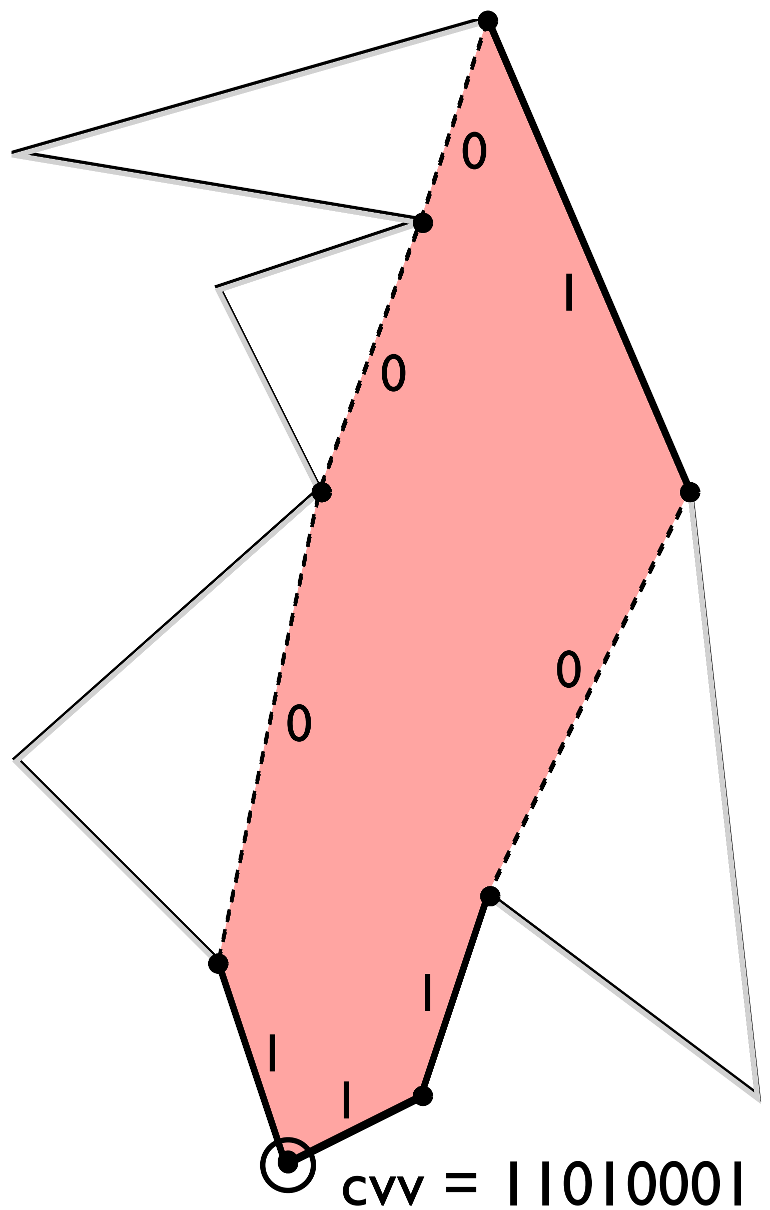


visibility graph

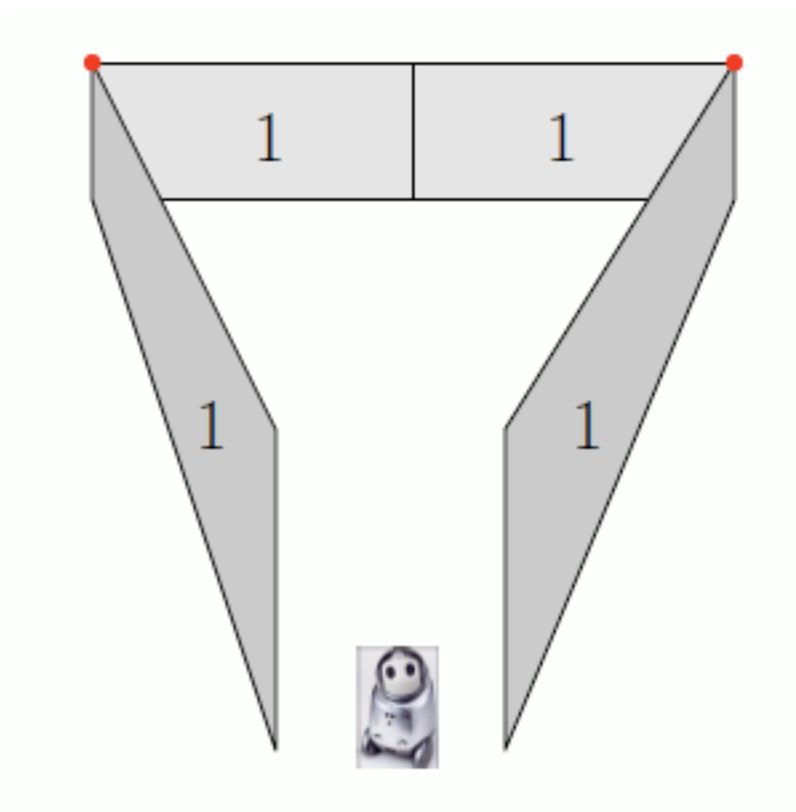
- polygon corner is graph vertex
- edge iff corners see each other

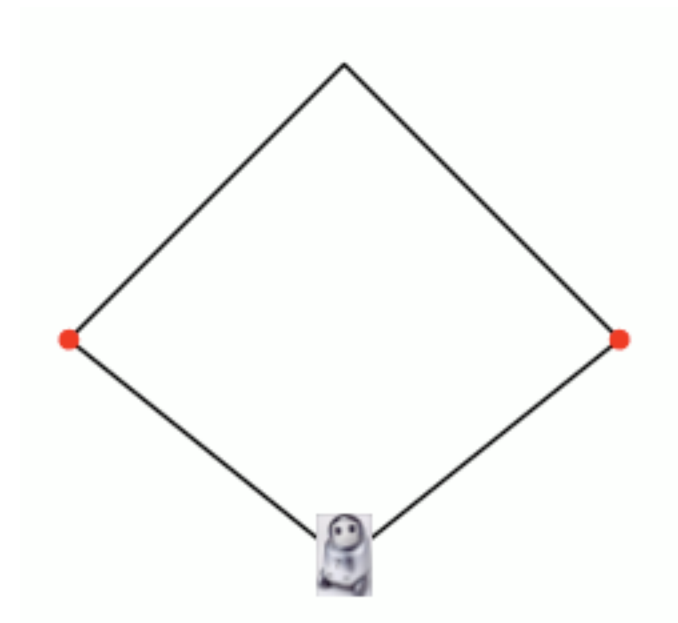
Combinatorial Visibility

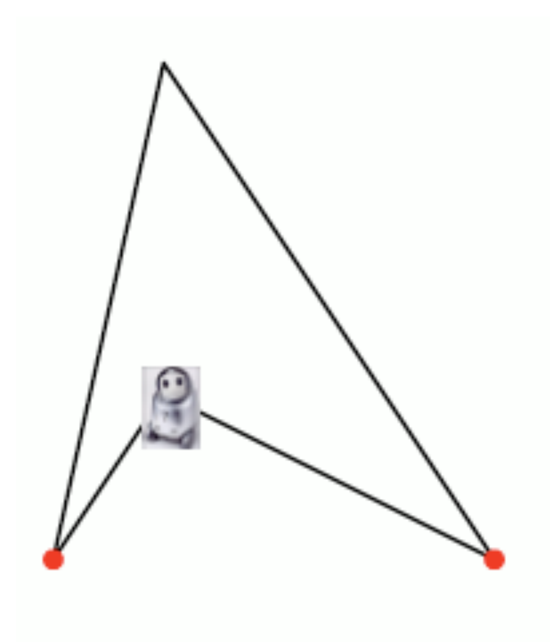
- For every consecutive pair of visible vertices:
  - 1 if neighbors in *P*
  - 0 else



What can robots do?

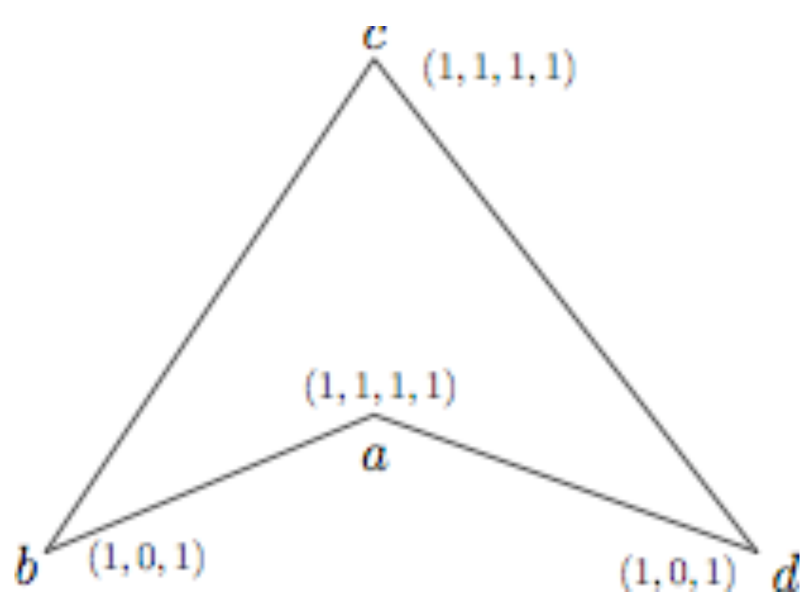






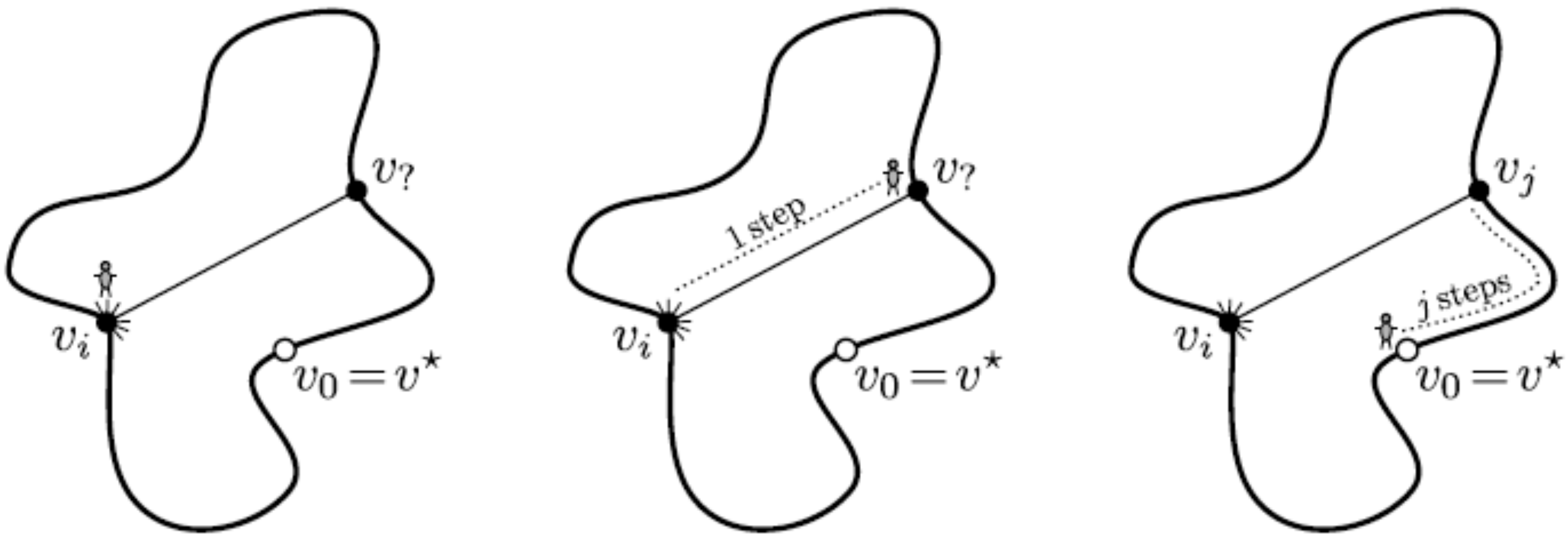
Global Convexity

- Polygon is convex iff every vertex's cvv is a vector of all 1's.



