What is in This Chapter?

This chapter explains how data is represented in memory. It begins with an explanation of how bits are stored using various bit models (i.e., magnitude only, sign magnitude, two’s compliment, fixed point, floating point, ASCII and UNICODE). It then discusses bit operators that help us understand how to manipulate memory data at the bit level which will allow us to make efficient use of storage space. Compound data types are introduced, such as Strings and Arrays (single and multi-dimensional). The chapter concludes with a discussion of custom type definitions such as structs and unions.
All data stored in a computer must somehow be represented numerically in some way whether it is numerical to begin with, a series of characters or an image. Ultimately, everything digitally breaks down to ones and zeros. We need to understand how data is stored (or represented) in the computer so that we interpret the 1’s and 0’s correctly. For example, consider this sequence of ones and zeros:

0100011 0100111 0101011

The data can be interpreted in different ways:

- Three unique integers: 67, 79, 87
- One large number: 4,411,223
- A word: COW
- A color (RGB):

The “correct” way to interpret the data depends on the context in which it is used.

Numerical values that we use normally every day are in base 10 ... the decimal system. In this system, a sequence of digits such as 62389 is easily understood to be "sixty-two thousand three hundred and eighty-nine". We know this because we perform the following operation in our head:

\[(9 \times 1) + (8 \times 10) + (3 \times 100) + (2 \times 1,000) + (6 \times 10,000)\]

Which is the same as doing this:

\[(9 \times 10^0) + (8 \times 10^1) + (3 \times 10^2) + (2 \times 10^3) + (6 \times 10^4)\]

When dealing with computers, we often use other number systems as well such as Hexadecimal, Octal and Binary. Here is a comparison of these number systems:

<table>
<thead>
<tr>
<th>Number System</th>
<th>Base</th>
<th>Digits/Characters Used</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary</td>
<td>2</td>
<td>0, 1</td>
<td>1001011</td>
</tr>
<tr>
<td>Octal</td>
<td>8</td>
<td>0, 1, 2, 3, 4, 5, 6, 7</td>
<td>113</td>
</tr>
<tr>
<td>Decimal</td>
<td>10</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9</td>
<td>75</td>
</tr>
<tr>
<td>Hexadecimal</td>
<td>16</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F</td>
<td>4B</td>
</tr>
</tbody>
</table>

Given a number in any of the non-decimal system formats, we can determine the value in the same manner as with our decimal number system, using the base as the multiplication factor.
Consider how to compute the value of the number $1001101_2$ in the various bases:

<table>
<thead>
<tr>
<th>Base</th>
<th>Calculation</th>
<th>Decimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$(1 \times 2^5) + (0 \times 2^4) + (1 \times 2^2) + (1 \times 2^3) + (0 \times 2^4) + (0 \times 2^5) + (1 \times 2^6)$</td>
<td>77</td>
</tr>
<tr>
<td>8</td>
<td>$(1 \times 8^8) + (0 \times 8^7) + (1 \times 8^2) + (1 \times 8^3) + (0 \times 8^4) + (0 \times 8^5) + (1 \times 8^6)$</td>
<td>262,721</td>
</tr>
<tr>
<td>10</td>
<td>$(1 \times 10^6) + (0 \times 10^5) + (1 \times 10^2) + (1 \times 10^3) + (0 \times 10^4) + (0 \times 10^5) + (1 \times 10^6)$</td>
<td>1,001,101</td>
</tr>
<tr>
<td>16</td>
<td>$(1 \times 16^6) + (0 \times 16^5) + (1 \times 16^2) + (1 \times 16^3) + (0 \times 16^4) + (0 \times 16^5) + (1 \times 16^6)$</td>
<td>16,781,569</td>
</tr>
</tbody>
</table>

Hopefully you can remember how to convert from one base to another base. If not, here is how to convert from **Binary** to **Decimal**, **Octal** and **Hex**:

### Binary to Decimal

<table>
<thead>
<tr>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$4 + 2 + 1 = 7_{10}$

<table>
<thead>
<tr>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$16 + 0 + 1 = 17_{10}$

<table>
<thead>
<tr>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$2 + 4 + 1 = 7_{10}$

### Binary to Octal

<table>
<thead>
<tr>
<th>2</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$4 + 2 + 1 = 7_8$

### Binary to Hexadecimal

<table>
<thead>
<tr>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$4 + 2 + 1 = 7_{16}$

### Octal to Decimal

<table>
<thead>
<tr>
<th>64</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

$(1 \times 64) + (2 \times 8) + (5 \times 1) = 85_{10}$

<table>
<thead>
<tr>
<th>64</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

$(3 \times 64) + (7 \times 8) + (2 \times 1) = 250_{10}$

<table>
<thead>
<tr>
<th>64</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

$(2 \times 64) + (0 \times 8) + (1 \times 1) = 129_{10}$

### Hexadecimal to Decimal

<table>
<thead>
<tr>
<th>4096</th>
<th>256</th>
<th>16</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>A</td>
<td>F</td>
</tr>
</tbody>
</table>

$(0 \times 4096) + (2 \times 256) + (10 \times 16) + (15 \times 1) = 687_{10}$

<table>
<thead>
<tr>
<th>4096</th>
<th>256</th>
<th>16</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>3</td>
<td>C</td>
</tr>
</tbody>
</table>

$(1 \times 4096) + (7 \times 256) + (3 \times 16) + (12 \times 1) = 5,948_{10}$

<table>
<thead>
<tr>
<th>4096</th>
<th>256</th>
<th>16</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>B</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

$(2 \times 4096) + (11 \times 256) + (0 \times 16) + (5 \times 1) = 11,013_{10}$

Here is how to convert from **Octal** and **Hex** to **Decimal**:

Here is how to convert from **Decimal** to **Binary** and **Hex**:
Decimal to Binary
(divide by 2 and get remainder)

<table>
<thead>
<tr>
<th>Dec</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>117</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>58</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>29</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Decimal to Hex
(divide by 16 and get remainder)

<table>
<thead>
<tr>
<th>Dec</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Here is a C example which shows how to specify literal values for decimal, octal, hex and binary numbers:

Code from bases.c

```c
#include <stdio.h>

int main() {
    unsigned char dec  = 27;  // = (2*10) + 7 = 27
    unsigned char oct  = 027; // = (2*8) + 7 = 23
    unsigned char hex  = 0xbf; // = (11*16) + 15 = 191
    unsigned int hex2 = 0xbad; // = (11*256) + (10*16) + 13 = 2989
    unsigned char bin  = 0b00111100; // = (32 + 16 + 8 + 4) = 60

    printf("%d %d %d %d %d\n", dec, oct, hex, hex2, bin);

    return 0;
}
```

Bit Models

Consider now binary numbers. Each 1 or 0 is called a bit. We can group bits together. A group of four consecutive bits is called a nibble and a group of eight consecutive bits is called a byte. Two nibbles can be grouped to form a byte. We often break things down into nibbles when we do hexadecimal calculations since each hexadecimal digit is represented with a nibble. We can then group two, four or 8 bytes together for form a word. We can thus have a 16-bit word, a 32-bit word or a 64-bit word. When we group bits of 1’s and 0’s together, there are a variety of methods for interpreting them. Each method of interpreting the sequence of bits is called a bit model. We will look at 6 models:

- Magnitude-only Bit Model
- Sign-Magnitude Bit Model
- Two’s Compliment Bit Model
- Fixed-Point Bit Model
- Floating-Point Bit Model
- ASCII and Unicode Bit Model
In C, the model that is used will depend on the data type. So, `unsigned int`, `signed int`, `float` and `char` will all have their own bit model. (Note: From here-on in the notes, it is assumed that sequences of 1’s and 0’s are binary numbers, and so the numbers will not have a base 2 subscript indication.)

