Algorithmics of Directional Antennae

By

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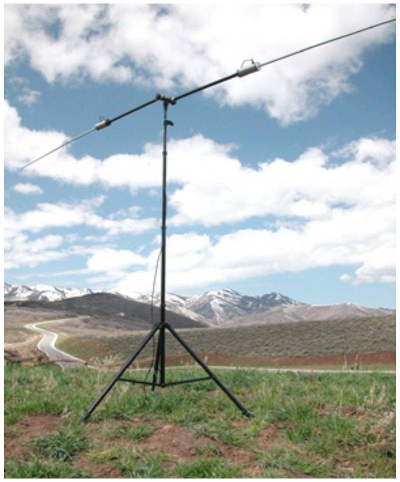
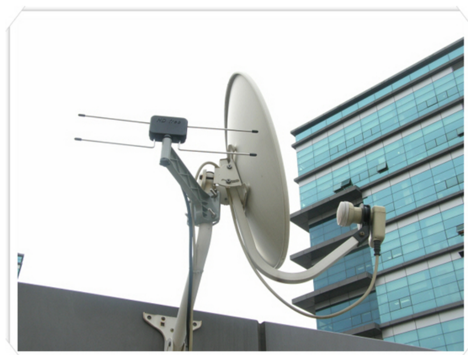
Outline

- Motivation
- Preliminaries
- Single Antenna
  - Upper Bounds
  - Lower Bounds
- Multiple Antennae
- New Ideas and Results
  - Robustness and consequenses
- Open Problems

Motivation

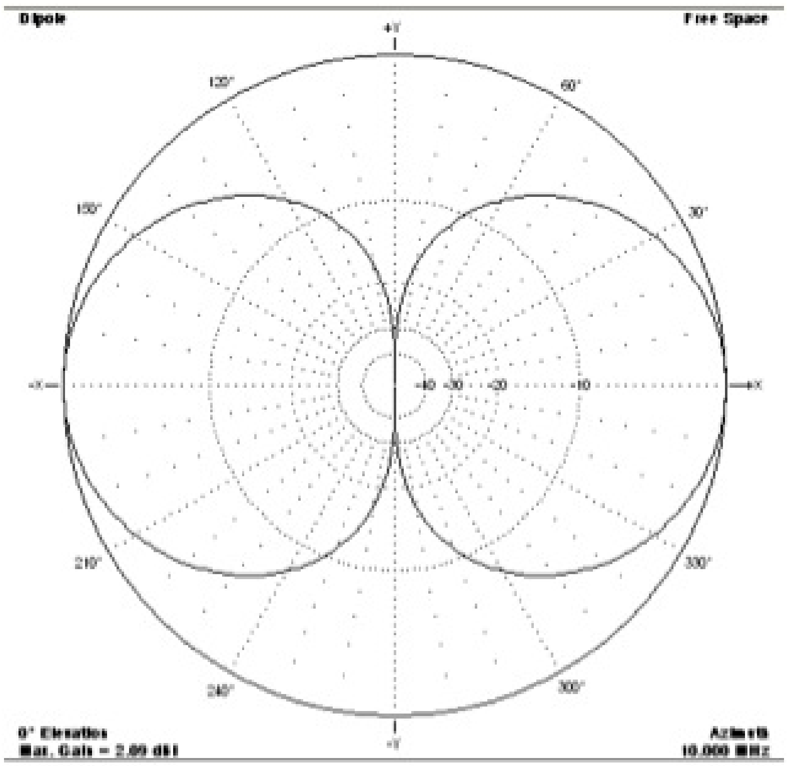
In the Real World...Antennae Everywhere

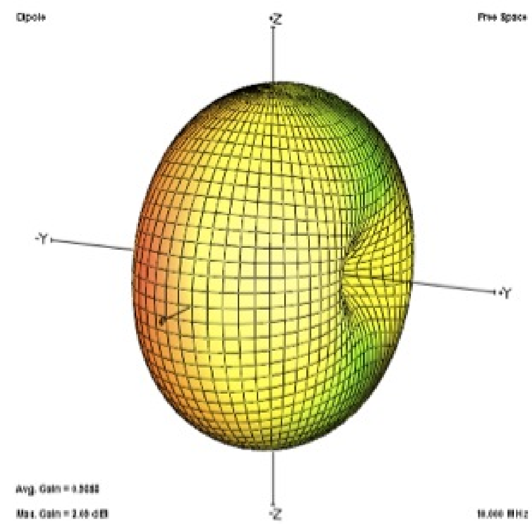




Radiation Patterns (Example: Dipole)

...are Complex, depending on the type of antenna.

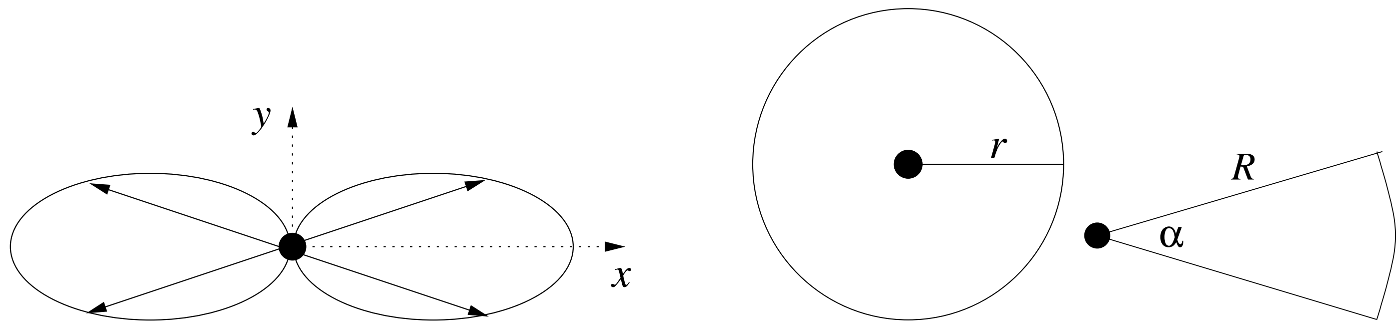




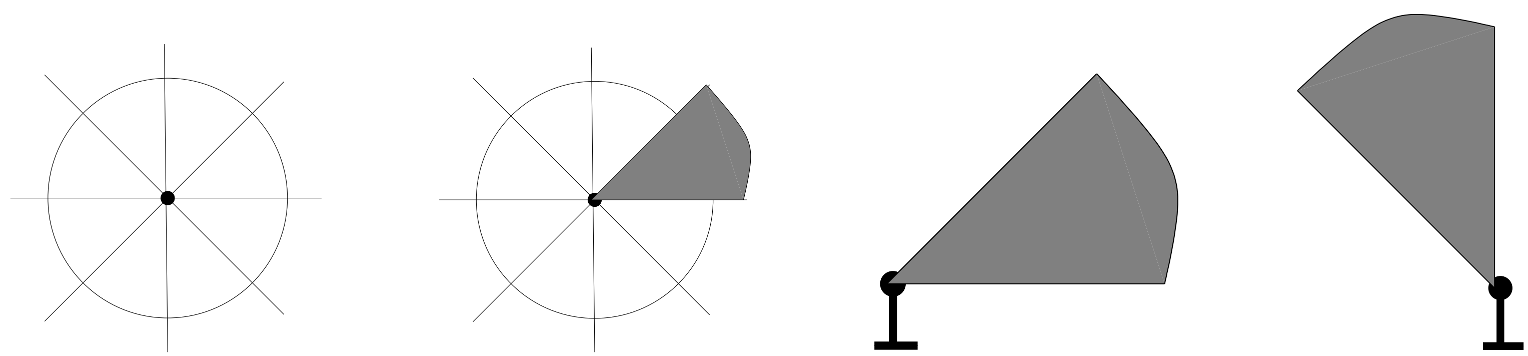
...there is extensive literature!

Idealized Antennae Models

- Lobes/Omnidirectional/Directional.



- Activating Sectors and/or Swiveling



Why Directional Antennae: Signal Protection

- Omnidirectional antennae transmit the signal everywhere in a 360 degree angle.
  - This makes it harder to protect the signal.
- Directional antennae restrict signal transmission within a bounded degree angle.
  - This makes it easier to prevent [a] attacks and detect malicious users. [b]
  - Employing authentication along a given direction along with localization can be beneficial.

[a] L. Hu, D. Evans, Using Directional Antennas to Prevent Wormhole Attacks, NDSS 2008.

[b] R. Maheshwari, J. Gao, Samir Das, Detecting wormhole attacks in wireless networks using connectivity information, INFOCOM 2007.

Why Directional Antennae: Capacity

- Consider a set of sensors that transmit W bits per second.
- For omnidirectional antennae, the network capacity[a] is

$$\sqrt{\frac{1}{2\pi}} W \sqrt{n}$$

- For directional antennae having transmission beam of width α and a receiving beam width of angle β the network capacity is [b]

$$\sqrt{\frac{2\pi}{\alpha\beta}} W \sqrt{n}$$

[a] Gupta and Kumar. The capacity of wireless networks. 2000.