- **Global knowledge despite local uncertainty.**

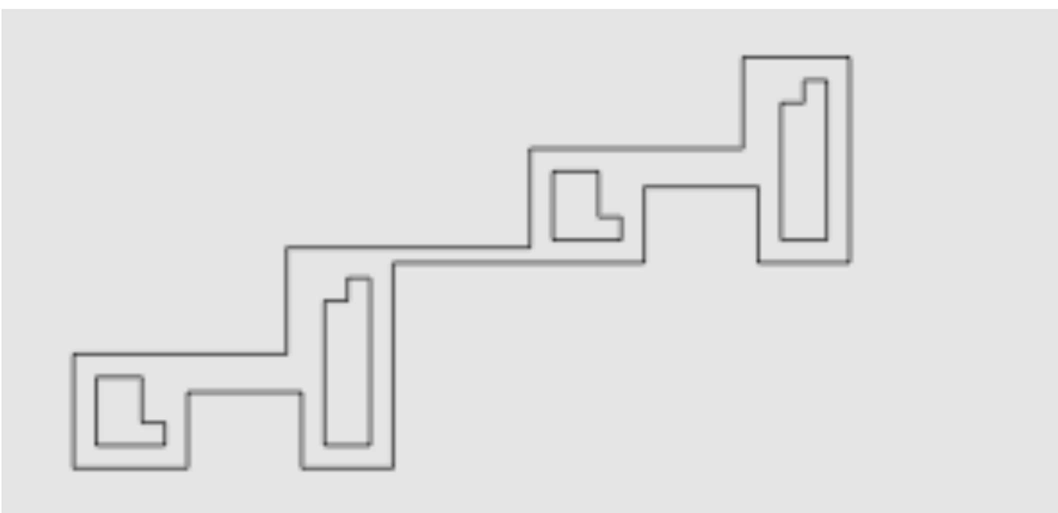
Navigation

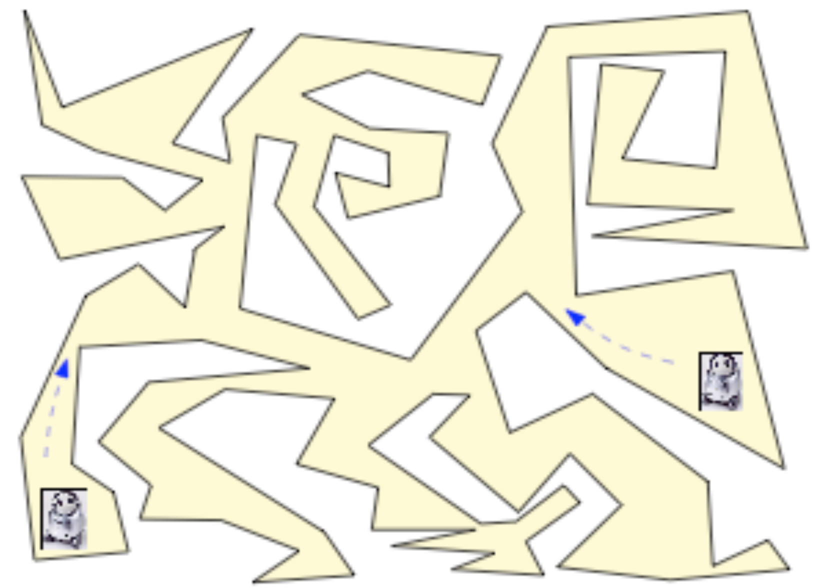


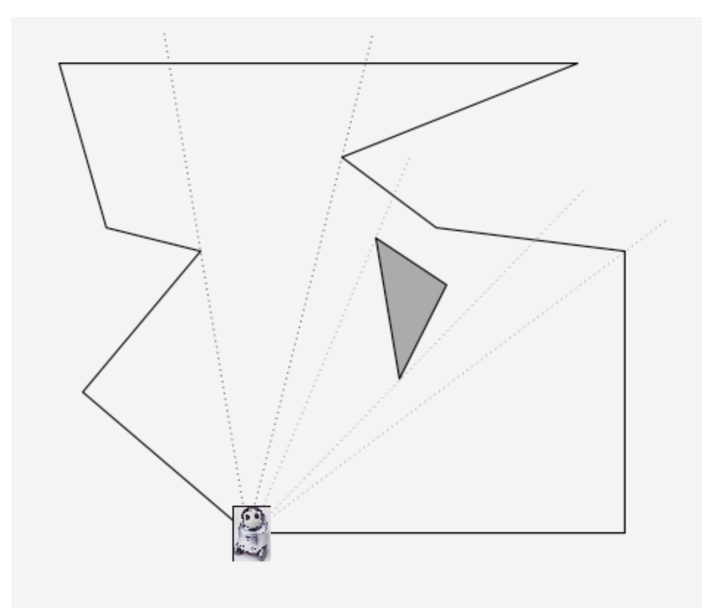
A distinguished vertex is enough ...

Topology: Global from local

- Are CVVs sufficient to decide if a polygon is simply-connected?
  - Does it have holes? How many?

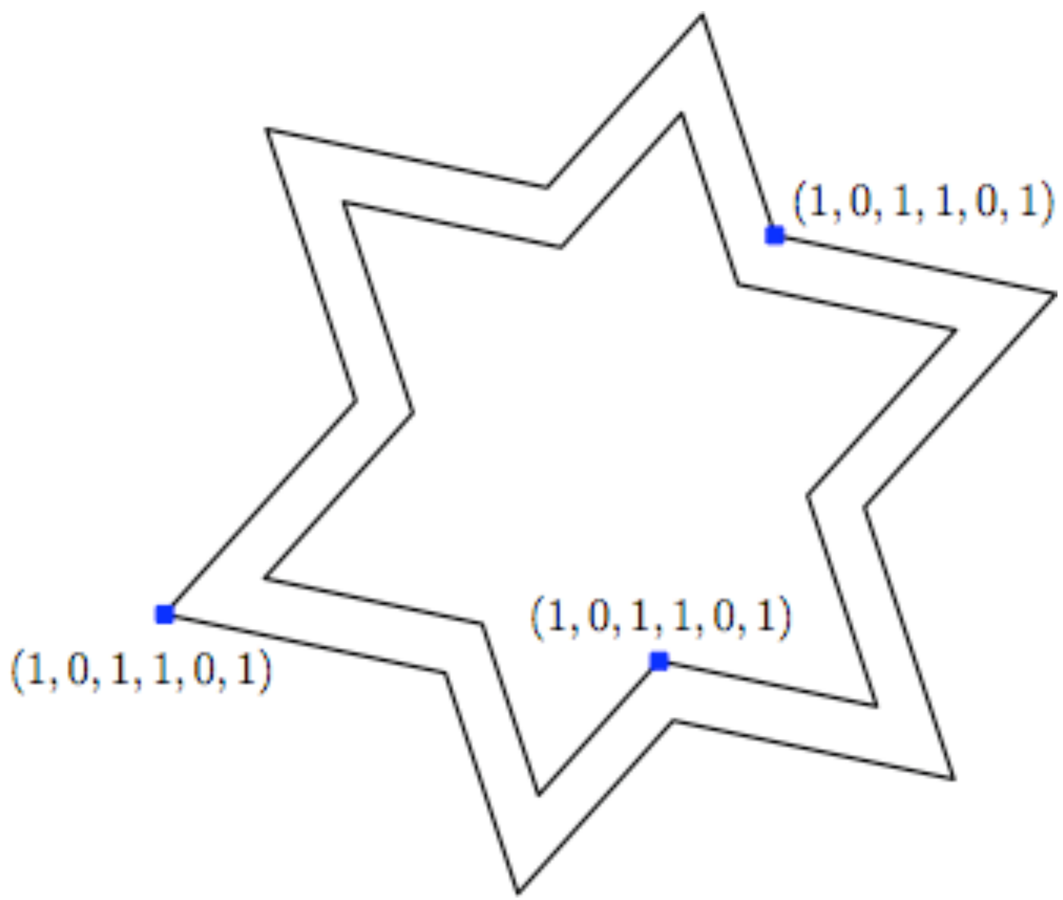






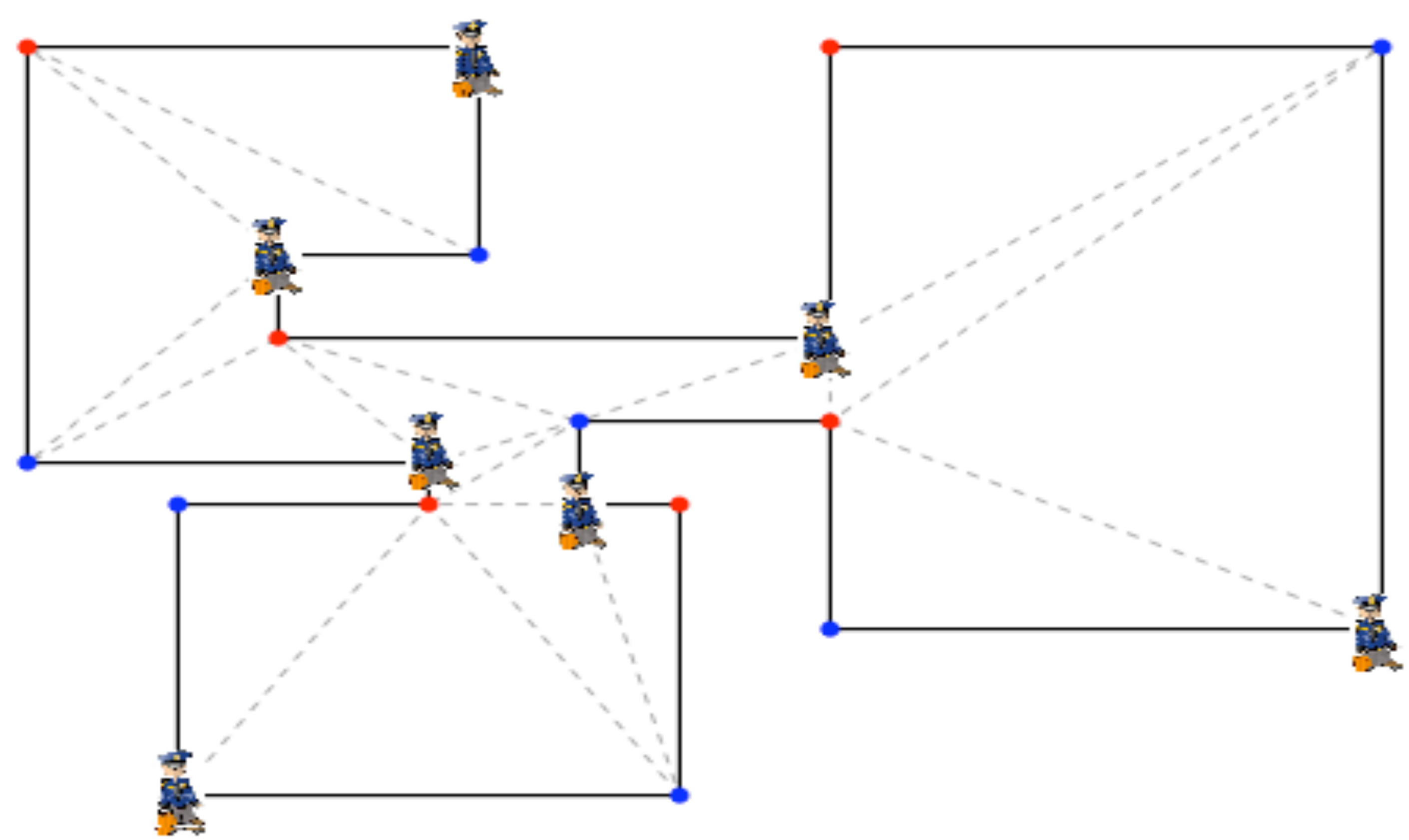
Local vs. Global

- Robots can detect multiple boundaries
- But cannot distinguish holes from outer boundary