**Magnitude-only Bit Model**

This model is for **non-negative decimal** numbers (whole numbers). It is the simplest model since each bit represents an integer power of 2. With an 8-bit value, we can store a value in the range of 0 to 255.

<table>
<thead>
<tr>
<th>Binary</th>
<th>Decade</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000 = 0</td>
<td></td>
</tr>
<tr>
<td>00000001 = 1</td>
<td></td>
</tr>
<tr>
<td>00000010 = 2</td>
<td></td>
</tr>
<tr>
<td>00000011 = 3</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>11111101 = 253</td>
<td></td>
</tr>
<tr>
<td>11111110 = 254</td>
<td></td>
</tr>
<tr>
<td>11111111 = 255</td>
<td></td>
</tr>
</tbody>
</table>

In this representation, we consider the leftmost bit to be considered the **most-significant bit** and the rightmost bit as the **least-significant bit**. (e.g., 10110101)

Interestingly, when adding numbers, we simply add the bits together starting from the least-significant bit to the most-significant bit:

<table>
<thead>
<tr>
<th>Example of (10 + 7)</th>
<th>Example of (254 + 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>00001010 + 00000111 = 00010001</td>
<td>11111110 + 00000111 = 10000101 OVERFLOW!</td>
</tr>
<tr>
<td>= 17</td>
<td>= 5</td>
</tr>
</tbody>
</table>

Notice that there can be an overflow. Ultimately, there is a limit to the values that we can store with just one byte. Hence, we often use more than one byte to represent numbers in our software.

When dealing with combinations of bytes, we can have groups of 8 bits to store words to provide a larger range of values. For example, a sequence of 4 bytes can represent a much larger number:

\[
\begin{align*}
10011001 & \ 00011110 & \ 01111001 & \ 00111001 \\
& = (10011001 \times 2^{24}) + (00011110 \times 2^{16}) + (01111001 \times 2^{8}) + (00111001 \times 2^{0}) \\
& = (153 \times 16,777,216) + (14 \times 65,536) + (121 \times 256) + (57 \times 1) \\
& = 2,567,862,585
\end{align*}
\]
Here is a table that shows the range of numbers that can be stored when using combinations of bytes together using the magnitude-only bit model:

<table>
<thead>
<tr>
<th>Word Size</th>
<th>Bits Used</th>
<th>Bytes Used</th>
<th>Number Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>byte</td>
<td>8</td>
<td>1</td>
<td>0 to 255</td>
</tr>
<tr>
<td>16-bit word</td>
<td>16</td>
<td>2</td>
<td>0 to 65,535</td>
</tr>
<tr>
<td>32-bit word</td>
<td>32</td>
<td>4</td>
<td>0 to 4,294,967,295</td>
</tr>
<tr>
<td>64-bit word</td>
<td>64</td>
<td>8</td>
<td>0 to 18,446,744,073,709,551,615</td>
</tr>
</tbody>
</table>

In C, the magnitude-only bit model is used with unsigned types:

<table>
<thead>
<tr>
<th>C – data type</th>
<th>Bits Used</th>
<th>Bytes Used</th>
<th>Number Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>unsigned char</td>
<td>8</td>
<td>1</td>
<td>0 to 255</td>
</tr>
<tr>
<td>unsigned short int</td>
<td>16</td>
<td>2</td>
<td>0 to 65,535</td>
</tr>
<tr>
<td>unsigned int</td>
<td>32</td>
<td>4</td>
<td>0 to 4,294,967,295</td>
</tr>
<tr>
<td>unsigned long int</td>
<td>64</td>
<td>8</td>
<td>0 to 18,446,744,073,709,551,615</td>
</tr>
</tbody>
</table>

**Sign-Magnitude Bit Model**

This model allows negative decimal numbers (whole numbers). It is the simplest strategy for representing negative numbers. It has a smaller magnitude range though … allowing numbers only in the range of -127 to +127 for a single byte. The idea is to simply use the most-significant bit to be the sign bit (which is the reason for the reduction in magnitude using this bit model). By convention, a value of 0 in the sign bit indicates a positive value, while a 1 indicates a negative value.

00000000 = 0  
00000001 = 1  
00000010 = 2  
...  
01111110 = 126  
01111111 = 127  
10000000 = -0  
10000001 = -1  
10000010 = -2  
...  
11111110 = -126  
11111111 = -127

This representation is not used often because of a couple of reasons. First, there are two values for zero … +0 and -0. That is weird. Even more of a hassle is that the math does not work out evenly. If the signs of the two numbers are the same, we simply add the magnitudes as unsigned numbers and leave the sign bit intact.

However, we need to be careful about overflow:
If the signs differ, we need to subtract the smaller magnitude from the larger magnitude, and keep the sign of the larger magnitude:

<table>
<thead>
<tr>
<th>Example of (10 + 7)</th>
<th>Example of (-10 + -7)</th>
<th>Example of (126 + 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>00001010 + 00000111 = 00100001 = 17</td>
<td>10001010 + 10000111 = 10001001 = -17</td>
<td>01111110 + 00000111 = 00111001 OVERFLOW!</td>
</tr>
</tbody>
</table>

But this is unpleasant and a bit ugly. Because of the two-zero problem and the need to subtract instead of add … the sign-magnitude bit model is not often used. It is not used in C.

**Two’s Complement Bit Model**

This model allows both positive and negative decimal numbers (whole numbers). Regarding positive and zero numbers … things are represented the same way as a magnitude-only bit model. It has a smaller magnitude range though … allowing numbers only in the range of -128 to +127.

To represent negative numbers … we take the bits that represent the number as if it were positive, then invert (or flip) all of the bits and then add 1.

\[
\begin{align*}
00010011 &= 19 \\
11101100 \text{ flipped bits} \\
11101101 \text{ added one} \\
\hline
11101101 &= -19
\end{align*}
\]

As it turns out, if the most significant bit is 1, then the number is negative, just like the sign-magnitude bit model. To determine the magnitude of a negative number, we perform the exact same steps:

\[
\begin{align*}
11101101 &= -19 \\
00010010 \text{ flip bits} \\
00010011 \text{ add one} \\
\hline
00010011 &= 19 \text{ magnitude}
\end{align*}
\]

We add numbers in the same way:
Example of \((10 + 7)\)  
\[
\begin{array}{c}
00001010 \\
+ \quad 00000111 \\
\hline \\
= \quad 00010001 \\
= \quad 17
\end{array}
\quad \begin{array}{c}
11110110 \\
+ \quad 11111001 \\
\hline \\
= 11110111 \text{ extra 1 carried out, no problem} \\
= \quad -17
\end{array}
\]

Example of \((-10 + (-7))\)

It is possible, however, that there can be an overflow ... resulting in the answer being invalid. It is easy to tell if an overflow has occurred. There are two cases:

- If the sum of two positive numbers results in a negative result.
- If the sum of two negative numbers results in a positive result.