[b] Yi, Pei and Kalyanaraman. On the capacity improvement of ad hoc wireless networks using directional antennas. 2003.

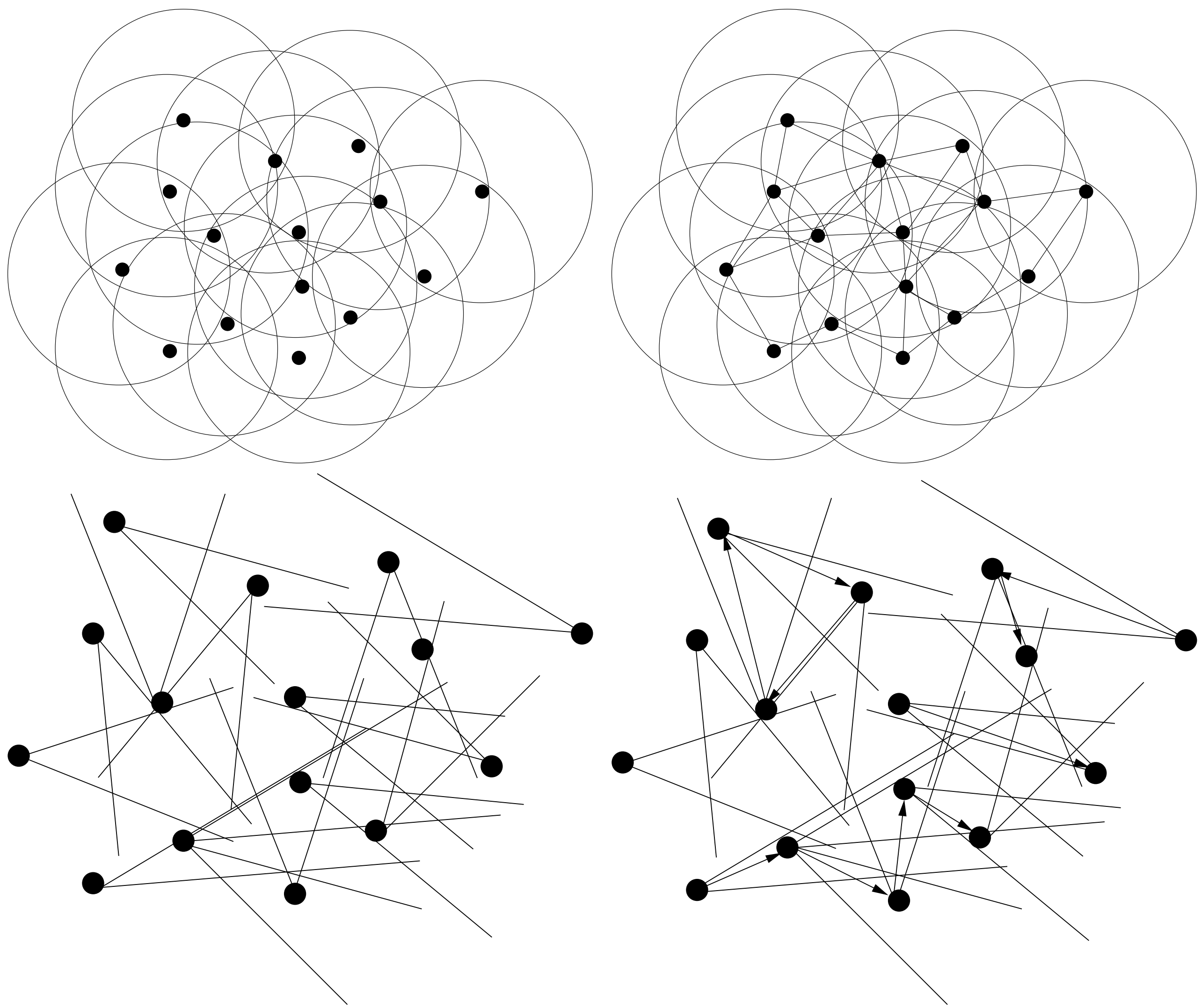
Why Directional Antennae: Energy Consumption

- **Directional** antennae with angle α and range R consume energy proportional to $\frac{\alpha}{2} \cdot R^2$. **Omnidirectional** $\alpha = 2\pi$.
- The smaller the angle the further you can reach: If energy is E
  - a **directional** antenna can reach distance $\sqrt{2E/\alpha}$ and an **ominidirectional** $\sqrt{E/\pi}$
- For a network of n **omnidirectional** sensors having radius r_i, for $i = 1, 2, \ldots, n$: total energy consumed is $\sum_{i=1}^{n} \pi \cdot r_i^2$.
- For a network of n **directional** sensors having angular spread α_i and range R_i, the total energy consumed is $\sum_{i=1}^{n} \frac{\alpha_i}{2} \cdot R_i^2$.
- For the same energy, the shorter the angle the bigger the range!
- Savings can be significant!

Directional Antennae: Main Issues

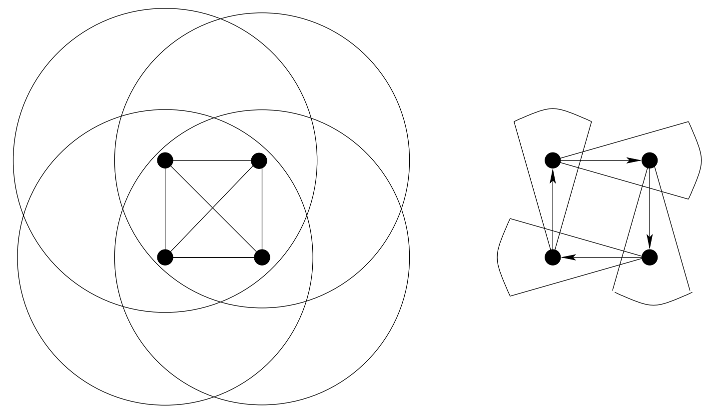
- Directional antennae seem to improve
  1. Security
  2. Capacity
  3. Energy Consumption
  4. ...and more!
- How do we attain good topology control?
  1. Connectivity
  2. Coverage
  3. Routing Stretch Factor
  4. ...and more!

Directional Antennae Affect Connectivity



Connectivity Problem and More

- Network topology changes!



- **Main Problem:**

  For a set of sensors located in the plane and a given angular spread provide algorithms that minimize the range required so that by an appropriate rotation of each of the antennae the resulting network becomes strongly connected.

- What are the angle/range/stretch-factor trade-offs?

Communication: $(Sender, Receiver)$ Model

- Communication must address the (T, L) access control issues:
  - How/when do sensors Talk and Listen?
  - Think of two phases: first you talk and then you listen.
- In this talk we use the (D, O) **Model**
  - Use Directional antenna to talk.
  - Use Omnidirectional antenna to listen.
  - In a way, this is how humans communicate!
- Other options: $(O, D), (D, D), (O, O)$ **Models**.

Comparison of Omnidirectional & Directional Antennae

	Omnidirectional	Directional
Energy	More	Less
Throughput	More	Less
Capacity	Less	More
Collisions	More	Less
Interference	More	Less
Connectivity	Stable	Intermittent
Discovery	Easy	Difficult
Coverage	Stable	Intermittent
Routing $SF^{(*)}$	Less	More
Security	Less	More