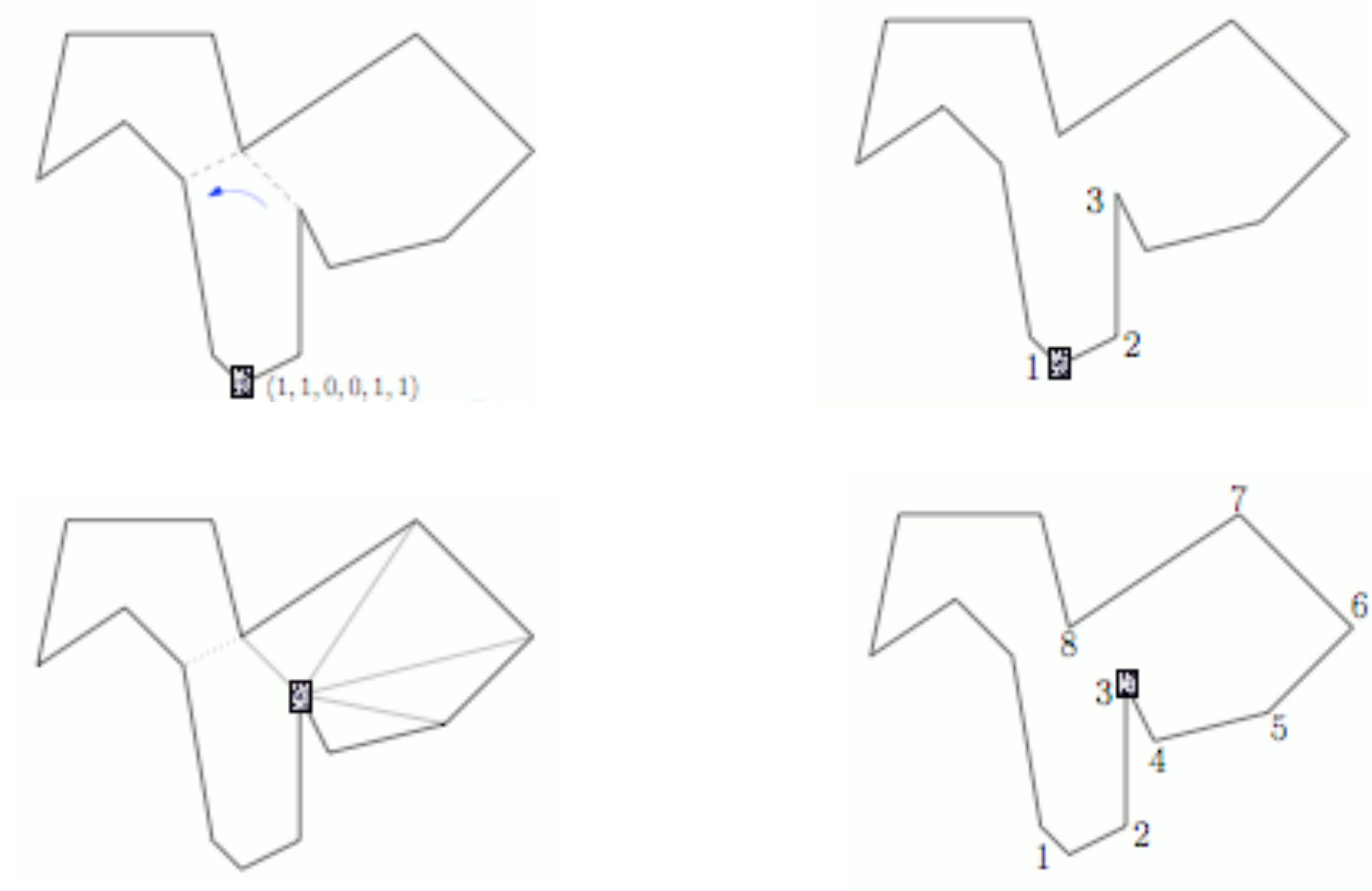
Monitoring: Visibility Coverage

- Can a group of robots self-deploy to cover a polygon?
- Art Gallery Theorem: Any n-gon can be guarded by n/3 guards.
- Fact: Guarding problem can be solved in the CVV model.



CVV triangulation

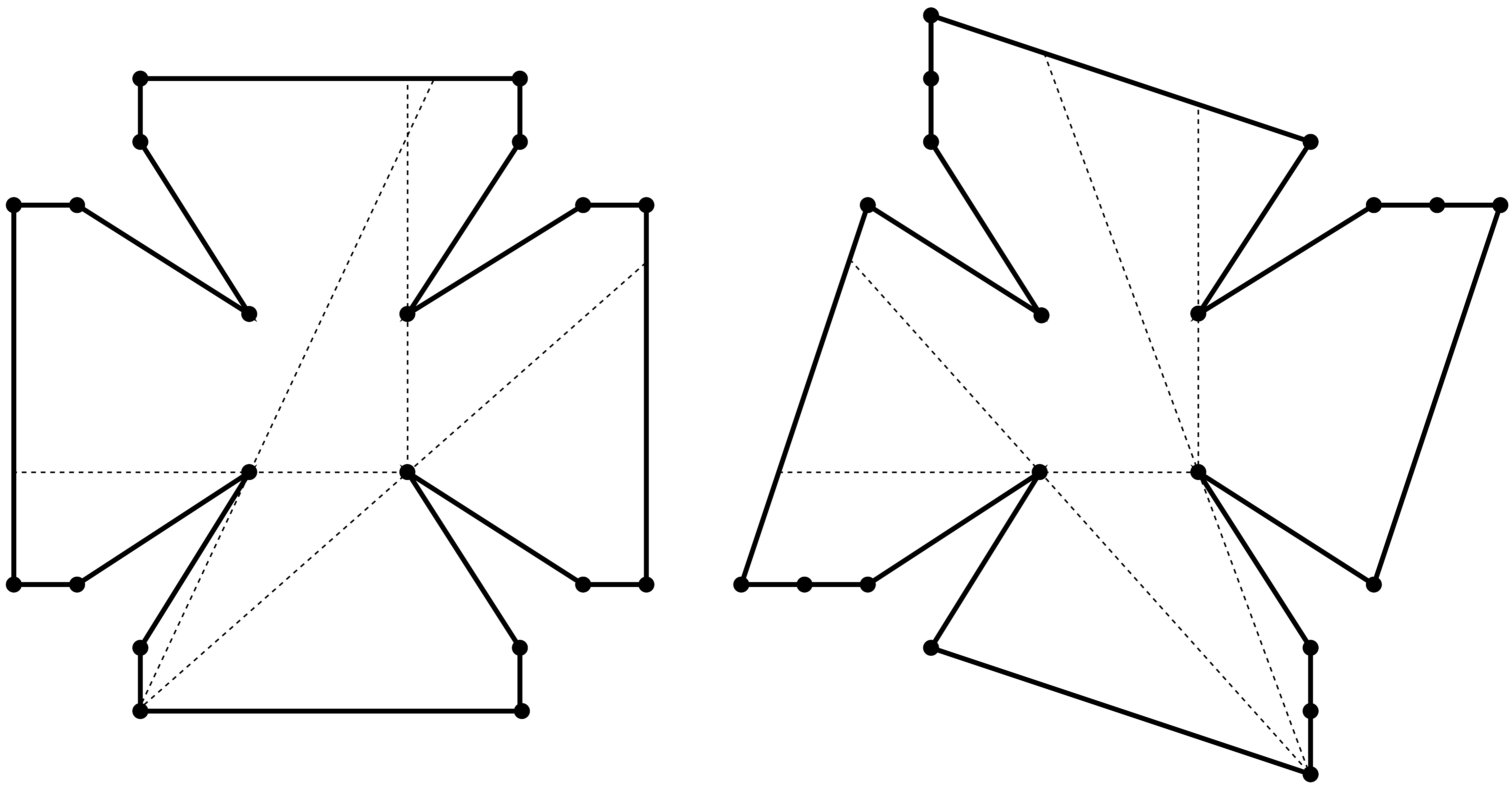
- Triangulation is a topological structure: dependent on visibility, but not coordinates.
- Fact: Triangulation is possible in the CVV model.



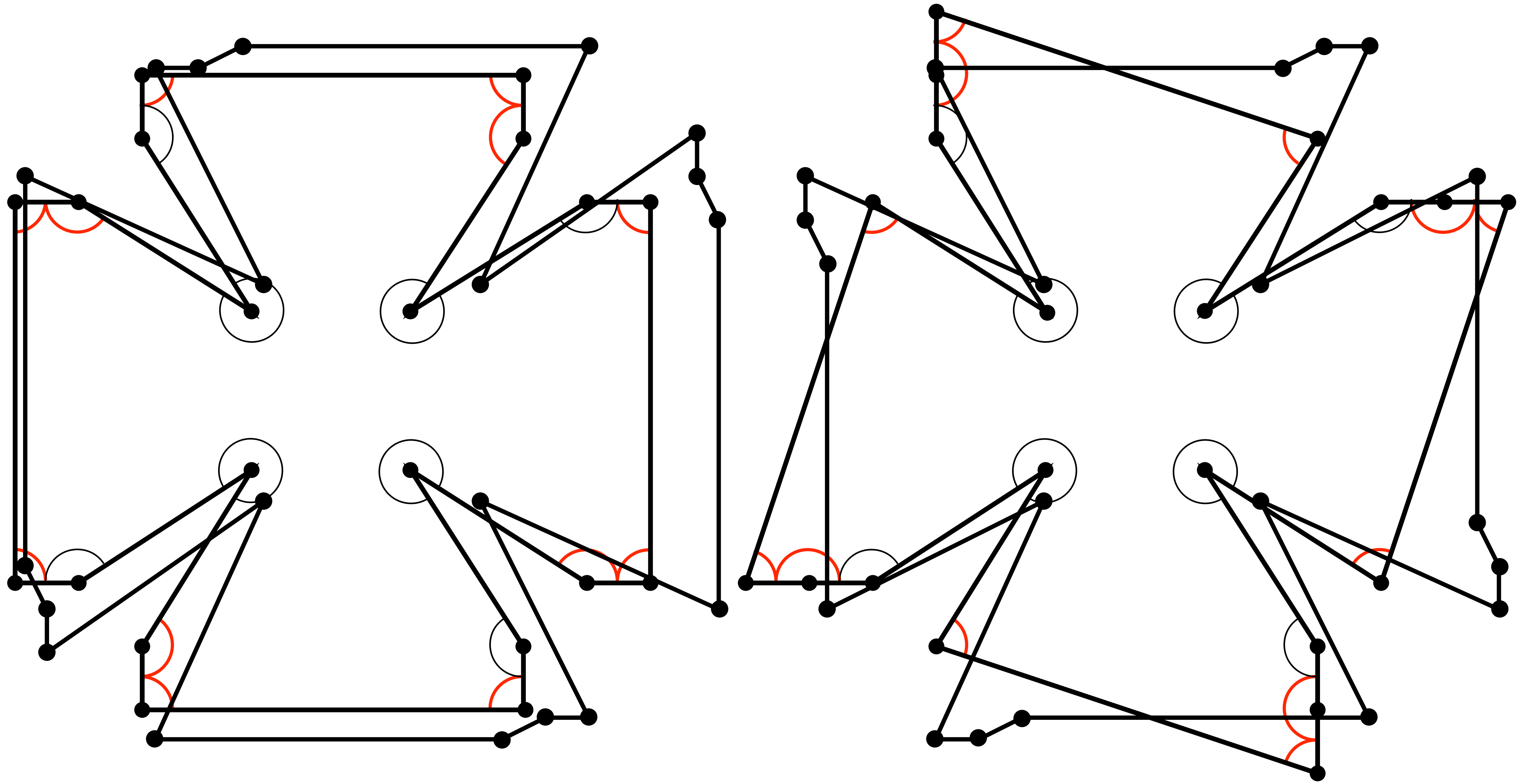
Map construction problem

Combinatorial Visibilities $\rightarrow$ Visibility Graph?

Combinatorial Visibilities → Visibility Graph?



Combinatorial Visibilities + Boundary Angles → Visibility Graph?