Example of \((126 + 7)\)

\[
\begin{array}{c}
01111110 \\
+ \quad 00000111 \\
\hline \\
= \quad 10000101 \text{ overflow ... sign bit changed!} \\
= \quad -123 \text{ in two's-complement}
\end{array}
\]

Example of \((-80 + -100)\)

\[
\begin{array}{c}
10110000 \\
+ \quad 10011100 \\
\hline \\
= 10100100 \text{ overflow ... sign bit changed!} \\
= \quad 76 \text{ in two's-complement}
\end{array}
\]

In C, the two’s complement bit model is used with signed types:

<table>
<thead>
<tr>
<th>C – data type</th>
<th>Bits Used</th>
<th>Bytes Used</th>
<th>Number Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>char signed char</td>
<td>8</td>
<td>1</td>
<td>-128 to +127</td>
</tr>
<tr>
<td>short int signed short int</td>
<td>16</td>
<td>2</td>
<td>-32,768 to +32,767</td>
</tr>
<tr>
<td>int signed int</td>
<td>32</td>
<td>4</td>
<td>-2,147,483,648 to +2,147,483,647</td>
</tr>
<tr>
<td>long signed long signed long int</td>
<td>64</td>
<td>8</td>
<td>-9,223,372,036,854,775,808 to +9,223,372,036,854,775,807</td>
</tr>
</tbody>
</table>

Here is a program that stores some positive and negative values in \(\text{signed}\) and \(\text{unsigned}\) chars (i.e., bytes). Since the range of signed chars is smaller than unsigned, you will notice some interesting results for values above 127. You will also notice some interesting results for values that go beyond the storage range of a \text{char} or \text{byte}. The compiler actually provides a warning, but allows the code to compile. Near the end of the code, some of these overflow values are stored in short data types ... and you can see that there are no problems.

It is important to understand that things may not always “appear” as you want them to when it comes to mixing signed and unsigned values. Be careful when printing out such values ... especially when debugging:
### Code from `twosCompliment.c`

```c
#include <stdio.h>

int main() {
    char sc;
    unsigned char uc;

    sc = uc = 0;  // 00000000
    printf("0 signed: %d\n", sc);  // 00000000
    printf("0 unsigned: %d\n", uc);  // 00000000

    sc = uc = 1;  // 00000001
    printf("1 signed: %d\n", sc);  // 00000001
    printf("1 unsigned: %d\n", uc);  // 00000001

    sc = uc = 7;  // 00000111
    printf("7 signed: %d\n", sc);  // 00000111
    printf("7 unsigned: %d\n", uc);  // 00000111

    sc = uc = 126;  // 01111110
    printf("126 signed: %d\n", sc);  // 01111110
    printf("126 unsigned: %d\n", uc);  // 01111110

    sc = uc = 127;  // 01111111
    printf("127 signed: %d\n", sc);  // 01111111
    printf("127 unsigned: %d\n", uc);  // 01111111

    sc = uc = 128;  // 10000000
    printf("128 signed: %d\n", sc);  // 10000000
    printf("128 unsigned: %d\n", uc);  // 10000000

    sc = uc = 255;  // 11111111
    printf("255 signed: %d\n", sc);  // 11111111
    printf("255 unsigned: %d\n", uc);  // 11111111

    sc = uc = 256;  // 100000000  *overflow
    printf("256 signed: %d\n", sc);  // 100000000
    printf("256 unsigned: %d\n", uc);  // 100000000

    sc = uc = 260;  // 100000100  *overflow
    printf("260 signed: %d\n", sc);  // 100000100
    printf("260 unsigned: %d\n", uc);  // 100000100

    sc = uc = -0;  // 100000000  *overflow
    printf("-0 signed: %d\n", sc);  // 100000000
    printf("-0 unsigned: %d\n", uc);  // 100000000

    sc = uc = -1;  // 11111111
    printf("-1 signed: %d\n", sc);  // 11111111
    printf("-1 unsigned: %d\n", uc);  // 11111111

    sc = uc = -7;  // 11111001
    printf("-7 signed: %d\n", sc);  // 11111001
    printf("-7 unsigned: %d\n", uc);  // 11111001

    sc = uc = -126;  // 10000010
    printf("-126 signed: %d\n", sc);  // 10000010
    printf("-126 unsigned: %d\n", uc);  // 10000010
```

### Output

<table>
<thead>
<tr>
<th>Code</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>sc = uc = 0;  // 00000000</td>
<td>0 signed: 0 0 unsigned: 0</td>
</tr>
<tr>
<td>sc = uc = 1;  // 00000001</td>
<td>1 signed: 1 1 unsigned: 1</td>
</tr>
<tr>
<td>sc = uc = 7;  // 00000111</td>
<td>7 signed: 7 7 unsigned: 7</td>
</tr>
<tr>
<td>sc = uc = 126;  // 01111110</td>
<td>126 signed: 126 126 unsigned: 126</td>
</tr>
<tr>
<td>sc = uc = 127;  // 01111111</td>
<td>127 signed: 127 127 unsigned: 127</td>
</tr>
<tr>
<td>sc = uc = 128;  // 10000000</td>
<td>128 signed: -128 128 unsigned: 128</td>
</tr>
<tr>
<td>sc = uc = 255;  // 11111111</td>
<td>255 signed: -1 255 unsigned: 255</td>
</tr>
<tr>
<td>sc = uc = 256;  // 100000000  *overflow</td>
<td>256 signed: 0 256 unsigned: 0</td>
</tr>
<tr>
<td>sc = uc = 260;  // 100000100  *overflow</td>
<td>260 signed: 4 260 unsigned: 4</td>
</tr>
<tr>
<td>sc = uc = -0;  // 100000000  *overflow</td>
<td>0 signed: 0 0 unsigned: 0</td>
</tr>
<tr>
<td>sc = uc = -1;  // 11111111</td>
<td>-1 signed: -1 1 unsigned: 255</td>
</tr>
<tr>
<td>sc = uc = -7;  // 11111001</td>
<td>-7 signed: -7 7 unsigned: 249</td>
</tr>
<tr>
<td>sc = uc = -126;  // 10000010</td>
<td>-126 signed: -126 126 unsigned: 130</td>
</tr>
</tbody>
</table>
sc = uc = -127; //10000001
printf("-127 signed: %d\n", sc);
printf("-127 unsigned: %d\n", uc);

sc = uc = -128; //10000000
printf("-128 signed: %d\n", sc);
printf("-128 unsigned: %d\n", uc);

sc = uc = -255; /*weird eh?
printf("-255 signed: %d\n", sc);
printf("-255 unsigned: %d\n", uc);

sc = uc = -256; /*overflow
printf("-256 signed: %d\n", sc);
printf("-256 unsigned: %d\n", uc);

sc = uc = -260; /*overflow
printf("-260 signed: %d\n", sc);
printf("-260 unsigned: %d\n", uc);

short ss;
unsigned short us;

ss = us = 255; //0000000111111111
printf("255 signed short: %d\n", ss);
printf("255 unsigned short: %d\n", us);

ss = us = 256; //0000000010000000
printf("256 signed short: %d\n", ss);
printf("256 unsigned short: %d\n", us);

ss = us = 260; //0000000010000010
printf("260 signed short: %d\n", ss);
printf("260 unsigned short: %d\n", us);

ss = us = -255; //111111100000001
printf("-255 signed short: %d\n", ss);
printf("-255 unsigned short: %d\n", us);

ss = us = -256; //111111100000000
printf("-256 signed short: %d\n", ss);
printf("-256 unsigned short: %d\n", us);

ss = us = -260; //1111111011111110
printf("-260 signed short: %d\n", ss);
printf("-260 unsigned short: %d\n", us);

return 0;
}
The three bit-models that we just looked at are used for representing whole numbers. But how do we represent real (i.e., non-whole) numbers?