(*) SF = Stretch Factor

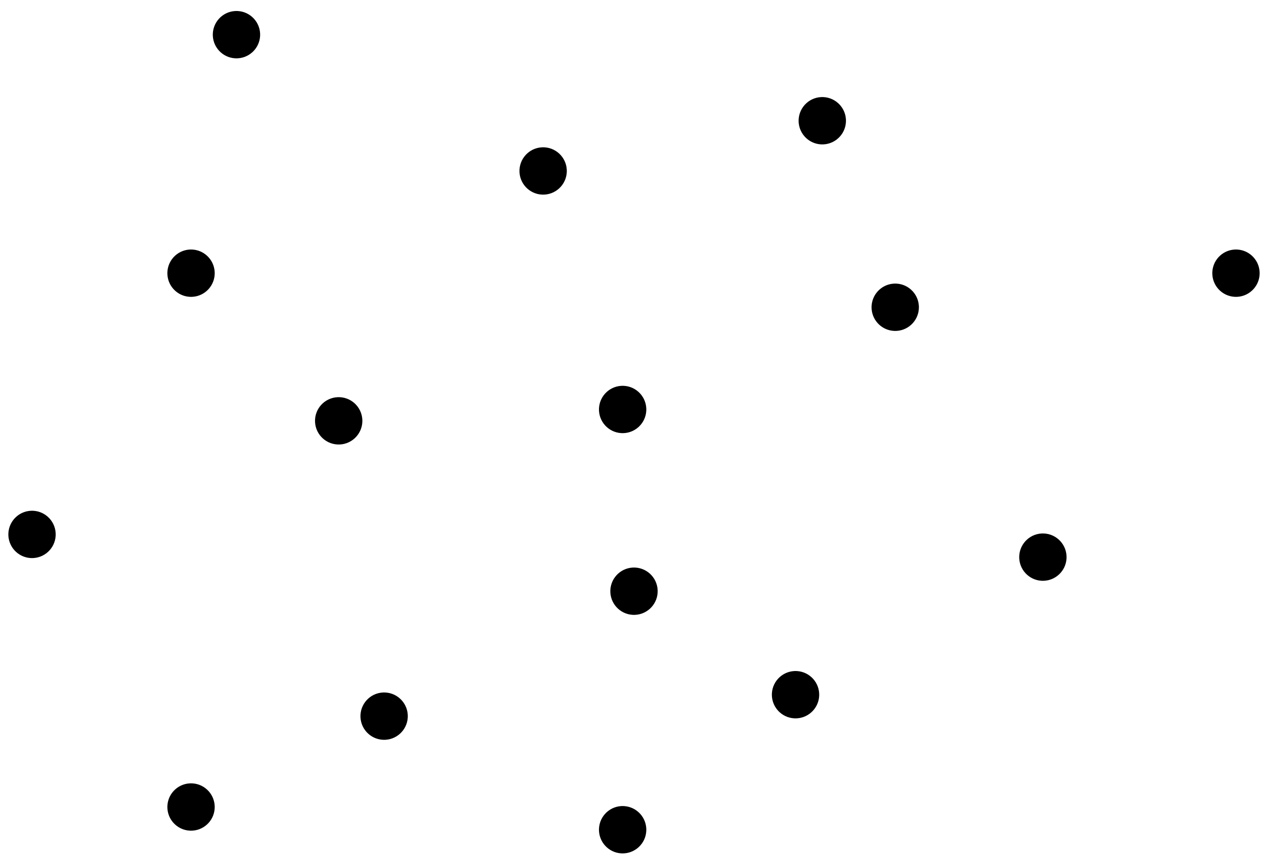
Goals

- Present algorithms for solvable cases of the problem.
- Understanding the limits and complexity of the problem.

Preliminaries

Setup: Set S of n Sensors in the Plane

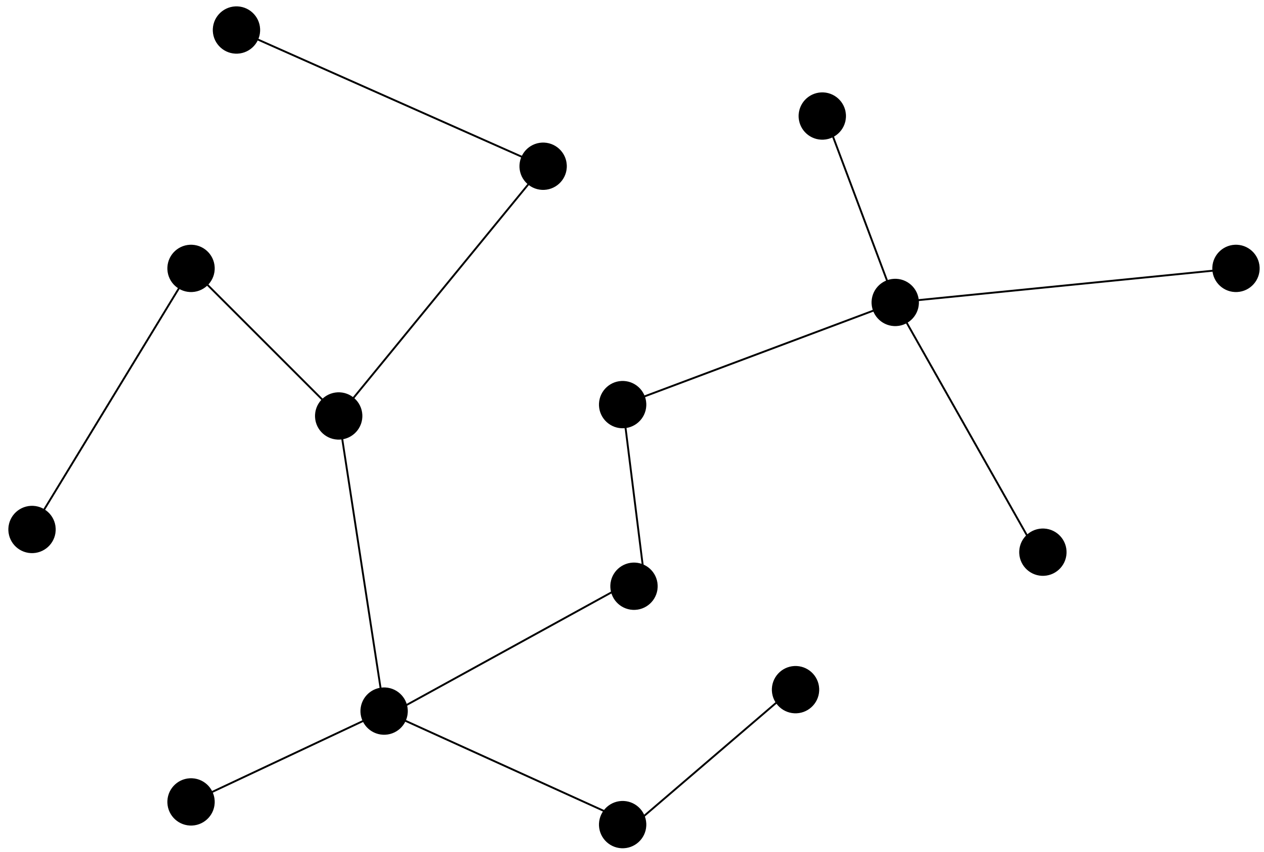
Consider a set S of n points in the plane.



Optimal Range for a Given Angle

- Given an angle φ.
- Directional antennae have identical range and angle φ.
- $r_k(S, \varphi)$ is the minimum range of directed antennae of angular spread at most φ so that if every sensor in S uses at most k such antennae (with adequate directioning) a strongly connected network on S results.
- When $\varphi = 0$ we use simpler notation $r_k(S)$ instead of $r_k(S, 0)$.
- $\mathcal{D}_k(S)$ is the set of all strongly connected graphs on S with out-degree at most k.

Importance of Minimum Spanning Tree on S (MST)



To attain connectivity, the sensors' range must exceed the longest edge of a MST.

Optimal Range and MST

- For any graph $G \in \mathcal{D}_k(S)$, let $r_k(G)$ be the maximum length of an edge in G.
- Let $MST(S)$ denote the set of all MSTs on S.
- For $T \in MST(S)$ let $r(T)$ denote the length of longest edge of T, and let $r_{MST}(S) = \min\{r(T) : T \in MST(S)\}$.
- For a set S of size n, it is easily seen that $r_{MST}(S)$ can be computed in $O(n^2)$ time.
- For any angle $\varphi \geq 0$, it is clear that

$$r_{MST}(S) \leq r_k(S, \varphi)$$

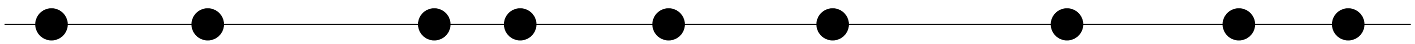
since every strongly connected, directed graph on S has an underlying spanning tree.

Single Antenna Problem:

Angle-Range Tradeoffs

1D: In a Line (Highway Model) (1/2)

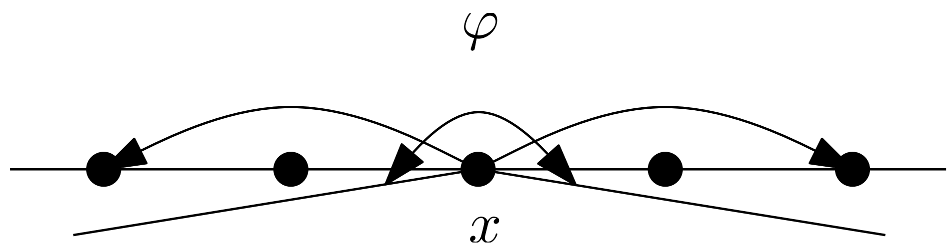
- Assume sensors are arranged on a line



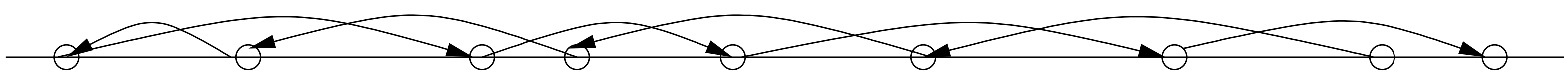
- **Theorem 1** *Consider a set of* $n > 2$ *points* $i = 1, 2, \ldots, n$ *sorted according to their location on the line. For any* $\phi \geq 0$ *and* $r > 0$*, there exists an orientation of sectors of angle* ϕ *and radius* r *at the points so that the transmission graph is strongly connected if and only if the distance between points* i *and* $i + 2$ *is at most* r*, for any* $i = 1, 2, \ldots, n - 2$*.*

1D: In a Line (Highway Model) (2/2)

- If the angle $\varphi \geq \pi$ then antennae behave like omnidirectional if properly oriented



- If the angle $\varphi < \pi$ then alternate antennae directions:



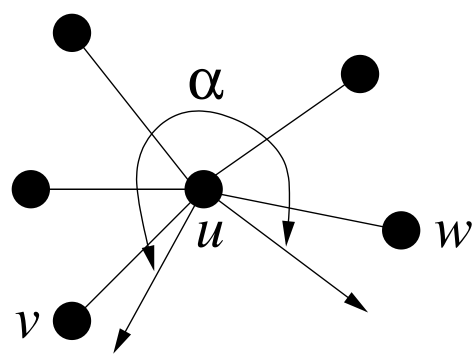
- Must be careful with antenna orientations, but same idea will work for a path that is not necessarily a straight-line.