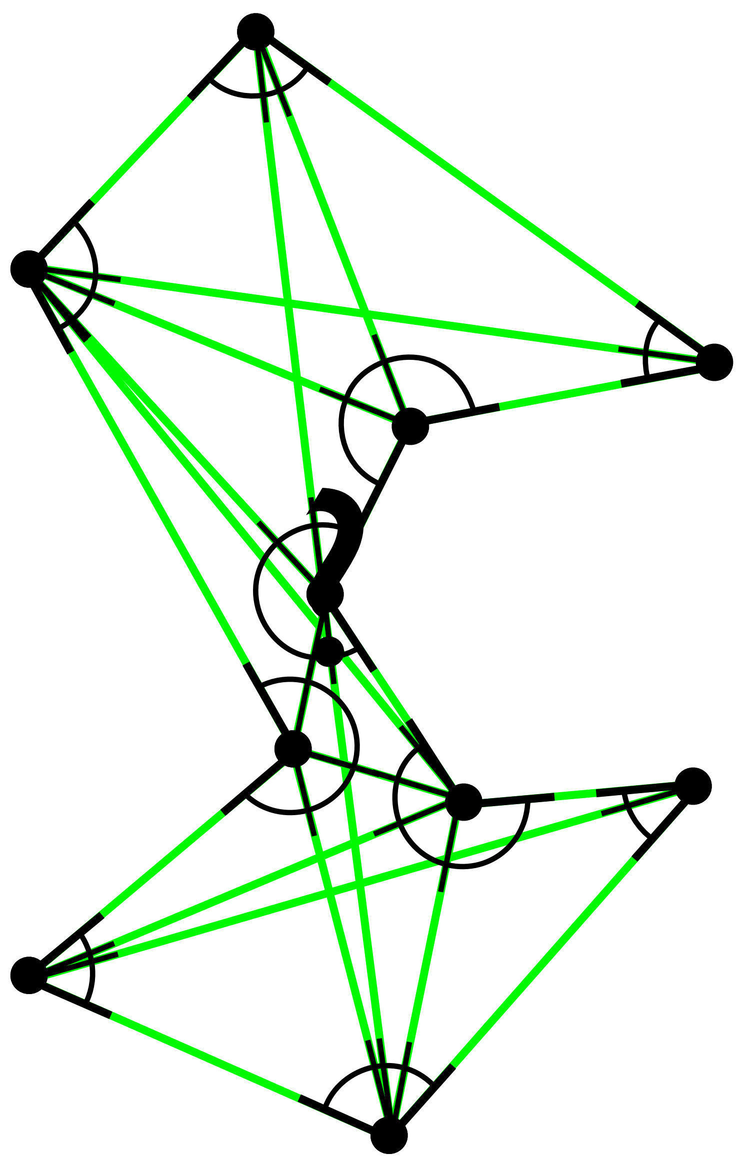


combinatorial visibilities
and boundary angles
are not enough

Combinatorial Sensors

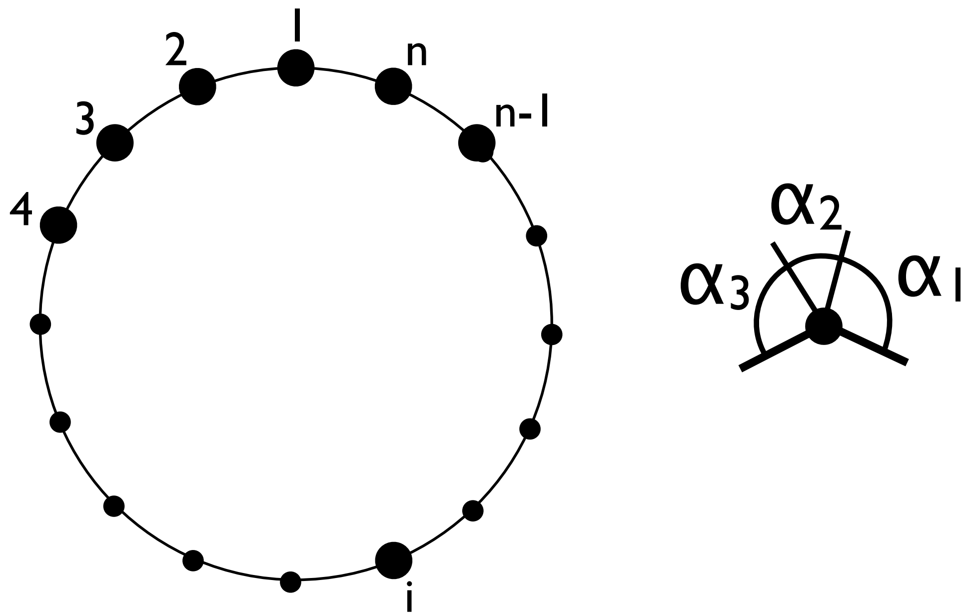
Geometrical Sensors

All inner angles

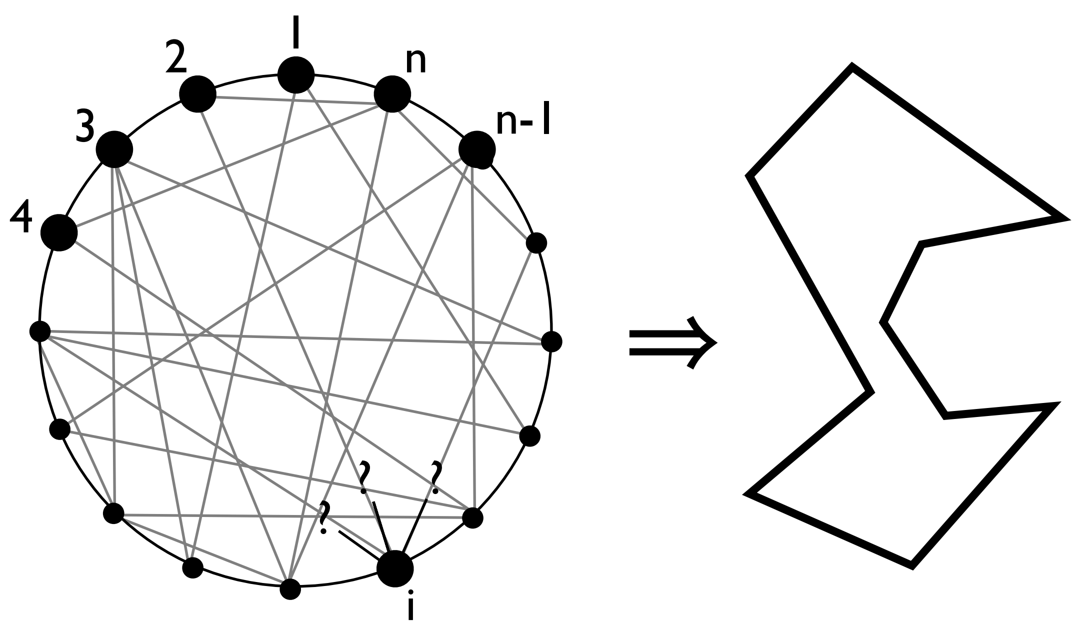


- Visibility graph + angles ⇔polygon shape
- Are angles alone enough?

Polygon reconstruction
problem definition



- Given:
  - boundary vertex order
  - angles at each vertex

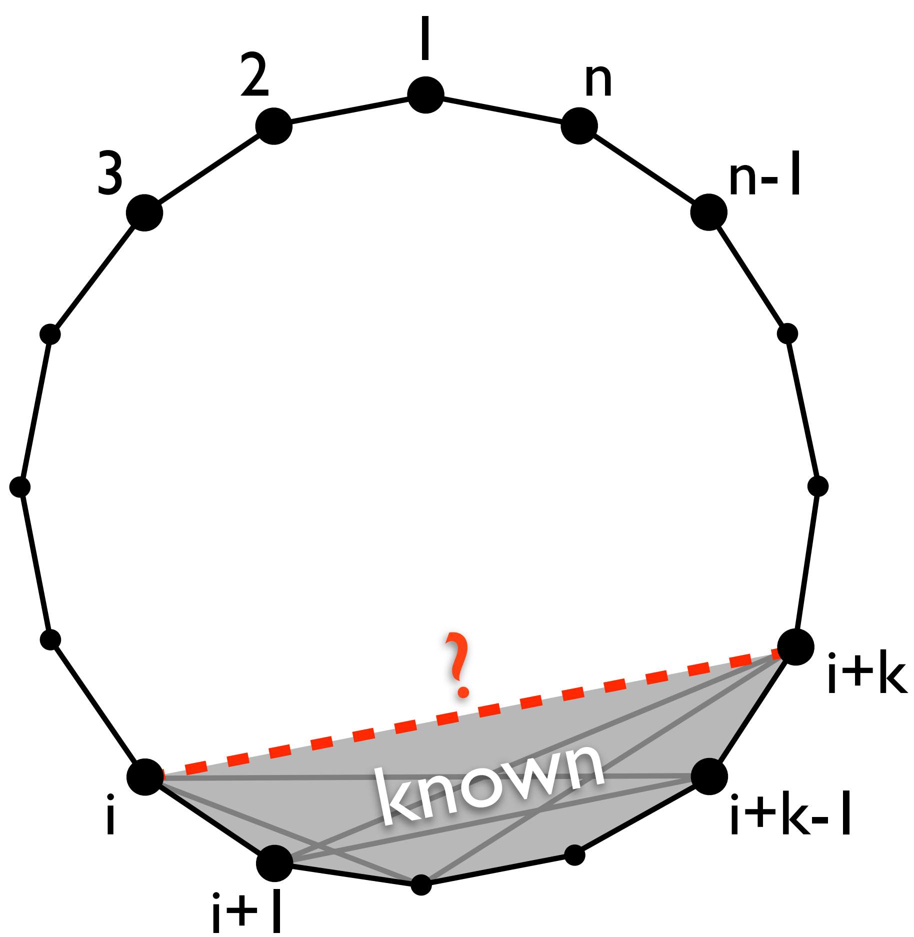


- Find:
  - visibility graph edges E

⇒polygon shape

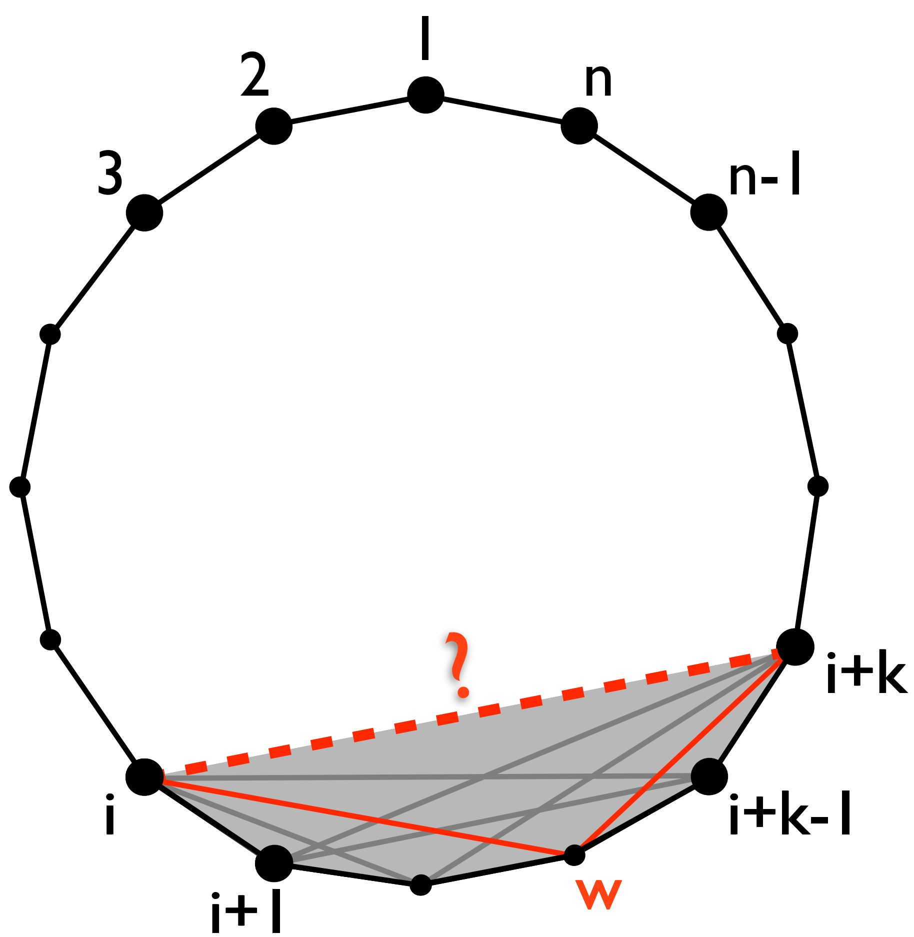
Polygon reconstruction

iterative reconstruction



- Reconstruct visibility graph iteratively
- For k = 1,...,n/2:
  find all edges (i, i+k) ∈ E
- k = 1:
  trivial (boundary)
- k > 1:
  need criterion to decide whether (i, i+k) ∈ E

Polygon reconstruction
criterion



- Criterion for (i, i+k) $\in$ E
- If (i, i+k) $\in$ E: there is a "witness" (sees both)

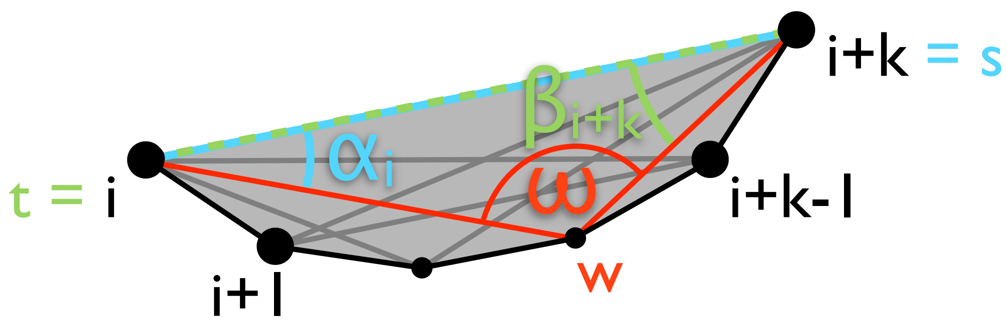
$\Rightarrow$ If there is no witness: (i, i+k) $\notin$ E

- And if there is a witness?