There are two main approaches:

1. **Fixed point**
   - **Pros:**
     - faster than floating-point arithmetic
     - Used in digital signal processing and game applications where performance is sometimes more important than precision
   - **Cons:**
     - loss of range for integer portion if more precise fraction is needed
     - works for fractional powers of 2 but not for powers of 10

2. **Floating point**
   - **Pros:**
     - much better precision and range
   - **Cons:**
     - slower than fixed-point arithmetic

**Fixed-Point Bit Model**

The fixed-point bit model is used to represent real numbers, such as floats and doubles. The key to the fixed-point bit model is based on the concept of a *binary point* ... which is like the decimal point in a decimal system that separates the integer part from the fractional part. Consider the binary point number as follows:

\[ 11010.101 \]

This can be calculated as follows:

\[
\begin{align*}
(1 \times 2^4) &+ (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \\
&= (1 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1) + (1 \times 0.5) + (0 \times 0.25) + (1 \times 0.125) \\
&= 16 + 8 + 2 + 0.5 + 0.125 \\
&= 26.625
\end{align*}
\]

Interestingly, if we shift the binary point to the left, we end up with a number half the size:

\[ 11010.101 = 13.3125 \]

and if we shift the binary point to the right, we end up with a number twice the size:

\[ 110101.01 = 53.25 \]

So ... the position of the binary point is crucial in determining the result. In a fixed-point representation, therefore, we must always know how many bits are being used (a.k.a. the *width*) ... and where to position the binary point (a.k.a. the *binary point position*).
Consider this number:

**11010101**

As we have seen it can represent various values, depending on where we place the binary point. We can use the notation \(\text{fixedPoint}(w, b)\) to represent a \(w\)-bit number with the binary point position set to have \(b\) digits to the right of the point:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Fixed-Point Binary</th>
<th>Real Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>fixedPoint(8,0)</td>
<td>11010101</td>
<td>213</td>
</tr>
<tr>
<td>fixedPoint(8,1)</td>
<td>1101010.1</td>
<td>105.5</td>
</tr>
<tr>
<td>fixedPoint(8,2)</td>
<td>110101.01</td>
<td>53.25</td>
</tr>
<tr>
<td>fixedPoint(8,3)</td>
<td>11010.101</td>
<td>26.625</td>
</tr>
<tr>
<td>fixedPoint(8,4)</td>
<td>1101.0101</td>
<td>13.3125</td>
</tr>
<tr>
<td>fixedPoint(8,5)</td>
<td>110.10101</td>
<td>6.65625</td>
</tr>
<tr>
<td>fixedPoint(8,6)</td>
<td>11.010101</td>
<td>3.328125</td>
</tr>
<tr>
<td>fixedPoint(8,7)</td>
<td>1.1010101</td>
<td>1.6640625</td>
</tr>
<tr>
<td>fixedPoint(8,8)</td>
<td>.11010101</td>
<td>0.83203125</td>
</tr>
</tbody>
</table>

Negative numbers are represented as either sign-magnitude or two’s compliment. Assuming two’s compliment, we represent the number -7.25 as follows:

- \(000111.01\) = 7.25 … positive number
- \(111000.10\) = flip bits …
- \(111000.11\) = add 1 to get negative number -7.25

Number addition is done the same way as two’s compliment. We basically just “ignore” the decimal when adding:

<table>
<thead>
<tr>
<th>Example of ((2.5 + 1.75))</th>
<th>Example of ((-2.5 + -7.25))</th>
</tr>
</thead>
<tbody>
<tr>
<td>000010.10</td>
<td>111101.10</td>
</tr>
<tr>
<td>+ 000001.11</td>
<td>+ 111000.11</td>
</tr>
<tr>
<td>000100.01</td>
<td>= 1110110.01</td>
</tr>
<tr>
<td>= 4.25</td>
<td>= -9.75</td>
</tr>
</tbody>
</table>

**Floating-Point Bit Model**

With the floating-point representation, we can use the available bits in different ways. As a result, the total precision using 4 bytes is around 8 digits. That means, we can represent numbers like this: \(0.123456\), \(123.456\) or \(123,456.0\) … but we cannot represent numbers like this: \(123,456.789012\). With the floating-point model, the binary point position is not fixed … but may move around (i.e., float).

In the decimal number system, a number is written in scientific notation like this:

\[284_{10} = 2.84 \times 10^2\]
In general, a number is written in scientific notation as:

\[ \pm M \times B^E \]

where \( M = \text{mantissa} \), \( B = \text{base} \) and \( E = \text{exponent} \)

In C, we have two types that represent real numbers:

<table>
<thead>
<tr>
<th>C – data type</th>
<th>Bits Used</th>
<th>Bits used - Exponent</th>
<th>Bits used - Mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>float</td>
<td>32</td>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>double</td>
<td>64</td>
<td>11</td>
<td>52</td>
</tr>
</tbody>
</table>

There are many ways to represent a floating-point number. Here is one way to represent the number 284:

\[ 284 = 100011100_2 = 1.000111 \times 2^8 \]

<table>
<thead>
<tr>
<th>1-bit sign</th>
<th>8-bit exponent</th>
<th>23-bit mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00001000</td>
<td>100 0111 0000 0000 0000 0000</td>
</tr>
</tbody>
</table>

Since the leading digit in the mantissa is always 1 (for non-zero values), we can assume that this is implied in an improved representation as follows:

<table>
<thead>
<tr>
<th>1-bit sign</th>
<th>8-bit exponent</th>
<th>23-bit mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00001000</td>
<td>000 1110 0000 0000 0000 0000</td>
</tr>
</tbody>
</table>

In the IEEE 754 32-bit floating-point standard, we add a bias of 127 to the exponent as follows:

<table>
<thead>
<tr>
<th>1-bit sign</th>
<th>8-bit biased exponent</th>
<th>23-bit mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10000111</td>
<td>000 1110 0000 0000 0000 0000</td>
</tr>
</tbody>
</table>

There are some special cases:

<table>
<thead>
<tr>
<th>Number</th>
<th>1-bit sign</th>
<th>8-bit biased exponent</th>
<th>23-bit mantissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>00000000</td>
<td>000 0000 0000 0000 0000 0000</td>
</tr>
<tr>
<td>±∞</td>
<td>0</td>
<td>11111111</td>
<td>000 0000 0000 0000 0000 0000</td>
</tr>
<tr>
<td>-∞</td>
<td>1</td>
<td>11111111</td>
<td>000 0000 0000 0000 0000 0000</td>
</tr>
<tr>
<td>NaN</td>
<td></td>
<td>11111111</td>
<td>Non-zero</td>
</tr>
</tbody>
</table>

\textit{NaN} is short for “Not a Number” and it is used to represent a number that does not exist.