2D: In the Plane

- **Theorem 2** *Given $\phi \geq 8\pi/5$, $r > 0$ and a set of points on the plane, an orientation of sectors of angle ϕ and radius r so that the transmission graph is strongly connected can be computed (if it exists) in polynomial time.*
- Given a set S of points on the plane and $r > 0$, consider the **proximity** graph $G_r(S)$ containing a node for each point of S and an edge for each pair of nodes if the distance of the corresponding points is at most r.

On the Plane: Proximity Graph

- **If the proximity graph is not connected:** clearly no orientation of the sectors that defines a strongly connected transmission graph can be found.
- **If the proximity graph is connected:** consider a MST.
- Since the edge costs are Euclidean, each node on this spanning tree has degree at most 5.
- For each node u, there are two consecutive neighbors v, w in the spanning tree so that the angle $\angle(vuw)$ is at least $2\pi/5$.



2D: Approximating the Range

- **Theorem 3 (Caragiannis et al., 2008)** *Given an angle* ϕ *with* $\pi \leq \phi < 8\pi/5$ *and a set of points in the plane, there exists a polynomial time algorithm that computes an orientation of sectors of angle* ϕ *and radius*

$$2\sin\left(\frac{\phi}{2}\right)\cdot r_1(S,\phi)$$

so that the transmission graph is strongly connected. [a]

[a] *Caragiannis, Kaklamanis, Kranakis, Krizanc and Wiese. Communication in Wireless Networks with Directional Antennae. 2008*

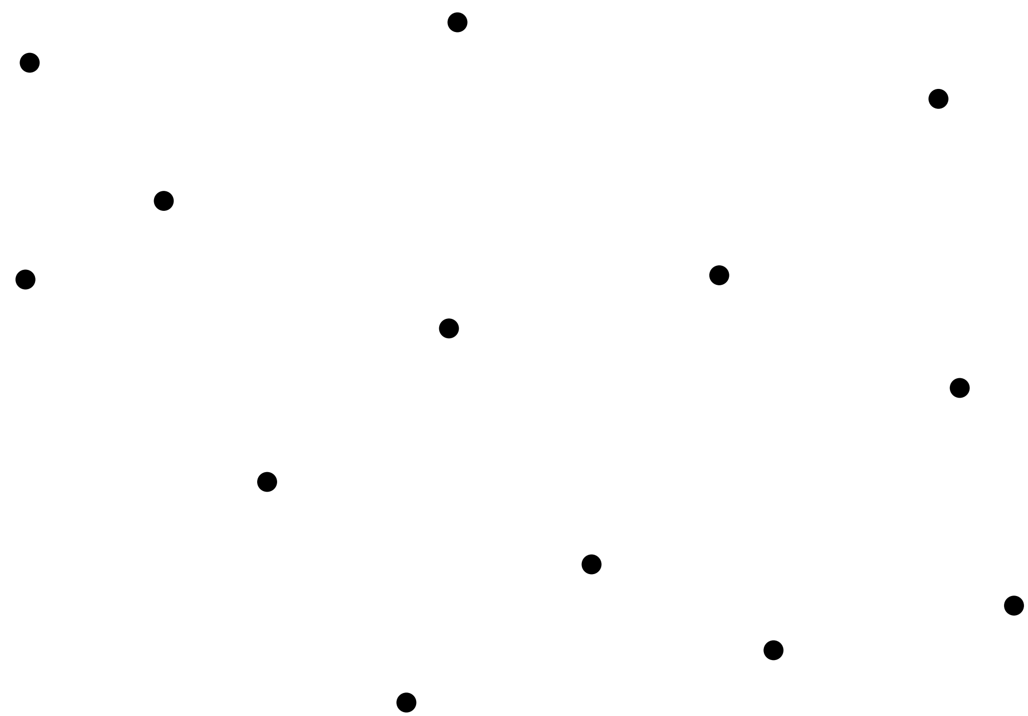
2D: Algorithm for Approximating the Range

Take an MST and construct a matching M such that any non-leaf node of T is adjacent to an edge of M.

1. Initially, M is empty.
2. We root T at an arbitrary node s.
3. We pick an edge between s and one of its children and insert it in M.
4. Then, we visit the remaining nodes of T in a BFS manner.
5. When visiting a node u,
   (a) if u is either a leaf-node or a non-leaf node such that the edge between it and its parent is in M, we do nothing;
   (b) otherwise, we pick an edge between u and one of its children and insert it to M.

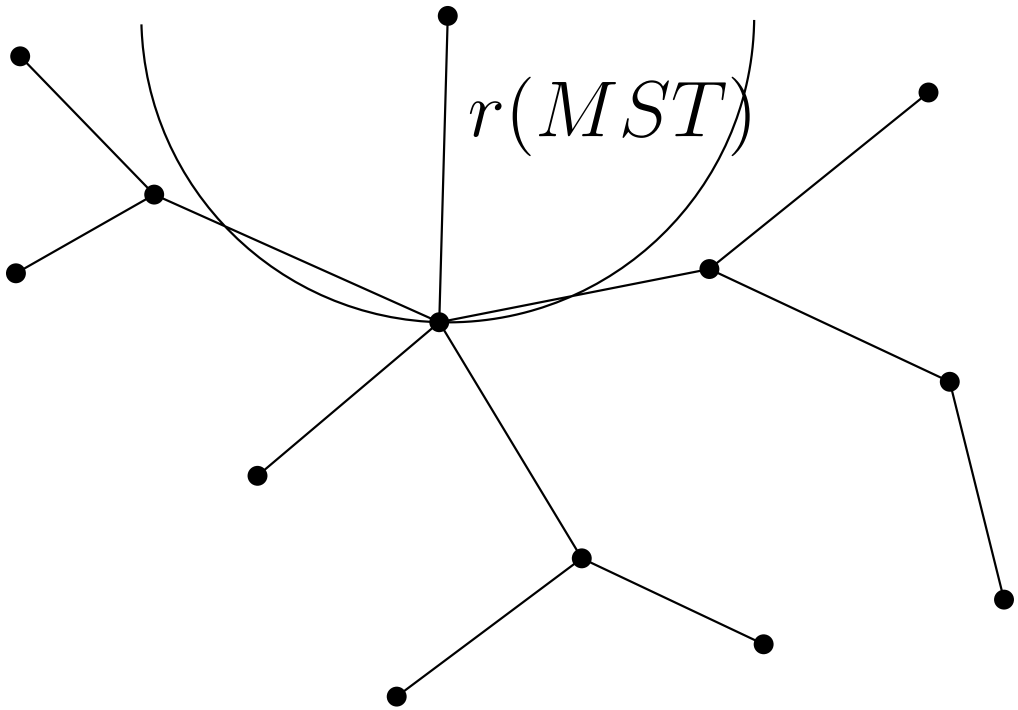
Antenna Orientation With Approximation Range

- Start with a set of n points in the plane:



Proof: MST (1/10)

Consider a Minimum Spanning Tree on the Set of Points.