Polygon reconstruction

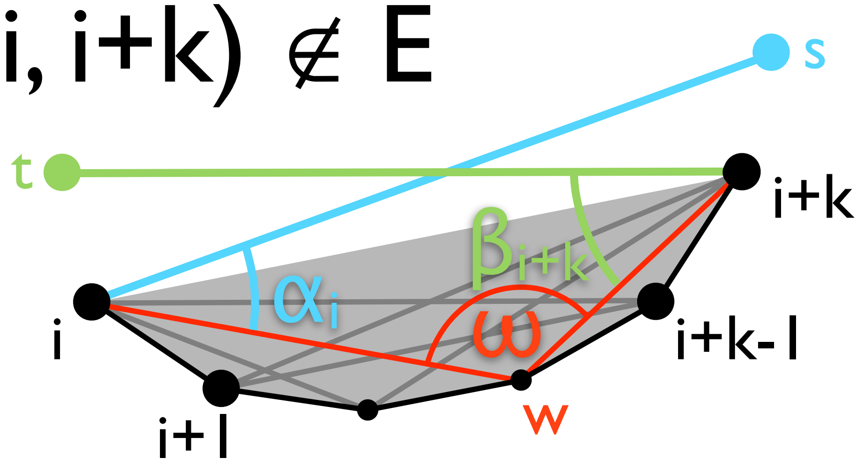
criterion

$(i, i+k) \in E$



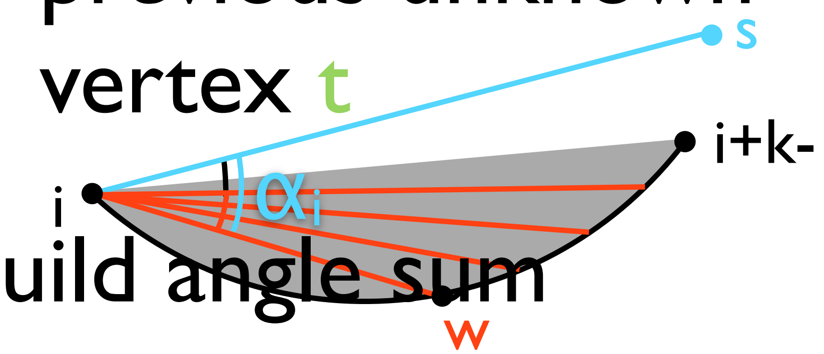
$$\omega + \alpha_i + \beta_{i+k} = \pi$$

$(i, i+k) \notin E$



$$\boxed{\omega + \alpha_i + \beta_{i+k} \neq \pi}$$

- Consider angles:
  - ω at w
  - α_i between w and the next unknown vertex s
  - β_{i+k} between w and the previous unknown vertex t



- Build angle sum

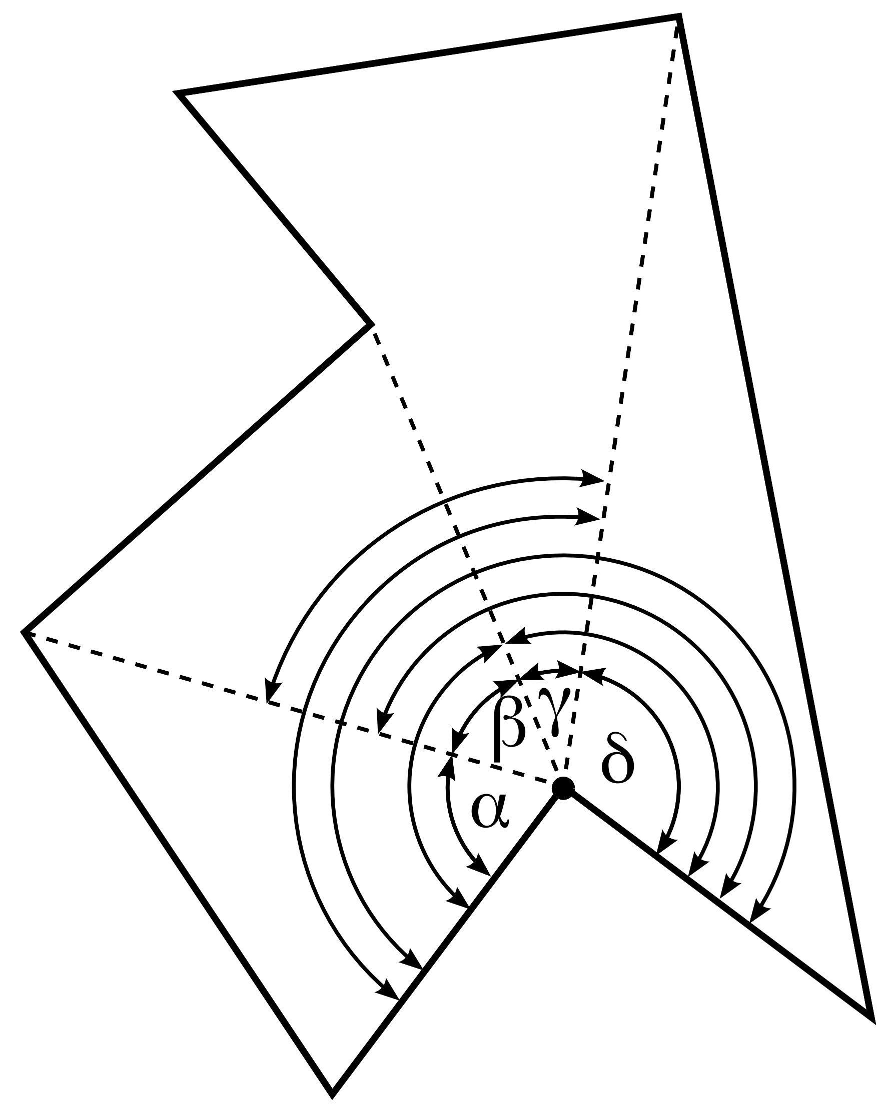
$\Rightarrow (i, i+k) \in E \Leftrightarrow \Sigma = \pi$

$\Rightarrow$ criterion: $\exists w \wedge \Sigma = \pi$

all inner angles
are enough

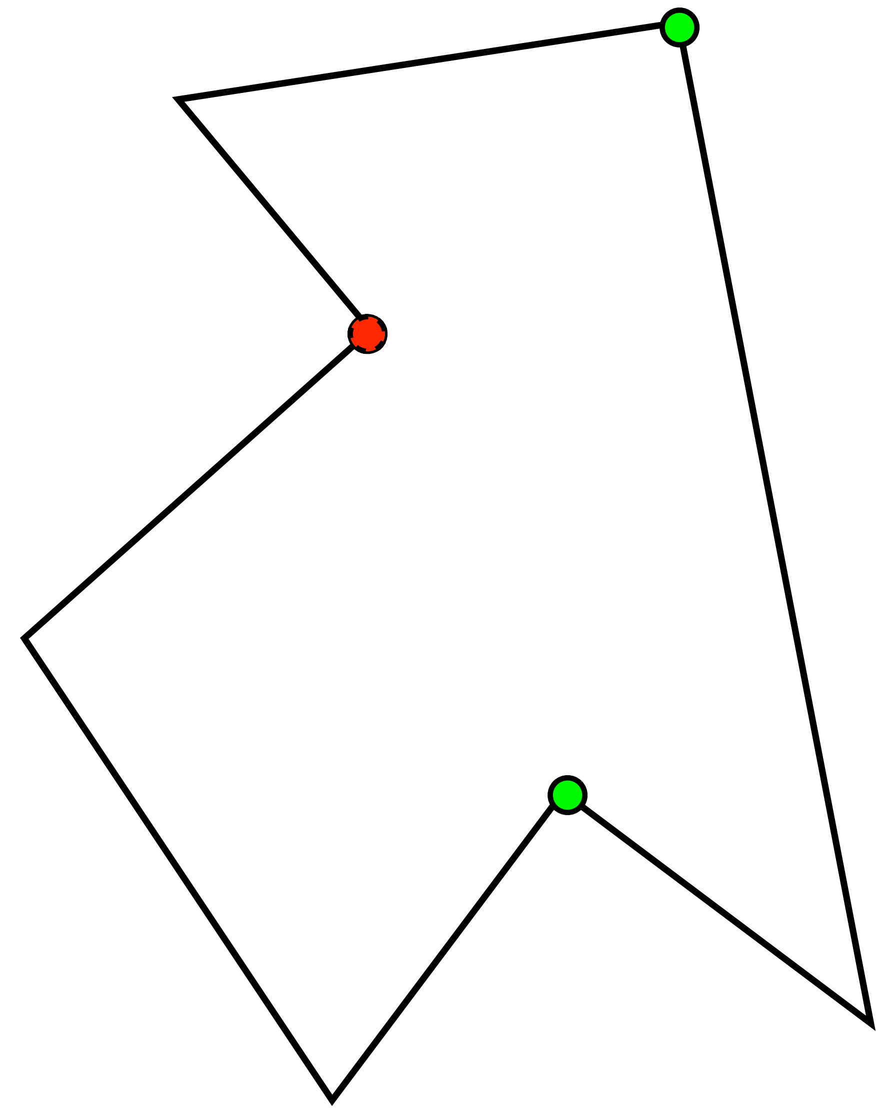
All inner angle types

- Are all inner angles really necessary? Or is less enough?
- **Types** of angles between all pairs of visible vertices



Looking Back

- Types of all inner angles is enough, if ...
- ... robot can look back



Algorithm

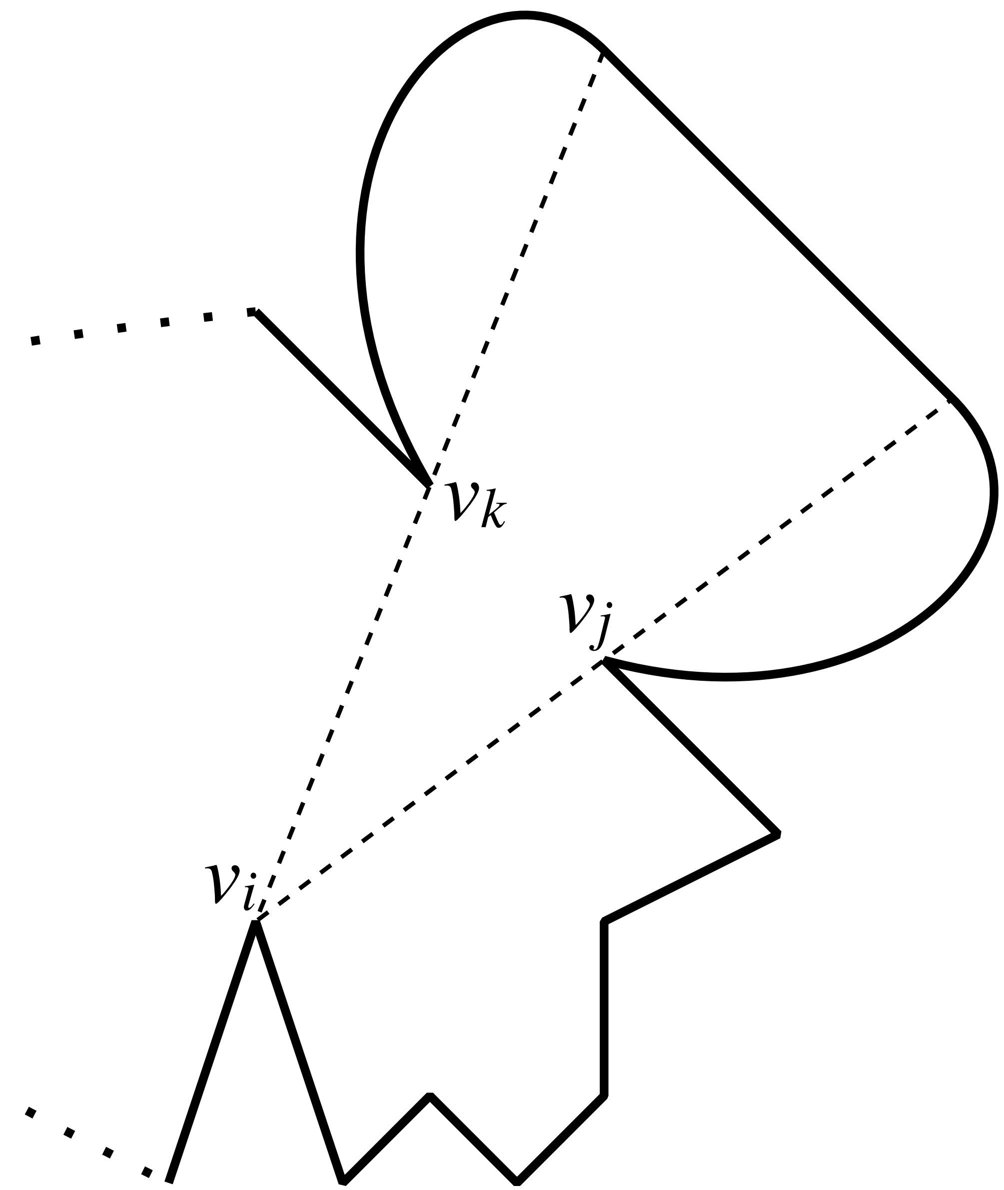
- Progressively identify distant vertices