When using \textit{double-precision} numbers, the bias used should be 1023 instead of 127. Otherwise things work the same way.

Adding floating point numbers is a little more work that non-floating-point numbers.
Consider adding the following two real numbers:

\[
\begin{align*}
345.7500 \\
+ 6,819.1875 \\
\hline
7,164.9375
\end{align*}
\]

How do we add these numbers using floating point notation? First, get them into binary and then scientific notation:

\[
\begin{align*}
0000101011001.1100 &= 1.010110011100 \times 2^8 \\
1101010100011.0011 &= 1.1010101000110011 \times 2^{12}
\end{align*}
\]

And now into floating-point representation:

\[
\begin{array}{c|cccccccccccc}
0 & 10000111 & 010 & 1100 & 1110 & 0000 & 0000 & 0000 \\
0 & 10001011 & 101 & 0101 & 0001 & 1001 & 1000 & 0000
\end{array}
\]

Now to add them, there are a few steps to take:

1. Get the two mantissas, putting the “1.” before both fractional parts:
   \[
   \begin{align*}
   N1 &= 1.0101100111 \\
   N2 &= 1.1010101000110011
   \end{align*}
   \]

2. Compare the exponents by subtracting the 2nd exponent from the 1st:
   \[
   135 - 139 = -4
   \]
   Since this is a negative number, we shift the binary point of N1 right by 4 bits so that the numbers both have exponent 2^{12} now:
   \[
   \begin{align*}
   N1 &= 0.0001010110011100 \\
   N2 &= 1.1010101000110011
   \end{align*}
   \]

3. Add the mantissas:
   \[
   \begin{align*}
   0.0001010110011100 \\
   + 1.1010101000110011 \\
   \hline
   1.1011111111001111
   \end{align*}
   \]

4. Normalize the mantissa if it has more than one digit to the left of the binary point

5. Round the result if need be (but should not need to if still fits in 23 bits)

6. Assemble the exponent and mantissa back into floating-point notation
7. Verify if answer is correct:

\[
7.1011111111001111000 \times 2^{12} = 110111111110.11110000000 = 7164 + 0.9375 = 7164.9375
\]

As you can see … it can be a little tricky! It is always important to verify that your calculations are correct.

**ASCII and Unicode Bit Model**

This model is for representing non-numerical values. It is a way of mapping characters to numbers. **ASCII** (American Standard Code for Information Interchange) and **Unicode** (not a real acronym but stands for a *Universal code* standard) are two ways of representing characters. The **ASCII** code standard was released in 1963, and was subsequently modified in 1967 and again in 1986. It is a subset of the **Unicode** standard which was released in 1991. **Unicode** incorporates characters from different languages.

Original ASCII code mapped non-accented English text characters and punctuations to numbers. It also mapped some control characters (e.g., NULL, whitespace, tabs, newline, separators) to numbers as well. Each character is contained in one byte. Here are the standard mappings from 0 to 255:

<table>
<thead>
<tr>
<th>ASCII control characters</th>
<th>ASCII printable characters</th>
<th>Extended ASCII characters</th>
</tr>
</thead>
<tbody>
<tr>
<td>00 NULL (Null character)</td>
<td>32 space</td>
<td>128 ç</td>
</tr>
<tr>
<td>01 SOH (Start of Header)</td>
<td>33 1 D5</td>
<td>129 û 161 i</td>
</tr>
<tr>
<td>02 STX (Start of Text)</td>
<td>34 66 A</td>
<td>130 é 162 ô</td>
</tr>
<tr>
<td>03 ETX (End of Text)</td>
<td>35 # 67 C</td>
<td>131 à 163 û</td>
</tr>
<tr>
<td>04 EOT (End of Trans.)</td>
<td>36 $ 68 D</td>
<td>132 a 164 ň</td>
</tr>
<tr>
<td>05 ENQ (Enquiry)</td>
<td>37 % 69 E</td>
<td>133 à 165 ň</td>
</tr>
<tr>
<td>06 ACK (Acknowledgement)</td>
<td>38 &amp; 70 F</td>
<td>134 ã 166 ň</td>
</tr>
<tr>
<td>07 BEL (Bell)</td>
<td>39 * 71 G</td>
<td>135 ã 167 ô</td>
</tr>
<tr>
<td>08 BS (Backspace)</td>
<td>40 ( 72 H</td>
<td>136 ã 168 ň</td>
</tr>
<tr>
<td>09 HT (Horizontal Tab)</td>
<td>41 ) 73 I</td>
<td>137 ã 169 ň</td>
</tr>
<tr>
<td>10 LF (Line feed)</td>
<td>42 ' 74 J</td>
<td>138 ã 170 ň</td>
</tr>
<tr>
<td>11 VT (Vertical Tab)</td>
<td>43 + 75 K</td>
<td>139 ã 171 ň</td>
</tr>
<tr>
<td>12 FF (Form feed)</td>
<td>44 . 76 L</td>
<td>140 ã 172 ň</td>
</tr>
<tr>
<td>13 CR (Carriage return)</td>
<td>45 - 77 M</td>
<td>141 ã 173 ň</td>
</tr>
<tr>
<td>14 SO (Shift Out)</td>
<td>46 . 78 N</td>
<td>142 ã 174 ň</td>
</tr>
<tr>
<td>15 SI (Shift In)</td>
<td>47 / 79 Õ</td>
<td>143 ã 175 ň</td>
</tr>
<tr>
<td>16 DLE (Data link escape)</td>
<td>48 * 80 P</td>
<td>144 ã 176 ň</td>
</tr>
<tr>
<td>17 DC1 (Device control 1)</td>
<td>49 81 Q</td>
<td>145 ã 177 ň</td>
</tr>
<tr>
<td>18 DC2 (Device control 2)</td>
<td>50 82 R</td>
<td>146 ã 178 ň</td>
</tr>
<tr>
<td>19 DC3 (Device control 3)</td>
<td>51 83 S</td>
<td>147 ã 179 ň</td>
</tr>
<tr>
<td>20 DC4 (Device control 4)</td>
<td>52 84 T</td>
<td>148 ã 180 ň</td>
</tr>
<tr>
<td>21 NAK (Negative ackn.)</td>
<td>53 85 U</td>
<td>149 ã 181 ň</td>
</tr>
<tr>
<td>22 SYN (Synchronous idle)</td>
<td>54 86 V</td>
<td>150 ã 182 ň</td>
</tr>
<tr>
<td>23 ETB (End of trans. block)</td>
<td>55 87 W</td>
<td>151 ã 183 ň</td>
</tr>
<tr>
<td>24 CAN (Cancel)</td>
<td>56 88 X</td>
<td>152 ã 184 ň</td>
</tr>
<tr>
<td>25 EM (End of medium)</td>
<td>57 89 Y</td>
<td>153 ã 185 ň</td>
</tr>
<tr>
<td>26 SUB (Substitute)</td>
<td>58 : 90 Z</td>
<td>154 ã 186 ň</td>
</tr>
<tr>
<td>27 ESC (Escape)</td>
<td>59 ; 91 [</td>
<td>155 ã 187 ň</td>
</tr>
<tr>
<td>28 FS (File separator)</td>
<td>60 &lt; 92 \</td>
<td>156 ã 188 ň</td>
</tr>
<tr>
<td>29 GS (Group separator)</td>
<td>61 = 93 ]</td>
<td>157 ã 189 ň</td>
</tr>
<tr>
<td>30 RS (Record separator)</td>
<td>62 &gt; 94 ^</td>
<td>158 ã 190 ň</td>
</tr>
<tr>
<td>31 US (Unit separator)</td>
<td>63 ? 95 –</td>
<td>159 f 191 ň</td>
</tr>
<tr>
<td>127 DEL</td>
<td></td>
<td>160 nsp</td>
</tr>
</tbody>
</table>
In C, there are 2 character-types that make use of ASCII:

<table>
<thead>
<tr>
<th>C – data type</th>
<th>Bits Used</th>
<th>Bytes Used</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>unsigned char</td>
<td>8</td>
<td>1</td>
<td>Decimal value converted to binary using magnitude-only</td>
</tr>
<tr>
<td>char</td>
<td>8</td>
<td>1</td>
<td>Decimal value converted to binary using two’s compliment</td>
</tr>
<tr>
<td>signed char</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

When dealing with UNICODE characters, there are various encoding schemes. UTF-8 (i.e., Unicode Transformation Format) is most commonly used to represent characters from other languages. Each character can take up to 4 bytes. But in C, we use a short int to hold a 2-byte Unicode value. There is a LOT to say about the Unicode text format, but we do not want to get into it too much in this course. We will focus mainly on ASCII code.

### 2.2 Bitwise Operations

Consider an internet company that has 1.2 million people using its services. Perhaps the company wants to keep track of some simple boolean values per customer … such as whether they are an adult, whether they are currently logged-on, or something as simple as whether their account is active. This can be done using an array:

```c
char loggedOn[1200000];
```

The char data type is the smallest primitive type in C, which uses 1 byte … or 8 bits. We would set each value of the array to be either TRUE (i.e., 1) or FALSE (i.e., 0), depending on the log-on-status of that customer. The only values that will be stored in the array are 1 and 0.

It is easy to see that this is a poor use of space. We would actually be making use of just 1 of the 8 bits in each byte. So we are wasting 87.5% of the storage space required by the array!!!

A simple solution is to allow 8 people’s log-on-status bools to be grouped together and stored in a single byte.

In addition to this memory storage problem, sometimes we might need to access data coming in from (or out to) a hardware port in which a byte (or set of bytes) contains portions of bit sequences that have particular meaning, such as on/off flags, error codes, data, instruction code, etc..

```
0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1
0 0 0 0 0 0 1 1
0 0 0 0 0 1 1 0
1 0 0 1 1 1 0 1
```

Wasted Memory

```
1 1 1 0 1 1 0 0 1 0 0 0 1 1 0 1
```

No Wasted Memory

```
1 1 1 1 1 1 1 1
```

OK Error Code Data Instruction Code
At any given moment, we many need to read or set the bits that are relevant for what we are trying to do. It is for good reason then that we will need a way of manipulating bytes at the bit-level. That is what this section of the notes is all about … understanding how to manipulate the bits.

A **bit operator** is an operator that takes one or two numbers and performs an operation on the bits of those numbers.

They are symbols that are used to manipulate individual bits of whole number data types such as **signed char**, **unsigned char**, **signed int** and **unsigned int**. These operators can be used on literals or variables. Here is a list of available bit-operators in C:

<table>
<thead>
<tr>
<th>Operator</th>
<th>Action</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>~</td>
<td>bitwise NOT</td>
<td>inverts every bit (works on one value)</td>
</tr>
<tr>
<td>&amp;</td>
<td>bitwise AND</td>
<td>performs AND between every bit (requires two values)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>performs OR between every bit (requires two values)</td>
</tr>
<tr>
<td>^</td>
<td>bitwise XOR</td>
<td>performs Exclusive OR between every bit (requires two values)</td>
</tr>
<tr>
<td>&gt;&gt;</td>
<td>right-shift</td>
<td>moves bits into lower-order (less significant) bit positions</td>
</tr>
<tr>
<td>&lt;&lt;</td>
<td>left-shift</td>
<td>moves bits into higher-order (more significant) bit positions</td>
</tr>
</tbody>
</table>

Here is a program that shows how to use them:

```c
#include <stdio.h>
#include <math.h>

void printAsBinary(unsigned char);

int main() {
    unsigned char n1 = 157;  // 10011101
    unsigned char n2 = 198;  // 11000110

    unsigned char answer;

    answer = ~n1;
    printf("\~157 = %d, \~10011101 = ", answer);printAsBinary(answer);
    answer = n1>>1;
    printf("\n\157 >> 1 = %d, 10011101 >> 1 = ", answer);printAsBinary(answer);
    answer = n1>>2;
    printf("\157 >> 2 = %d, 10011101 >> 2 = ", answer);printAsBinary(answer);
    answer = n1<<1;
    printf("\157 << 1 = %d, 10011101 << 1 = ", answer);printAsBinary(answer);
    answer = n1<<2;
    printf("\157 << 2 = %d, 10011101 << 2 = ", answer);printAsBinary(answer);
    answer = n1&n2;
    printf("\n\157 & 198 = %d, 10011101 & 11000110 = ", answer);printAsBinary(answer);
    answer = n1^n2;
    printf("\157 | 198 = %d, 10011101 | 11000110 = ", answer);printAsBinary(answer);
    answer = n1^n2;
    printf("\157 ^ 198 = %d, 10011101 ^ 11000110 = ", answer);printAsBinary(answer);
```
```c
char n3 = -100;       // 10011100
char answer2;

answer2 = n3<<1;
printf("\n-100 << 1 = %d, 10011100 << 1 = ", answer2);printAsBinary(answer2);
answer2 = n3<<4;
printf("-100 << 4 = %d, 10011100 << 4 = ", answer2);printAsBinary(answer2);
answer2 = n3>>1;
printf("-100 >> 1 = %d, 10011100 >> 1 = ", answer2);printAsBinary(answer2);
answer2 = n3>>4;
printf("-100 >> 4 = %d, 10011100 >> 4 = ", answer2);printAsBinary(answer2);
return 0;
}

// Convert an unsigned char to an integer that looks like binary
void printAsBinary(unsigned char n) {
    for (int i=7; i>=0; i--) {
        if ((int)(n/pow(2,i)) > 0) {
            printf("1");
            n = n - pow(2,i);
        }
        else
            printf("0");
    }
    printf("\n");
}
```

The output of the program is as follows:

```
~157 = 98, ~10011101 = 01100010
157 >> 1 = 78, 10011101 >> 1 = 01001110
157 >> 2 = 39, 10011101 >> 2 = 00100111
157 << 1 = 58, 10011101 << 1 = 00111010
157 << 2 = 116, 10011101 << 2 = 01110100
157 & 198 = 132, 10011101 & 11000110 = 10000100
157 | 198 = 223, 10011101 | 11000110 = 11011111
157 ^ 198 = 91, 10011101 ^ 11000110 = 01011011
-100 << 1 = 56, 10011100 << 1 = 00111000
-100 << 4 = -64, 10011100 << 4 = 11000000
-100 >> 1 = -50, 10011100 >> 1 = 11001110
-100 >> 4 = -7, 10011100 >> 4 = 11110001
```

Notice that during a right bit shift, zeros are added in on the left as the most significant bit. Similarly, when shifting left, zeros are added in on the right as the least significant bit. This is always the case when dealing with unsigned integers (i.e., magnitude only).