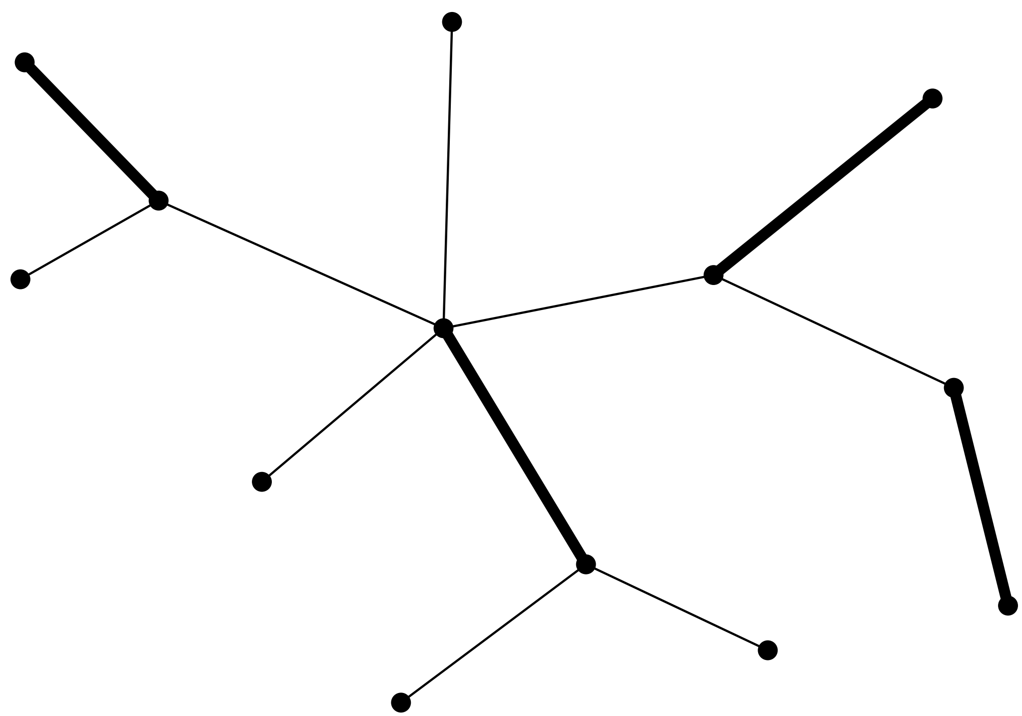


Proof: Range (2/10)

- Let $r^*(\varphi)$ be the optimal range when the angle of the antennae is at most φ.
- Let $r(MST)$ be the longest edge of the MST on the set of points.
- Observe that for $\varphi \geq 0$,
  - $r^*(\varphi) \geq r(MST)$.

Proof: Edge Selection (3/10)

Find a maximal matching such that each internal vertex is in the matching. This can be done by traversing T in BFS order.



Proof: Algorithm (4/10)

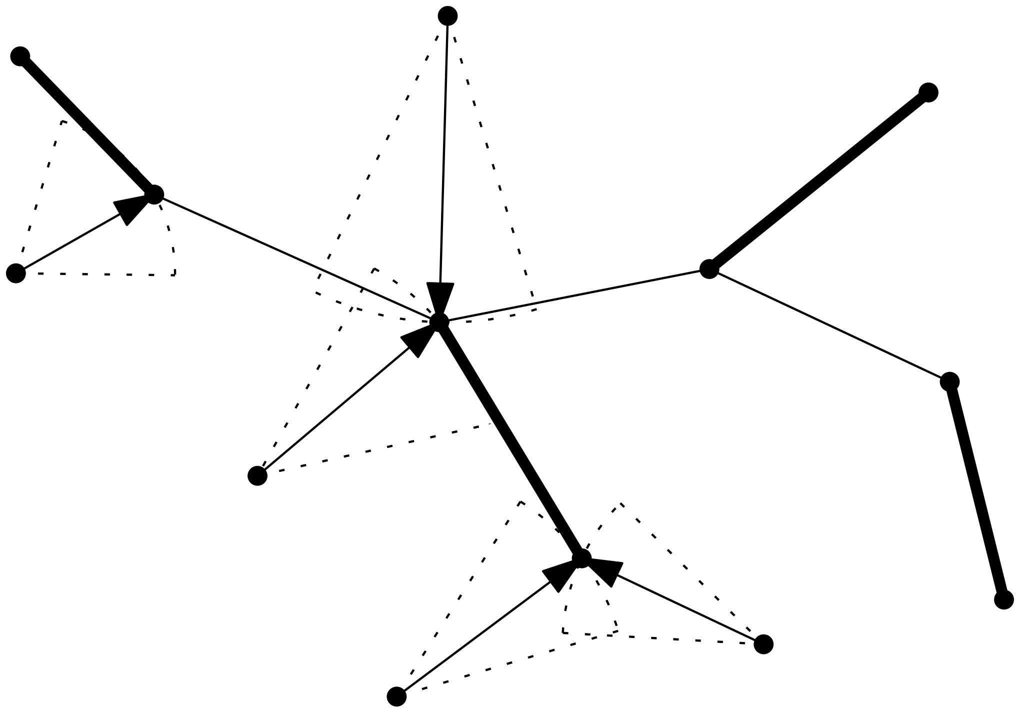
- A greedy algorithm works as follows:
  1. Label all the vertices with *unused* and an arbitrary leaf with *used*.
  2. While there exist *unused* vertices do

     2.1 Add $\{u, v\}$ to the matching only if u is *used* and v is *unused*.

     2.2 Set *used* to v and each neighbor of v.
- The algorithm returns a valid Matching.
- A leaf is marked as used if its neighbor is added to the matching.

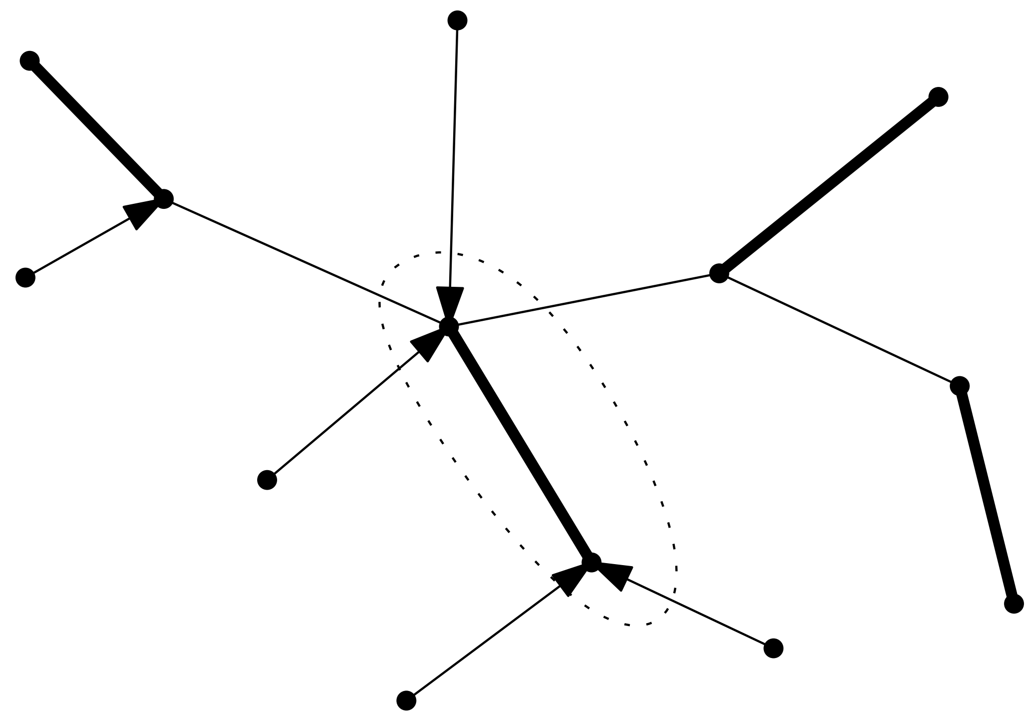
Proof: Antennae Orientation for Leaves (5/10)

Orient unmatched leaves to their immediate neighbors.



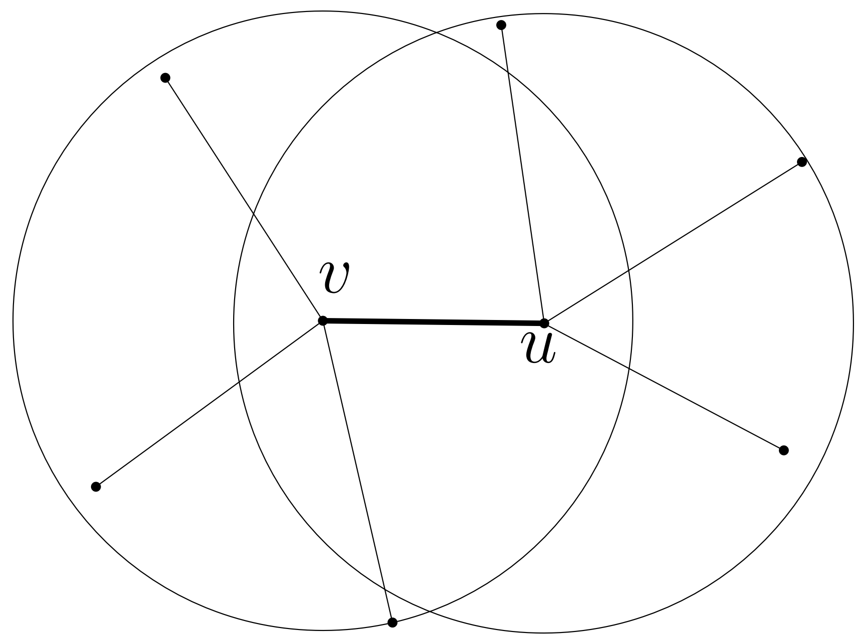
Proof: Connecting Matched Vertices (6/10)