  → assign global name

- Assume robot at v_i already has identified vertices up to v_j

- Vertex v_k is the next unidentified vertex

  → determine k

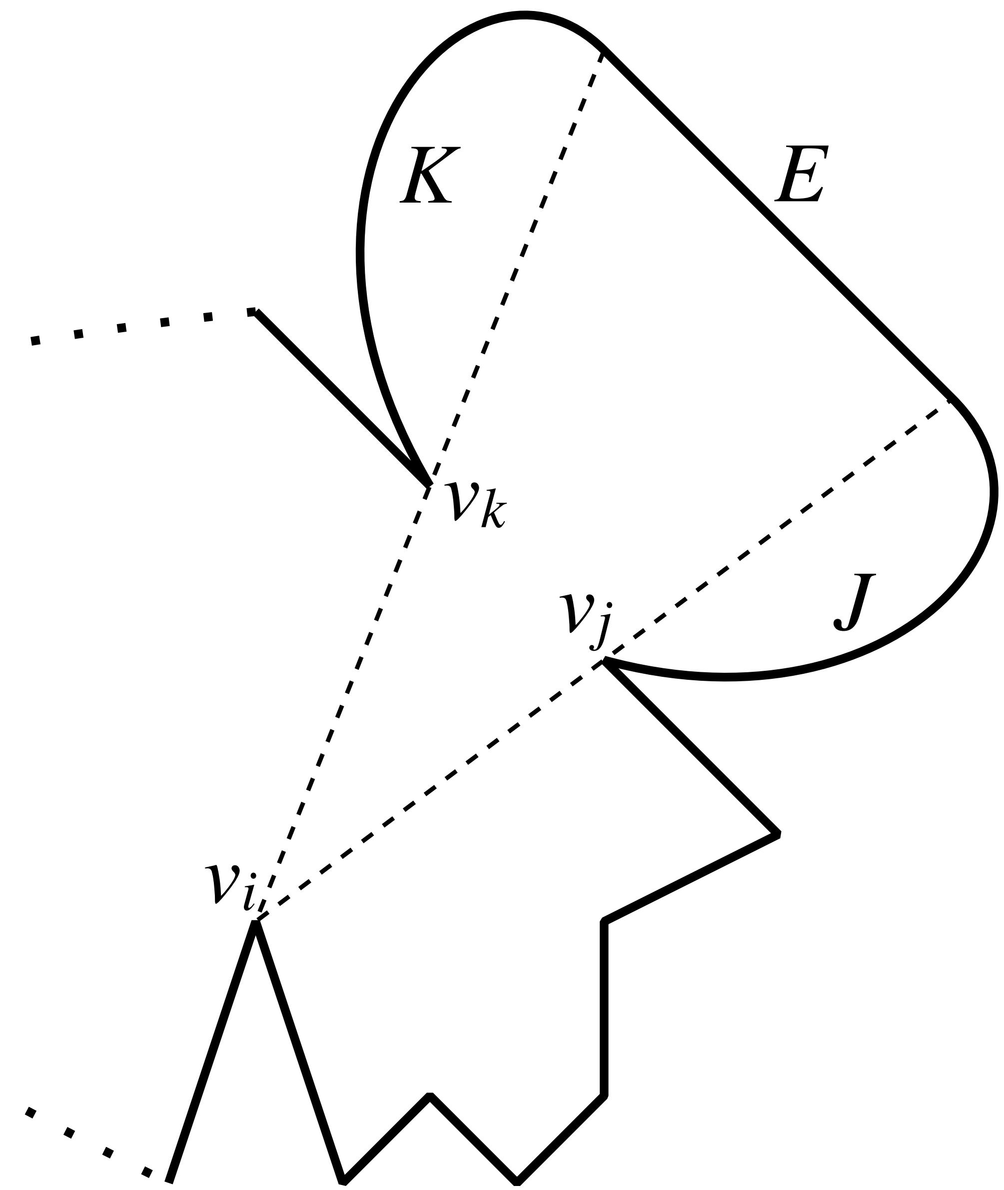


Algorithm

- *E* cannot contain vertices

  → $k = j + |J| + |K| + 1$

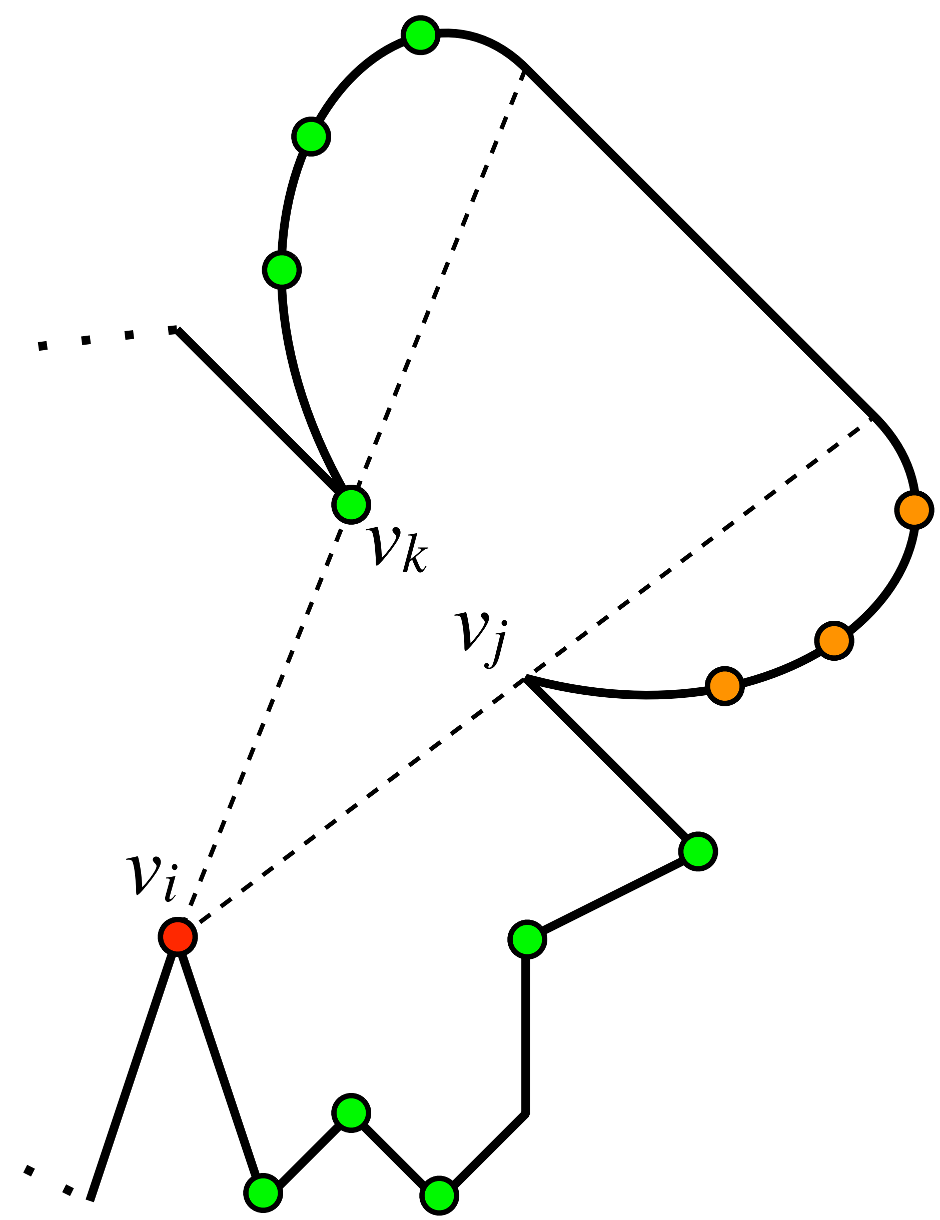
- Need to count vertices "behind" v_j and v_k



Algorithm

Count vertices behind v_j:

- Move to v_j and identify v_i by looking back
- Count vertices with reflex angle to v_i
- Count vertices behind those vertices recursively
- move back to v_i