However, when right-shifting with negative numbers (i.e., two’s complement), the bits coming in are 1’s … the highest order sign bit.
Examining bitmasks

In order to extract portions of bits from numbers, we need to use ...

A bitmask is a sequence of one or more bits that you apply to another binary number to read, set or clear the value of one or more bits.

The bitmask number is used to indicate which bits are to be affected by an operation. The following table shows how to read, set and clear a particular bit (i.e., the \(n\)th bit) of a number:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Solution</th>
<th>in C code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set (n)th bit</td>
<td>(x \text{ OR } 2^n)</td>
<td>(x = x \text{</td>
</tr>
<tr>
<td>Clear (n)th bit</td>
<td>(x \text{ AND } \text{ (NOT } 2^n)</td>
<td>(x = x \text{ &amp; (~ (1 \text{ &lt;&lt; } n)});</td>
</tr>
<tr>
<td>Read (n)th bit</td>
<td>(x \text{ AND } 2^n / 2^n)</td>
<td>(x = x \text{ &amp; (1 \text{ &lt;&lt; } n)} \text{ &gt;&gt; } n);</td>
</tr>
</tbody>
</table>

Here is an example of how we apply mask \(m\) to integer \(x\):

<table>
<thead>
<tr>
<th>Set bit 2 of (x)</th>
<th>Clear bit 4 of (x)</th>
<th>Read bit 4 of (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x = 19)</td>
<td>(m = 1&lt;&lt;2)</td>
<td>(x = 19)</td>
</tr>
<tr>
<td>00010111</td>
<td>00000100</td>
<td>00010011</td>
</tr>
<tr>
<td>(~m)</td>
<td>(11101111)</td>
<td>((x \text{ &amp; } m) \text{ &gt;&gt; } 4)</td>
</tr>
<tr>
<td>(x \text{</td>
<td>} m)</td>
<td>00000011</td>
</tr>
</tbody>
</table>

Let's look at a more interesting example. Consider our earlier example of a two-byte bit-sequence that can be used to send commands to a device and to read data from that device. The device allows up to 16 instructions, some of which require 8-bits of data. After the device performs the user's instruction, it will make use of the most significant bit (i.e., the OK bit) to indicate an error status. If all went well, this bit will be set to 0, otherwise it will be set to 1. The particular kind of error will be indicated by the device which will set the Error Code bits to indicate one of 8 possible errors that may have occurred. Here is how the bits are organized:

We can store this command in an unsigned short as follows:

```
unsigned short command;
```

Now, assume that we want to send instruction 0110 to the device with its corresponding data 1001101. How can we set the instruction and data bits accordingly? Since the OK and Error Code bits are to be set by the device, we don’t worry about what value they have when sending our instructions. So, it is easy to set the command.
We just need to shift the data left by 4 bits and add the instruction:

```c
command = instruction + (data << 4);
send(command);
```

Assuming that the command is then modified by the device to contain an answer in the data bits. How can we extract that data? We would need to provide a bitmask indicating which bits that we want to read as follows:

```c
unsigned char result = receive();
printf("The result from the device is: %d",
(result & (255<<4)) >> 4);
```

But we would have to first check the ok bit to make sure that the data is valid. If not, we should check the error code bits and inform the user:

```c
if ((result & 0b1000000000000000) > 0)
    printf("An error has occurred with code: %d\n",
    (result & (0b111 << 12)) >> 12);
```

Here is a completed program that simulates the sending of 5 instructions to a device and then checks the 5 received results for errors and prints out an appropriate message:

```c
#include <stdio.h>
#include <math.h>

void printAsBinary(unsigned short);
void send(int);
void receive(int);

unsigned char simulatedInstructions[] = {0b0110, // 6
0b0000, // 0
0b0011, // 3
0b1111, // 15
0b0001}; // 1

unsigned char simulatedData[] = {0b00110011, // 51
0b11100000, // 228
0b10101010, // 170
0b00000000, // 0
0b11111111}; // 255

unsigned short simulatedResults[] = {0b0000101000010110, // 161
0b1010000000000000, // error code 2
0b1101001110010011, // error code 5
0b0000111111101111, // 254
0b00000000111110001}; // 31

int main() {
    unsigned short result;
    
    // Send 5 commands and get 5 results
    for (int i=0; i<5; i++) {
        send(i);
        receive(i);
    }
```
return 0;
}

// Simulate the sending of an instruction with data to the device
void send(int i) {
    unsigned short command = simulatedInstructions[i] + (simulatedData[i] << 4);
    printf("Sending Command: ");
    printAsBinary(command);
}

// Simulate the receiving of a one byte data reply from the device. An error may have occurred, so this is checked for before the data is read.
void receive(int i) {
    unsigned short result = simulatedResults[i];
    if ((result & 0b1000000000000000) > 0)
        printf("An error has occurred with code: %d\n", (result & (0b111<<12)) >> 12);
    else
        printf("The result data from the device is: %d\n", (result & (255<<4)) >> 4);
}

// Convert an unsigned short to an integer that looks like binary
void printAsBinary(unsigned short n) {
    for (int i=15; i>=0; i--)
        if ((int)(n/pow(2,i)) > 0) {
            printf("1");
            n = n - pow(2,i);
        }
    else
        printf("0");
    printf("\n");
}

Here is the program output:

Sending Command: 0000001100110110
The result data from the device is: 161
Sending Command: 0000111000000000
An error has occurred with code: 2
Sending Command: 0000101010100011
An error has occurred with code: 5
Sending Command: 0000000000001111
The result data from the device is: 254
Sending Command: 0000111111110001
The result data from the device is: 31


### 2.3 Compound Data Types

Up until now, we have examined only primitive data types such as `char`, `long int`, `short int`, `int`, `float` and `double`. We have also looked at how these are broken down into bits to be stored in the computer’s memory. We will now look at working with compound data types:

A **compound data type** is a data type that stores multiple values under a single variable name.

Recall that in JAVA, we have primitive types and we were able to combine multiple primitive types into one compound type. For example ... in JAVA ...