Consider a pair of matched vertices



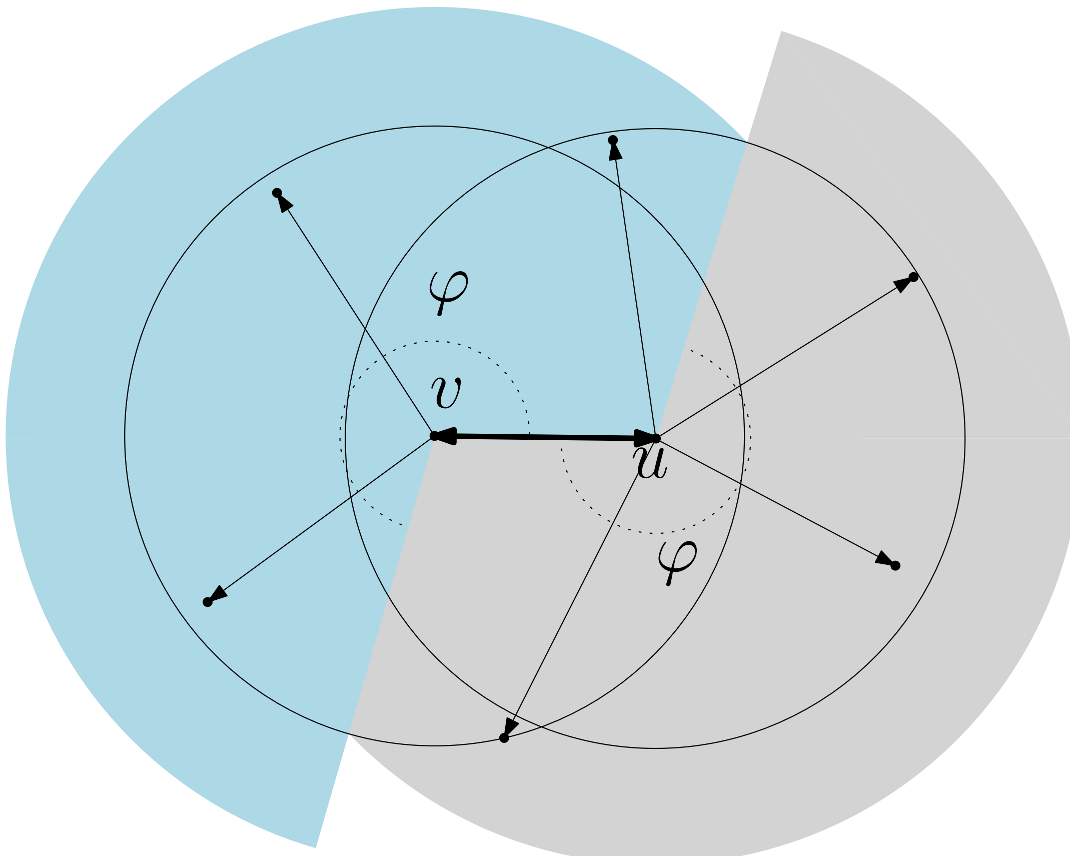
Proof: Sufficient Coverage (7/10)

Let $\{u, v\}$ be an edge in the matching. Consider the smallest disks of same radius centered at u and v that contain all the neighbors of u and v in the MST.



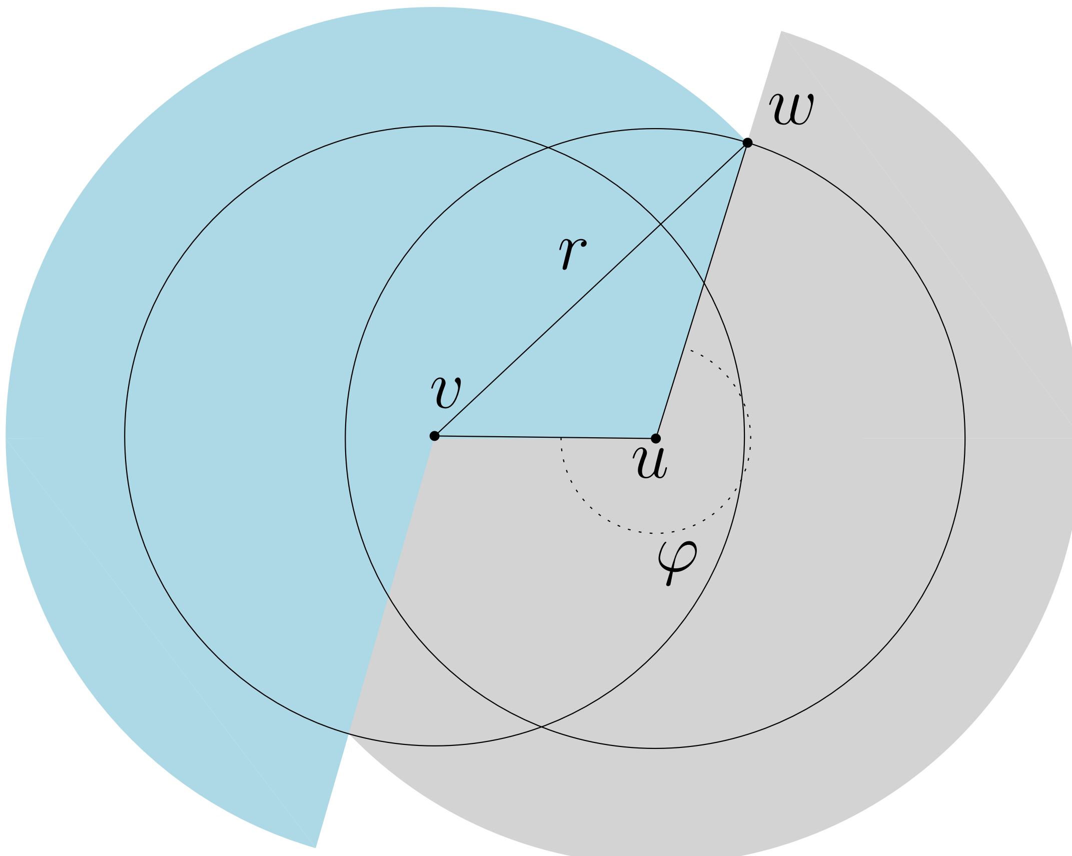
Proof: Orienting Antennae at Matched Vertices (8/10)

Orient the directional antennae at u and v with angle φ in such a way that both disks are covered.



Proof: Sufficient Antenna Range (9/10)

To calculate the smallest radius necessary to cover both disks, consider the triangle uvw.



Proof: Approximation of Antenna Range (10/10)

From the law of cosines we can determine r.

$$
\begin{aligned}
r &= \sqrt{|uv|^2 + |uw|^2 - 2|uv||uw|\cos(2\pi - \varphi)} \\
&\leq \sqrt{2 - 2\cos(2\pi - \varphi)} \\
&\leq 2\sin(\tfrac{2\pi-\varphi}{2}) \\
&\leq 2\sin(\pi - \varphi/2) \\
&= 2\sin(\varphi/2)
\end{aligned}
$$

Lower Bounds

- **Theorem 4 (Caragiannis et al.)** *Deciding whether there exists an orientation of one antenna at each sensor with angle less that $2\pi/3$ and optimal range is NP-Complete. The problem remains NP-complete even for the approximation range less than $\sqrt{3}$ times the optimal range.* [a]

[a] *Caragiannis, Kaklamanis,Kranakis, Krizanc and Wiese. Communication in Wireless Networks with Directional Antennae. 2008*

2D: One Anenna (Summary)

- For $\pi \leq \phi < 8\pi/5$, can we improve the approximation factor

$$2\sin\left(\frac{\phi}{2}\right)?$$

- Table of values

Angle	Approximation
$\pi \leq \phi \leq 2\pi$	$2\sin\left(\frac{\phi}{2}\right)$
$\phi = 0$	2
$\phi = 8\pi/5$	1.175...

- How about values $\phi < \pi$?

Multiple Antennae

The Multiple Antennae Problem

- **Recall:** $r_k(S, \varphi)$ is the minimum range of directed antennae of angular spread at most φ so that if every sensor in S uses at most k such antennae (with adequate directioning) a strongly connected network on S results.

- We are interested in the problem of providing an algorithm for orienting the antennae and ultimately for estimating the value of $r_k(S, \varphi)$.