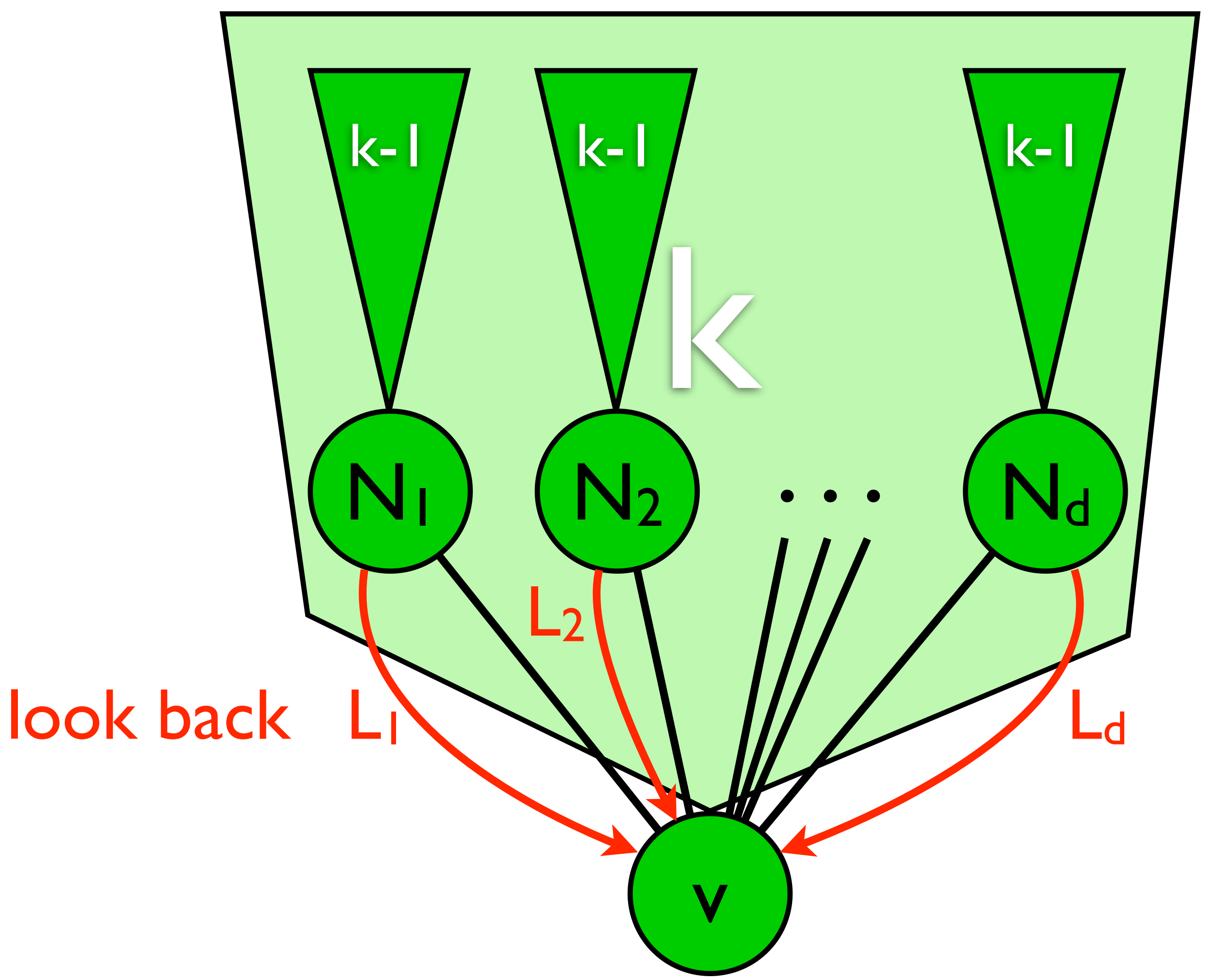


Works also if the number of vertices is not known

all inner angle types
and look back
are enough

Look Back, no angle types

view



- All information a robot can ever collect about v

  ⇒ ***view*** from v

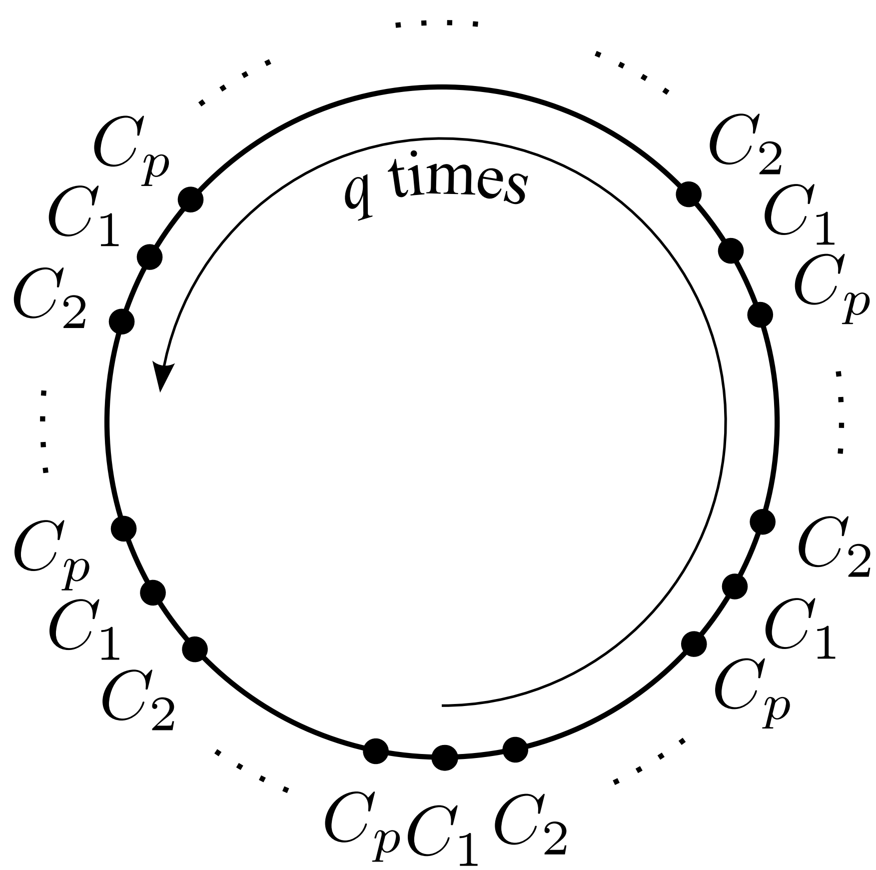
  ⇒ collection of all paths

- level-one-view:
  $v^1 = (L_1, L_2, ..., L_d)$

- level-k-view:
  $v^k = (N_1^{k-1}, N_2^{k-1}, ..., N_d^{k-1})$

Vertices as Viewpoints

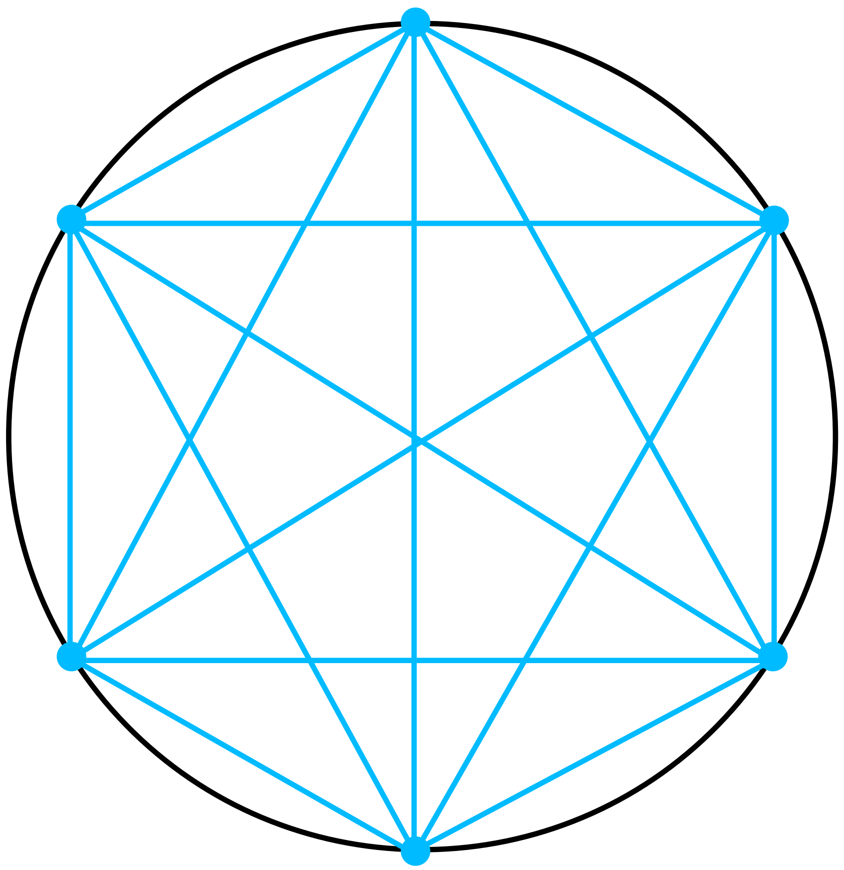
classes



- group all vertices with same v^∞ into classes C_i
  $\Rightarrow$ periodic on boundary
  $\Rightarrow |C_i| = |C_j| \; \forall i,j$
- $|C_i| = 1$: distinguishable ✓
- v^{n-1} is enough (same resulting classes)

The Unique Class C^*

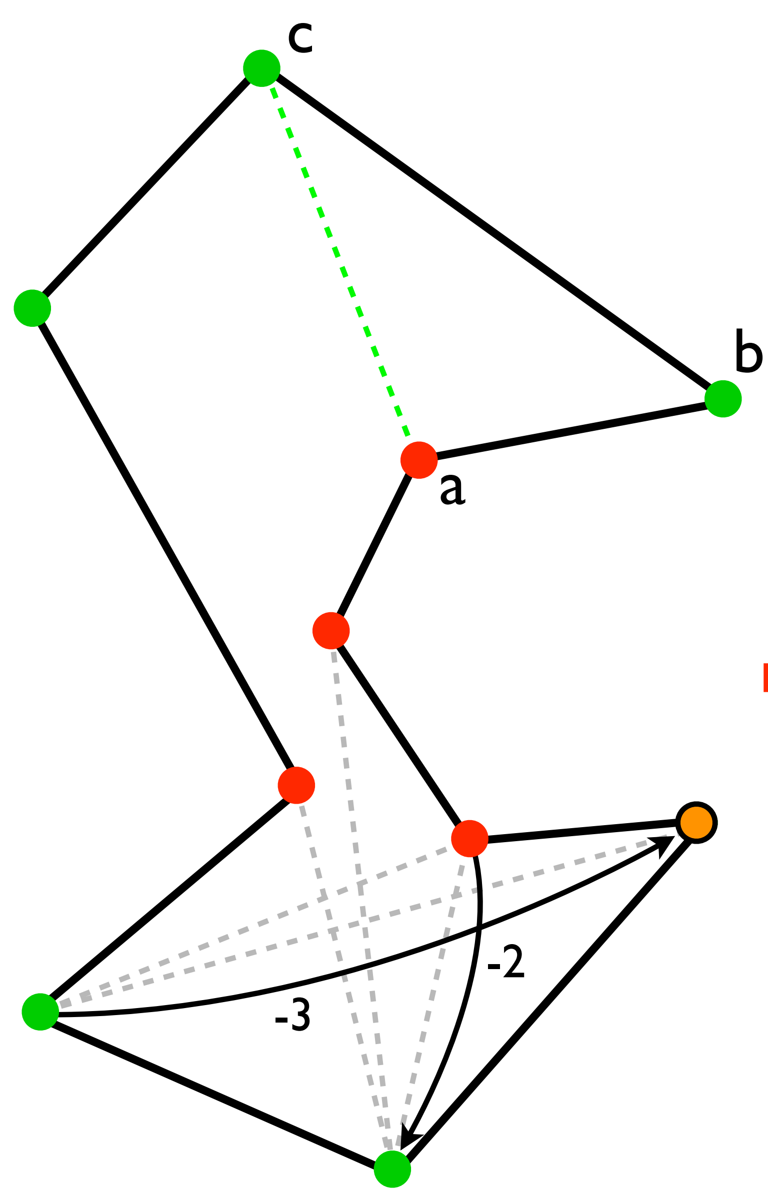
definition



- C^* is the lexicographically smallest class that forms a clique
- We will see:
  Every polygon has a class that forms a clique
- C^* is well defined, unique

The Class C*

ears



- Let a,b,c be a sequence of vertices on the boundary

  ⇒b is an *ear*, iff a sees c

- b is an ear, iff the move

  first left neighbor -1, 2, look back yields "-2" second left neighbor

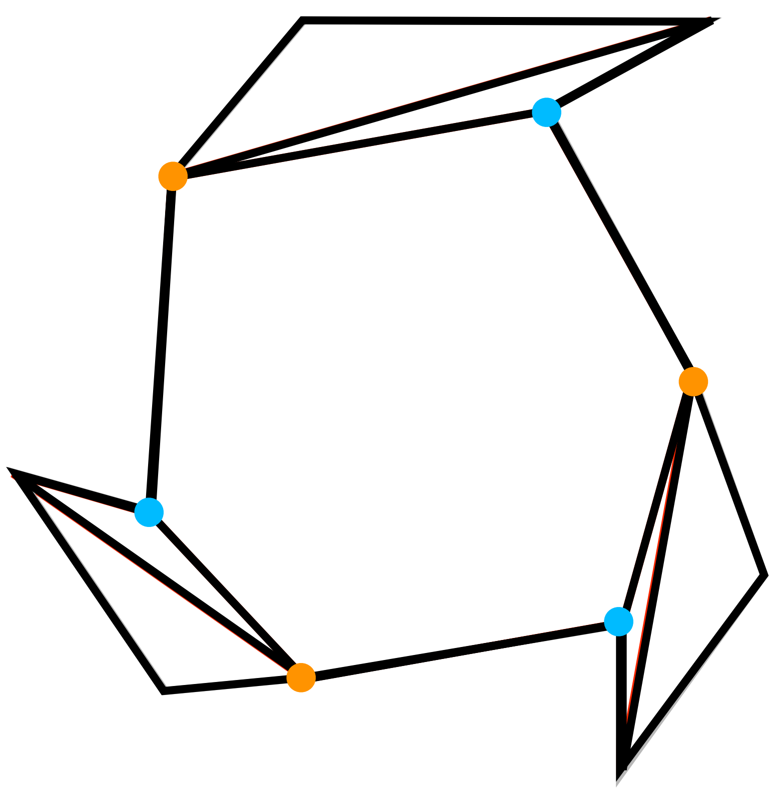
  second right neighbor

  ⇒vertices in the same class as an ear are ears

- Every polygon has an ear

The Class C^*

existence of a clique

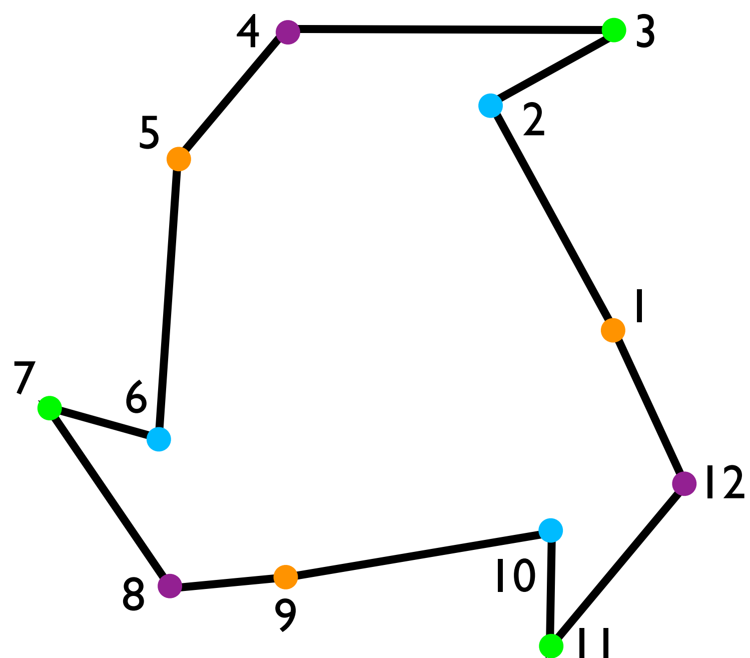


- Cut ears repeatedly...
  - ⇒cut the entire class
  - ⇒no class will split!
- ... until only one remains
  - ⇒must be a clique!
  - ⇒contains all vertices of some original class

⇒Every polygon has a class that is a clique!