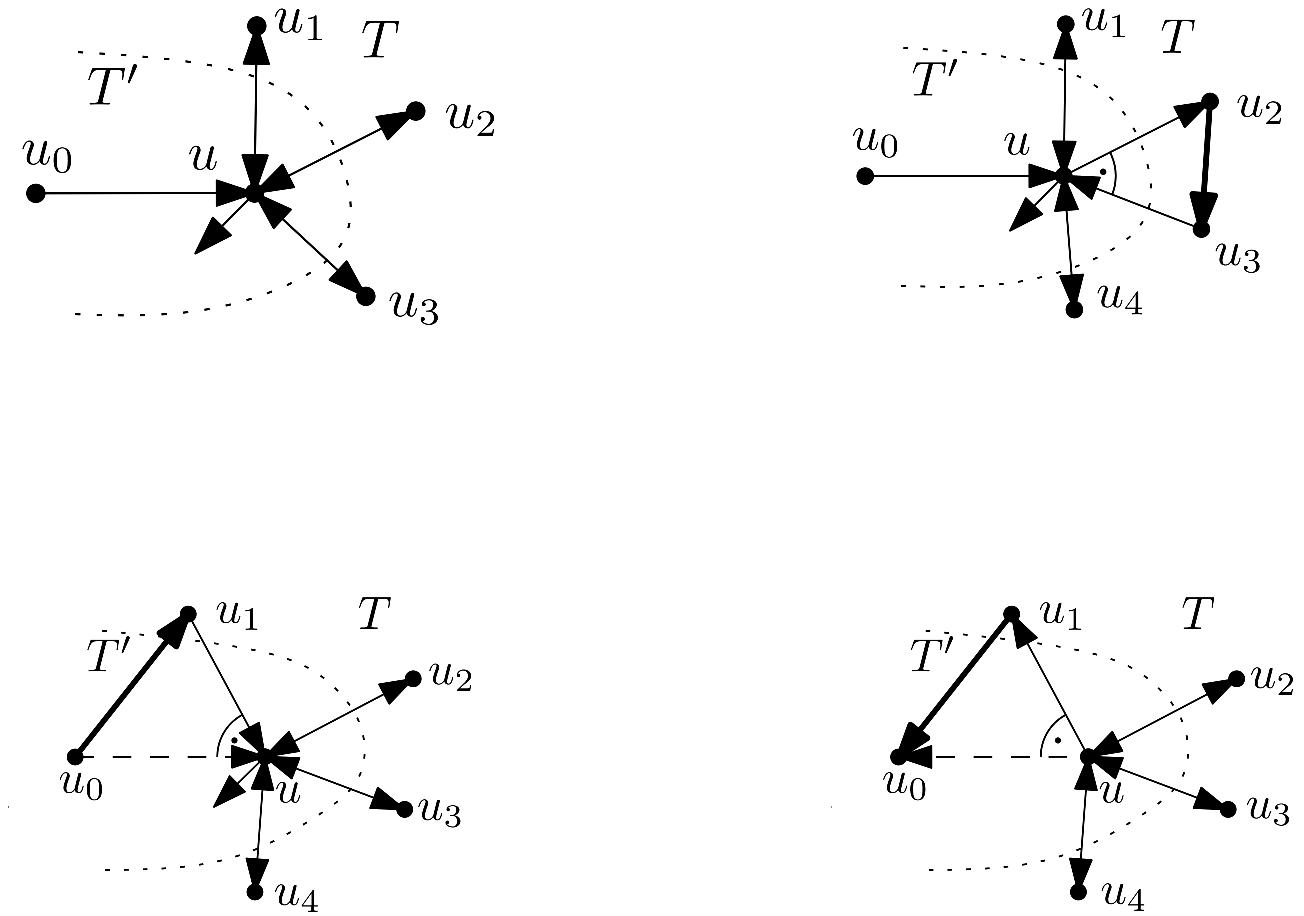
- Without loss of generality antennae ranges will be normalized, i.e., $r_{MST}(S) = 1$.

Main Theorem (Upper Bound)

- **Theorem 5 (Dobrev et al 2010)** *Consider a set S of n sensors in the plane and suppose each sensor has k, $1 \leq k \leq 5$, directional antennae.*[a]
  - *Then the antennae can be oriented at each sensor so that the resulting spanning graph is strongly connected and the range of each antenna is at most $2 \cdot \sin\left(\frac{\pi}{k+1}\right)$ times the optimal.*
  - *Moreover, given a MST on the set of points the spanner can be constructed with additional $O(n)$ overhead.*

[a] S. Dobrev, E. Kranakis, D. Krizanc, O. Morales Ponce, J. Opatrny, L. Stacho. Strong Connectivity in Sensor Networks with Given Number of Directional Antennae of Bounded Angle, DMAA 2012.

Inductive Step: 4 antennae, angle 0



Lower Bound

- **Theorem 6 (Dobrev et al 2010)** *For $k = 2$ antennae, if the angular sum of the antennae is less then α then it is NP-hard to approximate the optimal radius to within a factor of x, where x and α are the solutions of equations* $x = 2\sin(\alpha) = 1 + 2\cos(2\alpha)$. [a]
- Using the identity $\cos(2\alpha) = 1 - 2\sin^2\alpha$ and solving the resulting quadratic equation with unknown $\sin\alpha$ we obtain numerical solutions $x \approx 1.30, \alpha \approx 0.45\pi$.

[a] S. Dobrev, E. Kranakis, D. Krizanc, O. Morales Ponce, J. Opatrny, L. Stacho. Strong Connectivity in Sensor Networks with Given Number of Directional Antennae of Bounded Angle.

New Ideas

Useful Ideas

- **Connected Components:**
  - When replacing an omnidirectional with a directional antennae you create connected components. Can you limit their number?
  - MST is a tool for accomplishing this, but it is not necessarily optimal.
  - **Idea:** Use toughness, robustness!
- **Range of directional antenna:**
  - To what extent can you bound it?
  - **Idea:** Use Bottleneck Travelling Salesman!

Toughness and Robustness

- **Graph toughness:**
  - A graph is t-**tough** if for each subset $S \subseteq V$ the number of connected components obtained from $G \setminus S$ is at most $|S|/t$.
  - A graph is t-**weakly tough** if for each vertex v, the number of connected components obtained from $G \setminus \{v\}$ is at most t.
- **Robustness of UDGs:**
  - The t-**robustness** $\sigma_t(P)$ is the infimum taken over all radii $r > 0$ such that $UDG(P, r)$ is t-tough.
  - The t-**weak robustness** $\alpha_t(P)$ is the infimum taken over all radii $r > 0$ such that $UDG(P, r)$ is t-weakly tough.

Toughness and Robustness in UDGs

- How do we compute strong robustness? It may be NPC!
- Weak robustness can be computed in polynomial time.
  - Indeed in $O(n \log n)$ time!
- Every connected UDG is 1/5-strong tough.
- **Key Observation** of Strong versus Weak:
  - Can prove that $\sigma_{1/4} = \alpha_{1/4}$.
  - Unknown whether or not $\sigma_{1/3} = \alpha_{1/3}$.

Optimality for Single Antennae

- $\varphi = 0$: equivalent to finding a Hamiltonian cycle minimizing the max length of an edge (Bottleneck Travelling Salesman). Parker, Rardin. (1984) proved that no polynomial time $(2-\epsilon)$-approx algorithm is possible unless $P = NP$
- Results from [a]

Antenna Angle	Approximation ratio	Complexity
$\frac{4\pi}{3} \leq \phi$	1	$O(n^2)$
$\pi \leq \phi < \frac{4\pi}{3}$	$2\sin(5\pi/6 - \phi/2)$	$O(n^2)$
$\frac{2\pi}{3} \leq \phi < \pi$	$2\cos(\phi/2) + 2$	$O(n \log n)$
$\frac{\pi}{3} < \phi \leq \frac{2\pi}{3}$	$\min(3, 4\sin(\phi/2))$	$O(n^2)$
$\phi \leq \frac{\pi}{3}$	2	$O(n^2)$

[a] Optimality in Directional Antennae to Attain Strong Connectivity, E. Kranakis, F. MacQuaries, O. Morales-Ponce, 2012, to appear.

Optimality for Multiple Antennae

Results from [a] and [b]

Out-Dg	LBound	UBound	Approx.	Complexity
4	r_{MST}	$2\sin(\pi/5)r_{MST}$	$2\sin(\pi/5)$	$O(n\log n)$
3	r_{MST}	$2\sin(\pi/4)r_{MST}$	$\sqrt{2}$	$O(n\log n)$
2	r_{MST}	$2\sin(\pi/3)r_{MST}$	$\sqrt{3}$	$O(n\log n)$
2	-	-	≤ 1.3	NPC
4	$\sigma_{1/4}$	$\alpha_{1/4}$	1	$O(n\log n)$
3	$\sigma_{1/3}$	$2\sin(2\pi/9)\alpha_{1/3}$	$\leq 2\sin(2\pi/9)$	$O(n\log n)$

[a] B. Bhattacharya, Y. Hu, E. Kranakis, D. Krizanc, Q. Shi. Sensor Network Connectivity with Multiple Directional Antennae of a Given Angular Sum. IPDPS 2009

[b] S. Dobrev, E. Kranakis, O. Morales Ponce, M. Plzik. Robust Sensor Range for Constructing Strongly Connected Spanning Digraphs in UDGs, CSR 2012

Stretch Factor for a Single Antenna

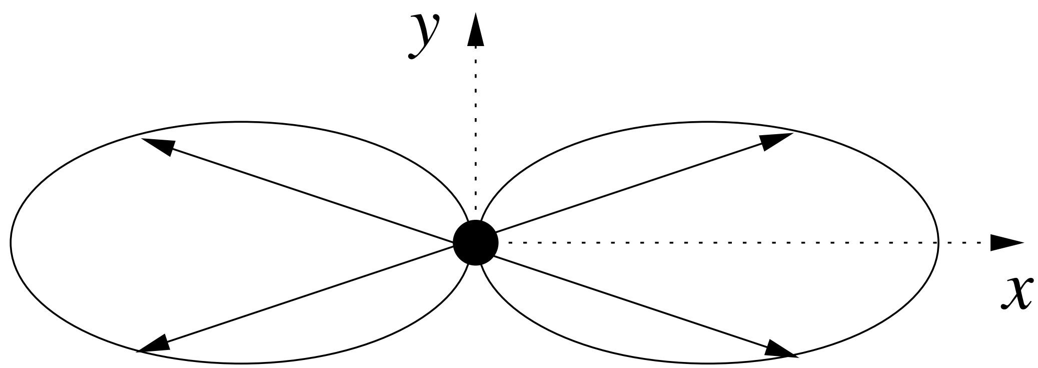
- Results from [a]

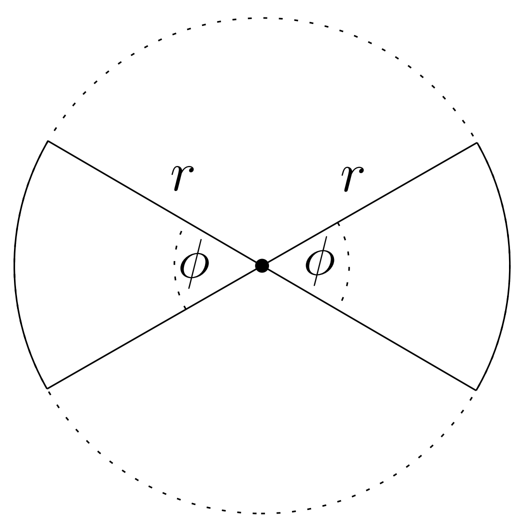
Beam Width	Approximation Ratio of r_s	Stretch Factor	Scope
$\frac{5\pi}{3} \leq \phi < 2\pi$	1	2	Global
$\pi \leq \phi < \frac{5\pi}{3}$	$2\sin(\frac{\phi}{2})$		
$\frac{\pi}{2} \leq \phi < \pi$	$4\cos(\frac{\phi}{2})+3$	3	Local
$\phi = \frac{\pi}{3}$	$36\sqrt{2}$	10	Global
$\phi < \frac{\pi}{3}$	$4\sqrt{2}(\frac{7\pi}{\phi}-6)$	$\lceil 8\log(\frac{2\pi}{\phi})\rceil - 1$	Global

[a] E. Kranakis, F. MacQuarie, O. Morales Ponce. Spanning Trade-offs in Wireless Sensor Networks with Directional Antennae. COCOA 2012.

Double Antennae

- Idealized Models





Double Antennae

- Results from[a]

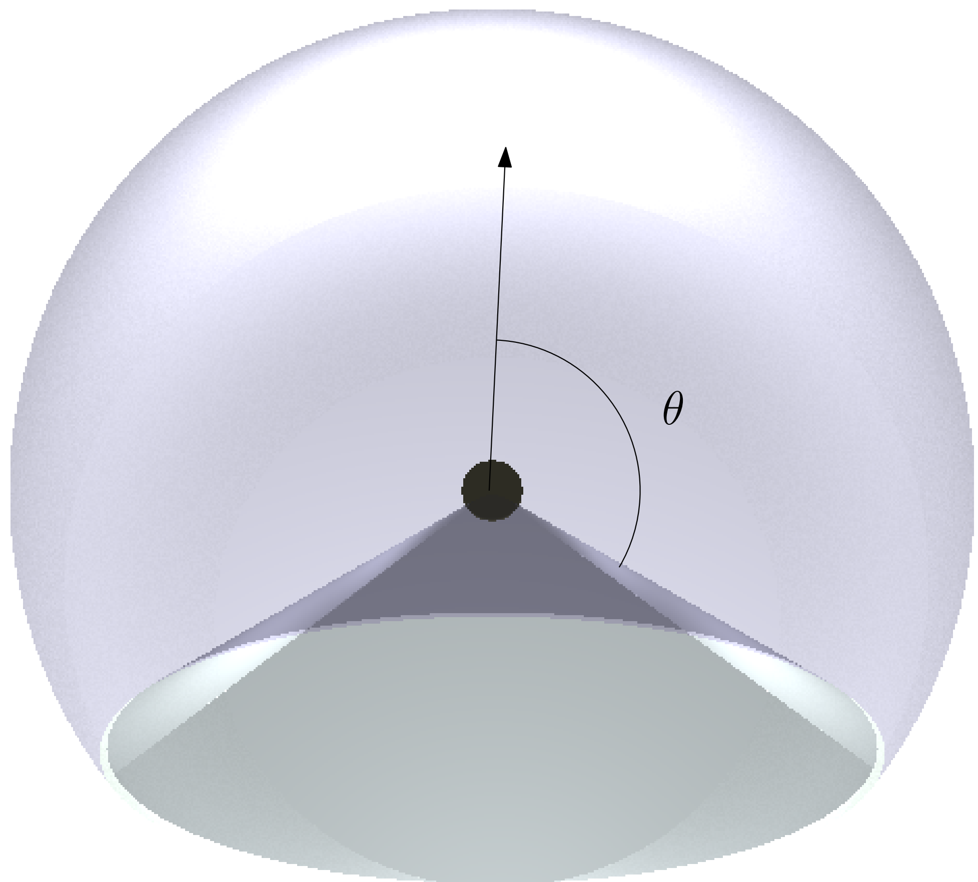
DA Angle	Approx Ratio	Complexity	Stretch Factor
$\frac{2\pi}{3} \leq \phi < \pi$	1	$O(n^2)$	-
$\frac{\pi}{2} \leq \phi \leq \frac{2\pi}{3}$	$\sqrt{3}$	$O(n \log n)$	-
$\frac{\pi}{2} \leq \phi < \pi$	$4 \sin(\frac{\pi}{4} + \frac{\phi}{2})$	$O(n)$	4
$0 \leq \phi < \frac{\pi}{2}$	3	$O(n \log n)$	-
$\phi < \frac{\pi}{3} - \epsilon$	$\sqrt{3} - \epsilon$	NP-Complete	-

Table 1: Double antenna connectivity and stretch-factor trade-offs on n sensors in the plane and for antennae of angle ϕ

[a] M. Eftekhari Hesari, E. Kranakis, O. Morales-Ponce, F. MacQuarrie, L. Narayanan. Strong Connectivity of Sensor Networks with Double Antennae., SIROCCO 2012

Antennae in 3D

- Idealized Model



Antennae in 3D

- Results from[a]

Theorem 7 *Given a set S of n points in 3D and a solid angle Ω such that $2\pi \leq \Omega < \frac{18\pi}{5}$, it is possible to orient the antennae at each sensor with solid angle Ω and range $r(\Omega)$ in $O((n \log(n))^{4/3})$ time so that the transmission graph is connected, where*

$$r(\Omega) = \frac{\sqrt{\Omega(4\pi - \Omega)}}{\pi} \cdot r_{MST}(S)$$

[a] E. Kranakis, D. Krizanc, A. Modi, O. Morales Ponce. Connectivity Trade-offs in 3D Wireless Sensor Networks Using Directional Antennae. In proceedings of IPDPS 2011

Open Problems

Open Problems

- **Single/Multiple Antenna Tradeoffs:**
  - Antennae models with realistic transceiver patterns.
  - k-Connectivity, Stretch factor, Angle, Range tradeoffs.
  - More realistic metrics (3D, Metric Spaces, Locality).
  - Energy consumption and capacity: angle/range tradeoffs.
- **Communication:**
  - Antenna Interference: SNIR (Signal Noise to Interference Ratio) in the context of directional antennae.
  - Broadcasting, gossiping, etc, in the (D, O) Model and its relation to the (O, O) Model.
- **Dynamic Models:**
  - Changing direction of antennae during the communication.

References

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- E. Kranakis, D. Krizanc, O. Morales. Maintaining Connectivity in Sensor Networks Using Directional Antennae. In Theoretical Aspects of Distr. Comp. in Sensor Networks, S. Nikoletseas and J. Rolim, editors, Springer, 2010.