Meeting and Mapping

problem re-definition



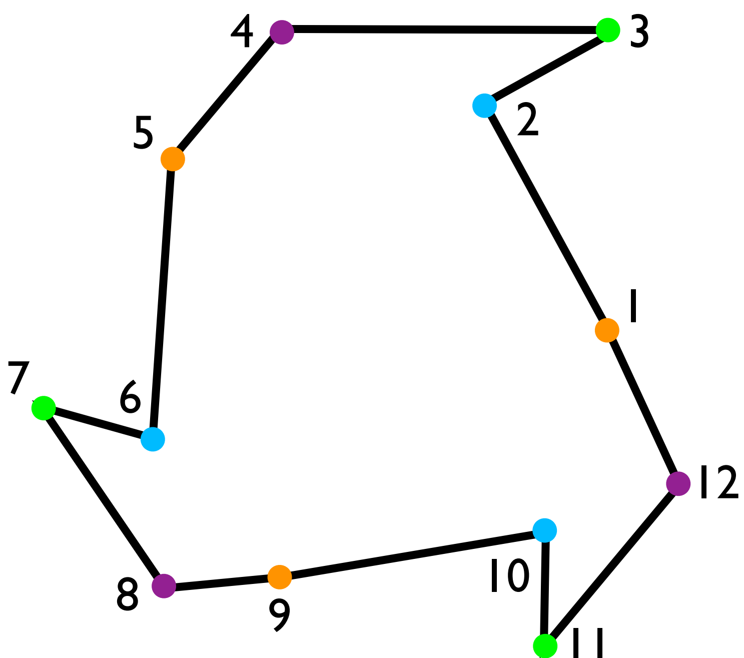
vert	class	neighbors
1	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
2	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
3	C_3	C_4,C_1,C_2
4	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$
5	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
6	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
7	C_3	C_4,C_1,C_2
8	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$
9	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
10	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
11	C_3	C_4,C_1,C_2
12	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$

- Views of level n-1 are sufficient to infer classes

  ⇒task in terms of classes
- Given:
  - classes along boundary
  - classes of neighbors
- Tasks:
  - meet other robots
  - infer visibility graph

Meeting and Mapping

meeting



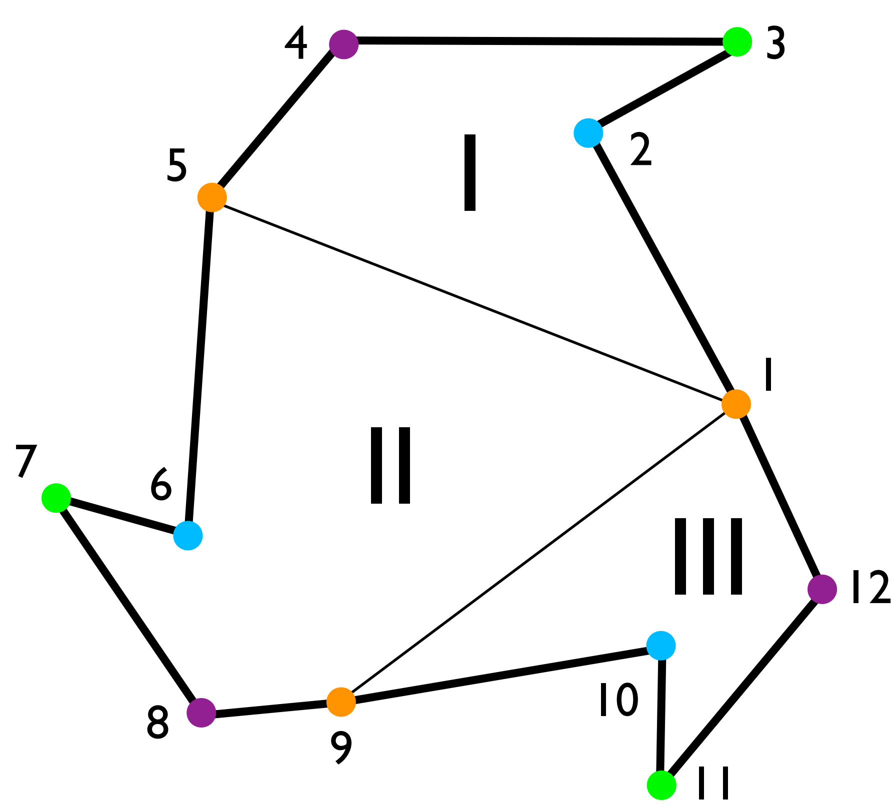
vert	class	neighbors
1	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
2	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
3	C_3	C_4,C_1,C_2
4	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$
5	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
6	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
7	C_3	C_4,C_1,C_2
8	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$
9	C_1	$C_2,C_4,C_1,C_2,C_4,C_1,C_2,C_3,C_4$
10	C_2	$C_3,C_4,C_1,C_2,C_4,C_1,C_2,C_4, C_1$
11	C_3	C_4,C_1,C_2
12	C_4	$C_1,C_2,C_4,C_1,C_2,C_4, C_1,C_2,C_3$

- C^* is unique
- C^* can be inferred

$\Rightarrow$ Meeting:
Move along boundary to a vertex in C^*

Meeting and Mapping

visibility graph reconstruction



vert	class	neighbors
1	C_1	$C_2, C_4, C_1, C_2, C_4, C_1, C_2, C_3, C_4$

? ? 5 ? ? 9 ? 12 ?

I II III

- need to identify vertices in the list of neighbors
- If own class is a clique:
  - classmates are easy
  - classmates can be used to segment neighbors
- Classes are periodic

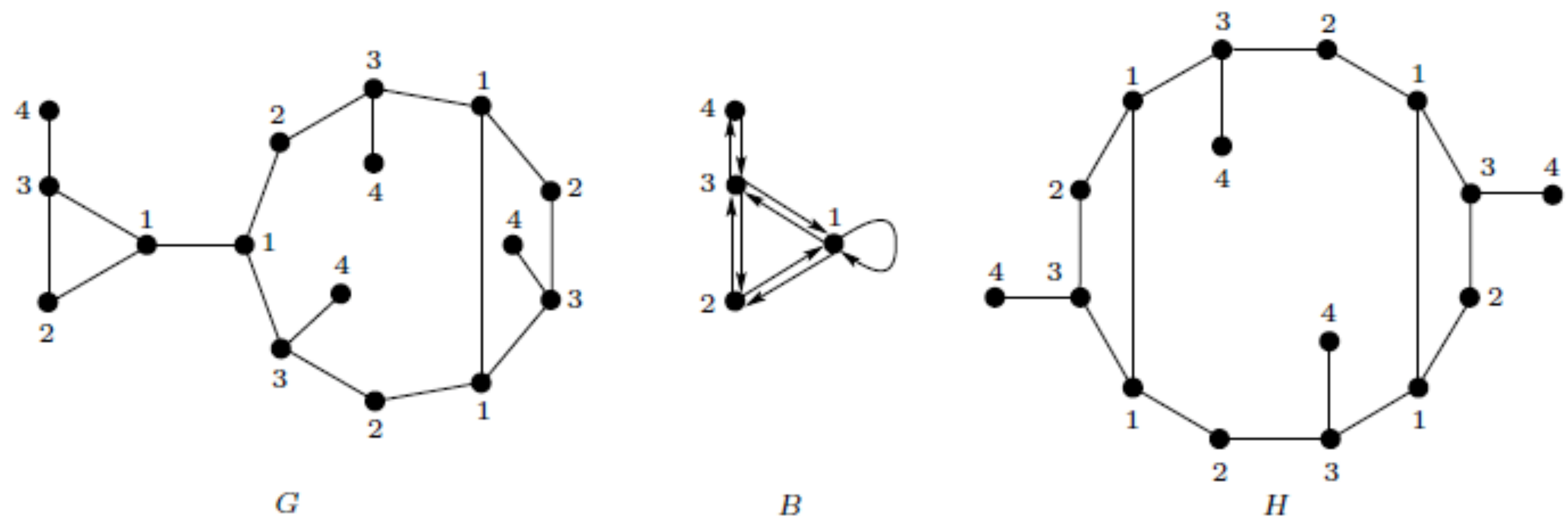
  ⇒ segment + class → ID

⇒ C^* vertices identified

⇒ others: a bit more work

look back
is enough

Contrast: Map construction for graphs is impossible



minimum base
is unique

for strongly connected, directed, edge labelled graphs:
  minimum base is the same for many graphs

for visibility graphs with labels reflecting all angle types:
  visibility graph is unique for minimum base

  algorithm:
    find minimum base
    reconstruct visibility graph

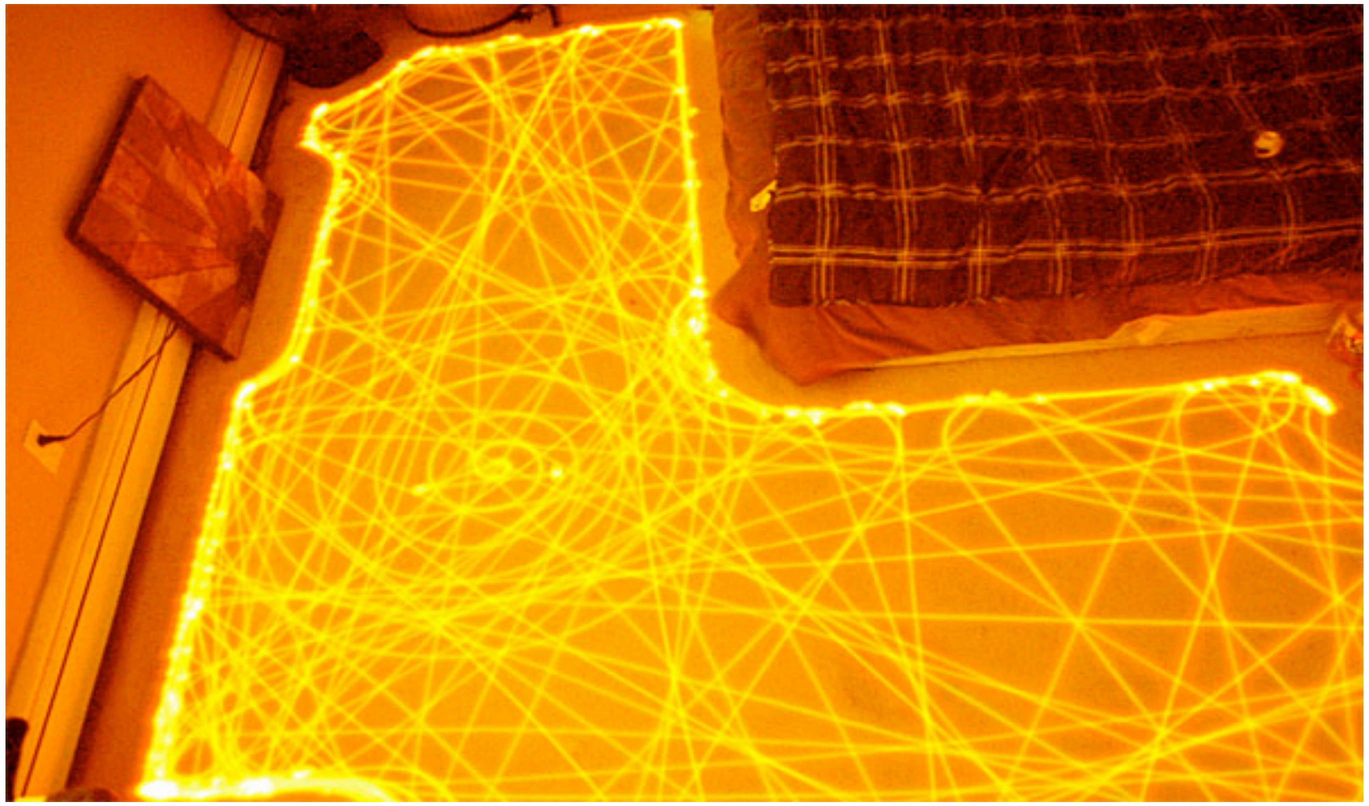
all inner angle types
are enough

Summary

- Boundary angles only: No map.
- All angles: Map, even if moves only on boundary.
- All angle types, look back: Map.
- Look back: Map.
- All angle types: Map.

Some Open Problems

- Many individual questions, such as: All lengths ?



- Robots: More realistic
- Environments: More realistic
- Solutions: More efficient