- **String** objects consist of an ordered-sequence of `char` primitives:
  - e.g., `String s = new String("Hello ");
    s = s + 'T' + 'h' + 'e' + 'r' + 'e';`

- **Array** objects consist of an ordered-sequence of `primitives` or other same-type objects:
  - e.g., `int[] scores = {33, 56, 21, 75, 82, 44, 91};
    Car[] cars = new Car[50];
    cars[0] = new Car("Red", "Porsche", 260);
    cars[2] = new Car("Yellow", "Ferrari", 252);`

- **Class** objects consist of a mixed set of attributes which may be `primitives` or `objects`:
  - e.g., `class Car {
      String owner;
      int accountNumber;
      float balance;
    }`

In the C programming language, we can do similar things. We can create Strings and Arrays as well as Structures (which are similar to the notion of a collection of a class’ attributes but without its methods). We will examine each of these now.

### 2.4 Strings and char Arrays

As in JAVA, strings in C are a sequence of characters. Unlike JAVA, which keeps track of a string’s length, in C we do not keep track of the length. Instead, the end of a string is identified by a special **null-terminating character** ... which is the null character "\0" ... which has ASCII value 0. It is a non-printable character ... so it cannot be displayed on the screen:

```
Hello There
```

NULL = '\0'
Recall that in JAVA, we cannot modify a string once it has been created. So, for example, consider appending to a string as follows:

```java
String s = new String("Hello ");
String c = s;
s = s + 'T' + 'h' + 'e' + 'r' + 'e';
System.out.println(s); // prints "Hello There"
System.out.println(c); // prints original string object "Hello"
```

In JAVA, we get a new string each time that the + operator is used. The original string remains intact. We are actually unable to modify a created String in JAVA.

In C, however, since strings are just a sequence of char primitives in memory, we can modify the string at any time, given that we know the memory location of the start of the string. In fact, strings are defined with respect to a starting location in memory. In C, we declare a string type in one of two ways as follows:

```c
char myString[4]; or char *myString;
```

In the first option, we are declaring an array of chars. We can set the characters individually and modify the array at any time, as with any array:

```c
myString[0] = 'J';
myString[1] = 'e';
myString[2] = 'n';
myString[3] = '\0'; // Make sure to end with a null terminator!!
```

We can also access the characters as a normal array, but be careful … we are allowed to access beyond the boundary:

```c
printf("\myString = %s\n", myString); // Jen
printf("\myString[0] = %c\n", myString[0]); // J
printf("\myString[1] = %c\n", myString[1]); // e
printf("\myString[2] = %c\n", myString[2]); // n
printf("\myString[3] = %c\n", myString[3]); // null char
printf("\myString[4] = %c\n", myString[4]); // garbage!
printf("\myString[5] = %c\n", myString[5]); // garbage!
printf("\myString[6] = %c\n", myString[6]); // garbage!
```

In addition to setting the characters of the array one at a time, we can also hard-code the values with a constant (i.e., literal) string as follows:

```c
char myString[] = "Jen";
```

Note that we do not need to specify the null-terminating character when creating a string in this manner.
Here is what it would look like as myString is stored in memory →

Note that it is the same as a char array in JAVA. The only difference is the use of the null-terminating character.

We can even reserve “extra” space for future characters by putting a size into the array:

```c
char myString[50];
```

But in this case, the string is uninitialized … and hence filled with garbage characters. It would not display properly.

```c
printf("myString = %s\n", myString); // myString = `\xH|`
```

Of course, we could always set it to be purposely blank (and hence display nothing) like this:

```c
char myString[50] = "";
```

Now, as for the 2\textsuperscript{nd} way of creating a string (i.e., `char *myString`), take note of the * character. This indicates that we want the variable to refer to the location (i.e., memory address) of the first character of the string.

This reference is also known as …

**A pointer is a reference to a memory location.**

We could assign a value to the string as before:

```c
char *myString = "Jen";
```

Again, the individual characters of the string can be accessed as if it were an array:

```c
printf("myString = %s\n", myString); // Jen
printf("myString[0] = %c\n", myString[0]); // J
printf("myString[1] = %c\n", myString[1]); // e
printf("myString[2] = %c\n", myString[2]); // n
printf("myString[3] = %c\n", myString[3]); // null char
printf("myString[4] = %c\n", myString[4]); // garbage!
printf("myString[5] = %c\n", myString[5]); // garbage!
printf("myString[6] = %c\n", myString[6]); // garbage!
```
Notice that the code is identical as when the string was declared as a `char[]`. At any time, we can determine the string’s location in memory by using the `&` character (which means address of) as follows:

    printf("address of myString = %p\n", (void *) &myString);

This will return the memory location of the start of the string which may look something like this: `0x7ffd9ee76644` ... which will of course depend on the amount of memory installed on your computer and how much is allocated to your program etc.. We will talk much more about pointers and addresses later on.

For now, here is a test program to summarize everything. Feel free to play around with the code and experiment:

<table>
<thead>
<tr>
<th>Code from strings.c</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>#include &lt;stdio.h&gt;</code></td>
<td></td>
</tr>
<tr>
<td><code>int main() {</code></td>
<td></td>
</tr>
<tr>
<td><code>char a = 'P';</code></td>
<td>a = P</td>
</tr>
<tr>
<td><code>char b[] = &quot;Jen&quot;;</code></td>
<td>b = Jen</td>
</tr>
<tr>
<td><code>char c[50] = &quot;&quot;;</code></td>
<td></td>
</tr>
<tr>
<td><code>char *d = &quot;Max&quot;;</code></td>
<td></td>
</tr>
<tr>
<td><code>printf(&quot;a = %c\n\n&quot;, a);</code></td>
<td></td>
</tr>
<tr>
<td><code>printf(&quot;b    = %s\n\n&quot;, b);</code></td>
<td>b[0] = J</td>
</tr>
<tr>
<td><code>printf(&quot;b[0] = %c\n&quot;, b[0]);</code></td>
<td></td>
</tr>
<tr>
<td><code>printf(&quot;b[1] = %c\n&quot;, b[1]);</code></td>
<td>b[1] = e</td>
</tr>
<tr>
<td><code>printf(&quot;b[3] = %c\n&quot;, b[3]);</code></td>
<td></td>
</tr>
<tr>
<td><code>printf(&quot;b[5] = %c\n\n&quot;, b[5]);</code></td>
<td>b[5] =</td>
</tr>
<tr>
<td><code>printf(&quot;c    = %s\n\n&quot;, c);</code></td>
<td>c</td>
</tr>
<tr>
<td><code>printf(&quot;d    = %s\n\n&quot;, d);</code></td>
<td>d = Max</td>
</tr>
<tr>
<td><code>printf(&quot;d[0] = %c\n&quot;, d[0]);</code></td>
<td>d[0] = M</td>
</tr>
<tr>
<td><code>printf(&quot;d[1] = %c\n&quot;, d[1]);</code></td>
<td>d[1] = a</td>
</tr>
<tr>
<td><code>printf(&quot;d[3] = %c\n&quot;, d[3]);</code></td>
<td>d[3] =</td>
</tr>
<tr>
<td><code>printf(&quot;d[5] = %c\n\n&quot;, d[5]);</code></td>
<td>d[5] =</td>
</tr>
<tr>
<td><code>printf(&quot;address of a = %p\n&quot;, (void *) &amp;a);</code></td>
<td>address of a = 0x7fffc5b2b871f</td>
</tr>
<tr>
<td><code>printf(&quot;address of b = %p\n&quot;, (void *) &amp;b);</code></td>
<td>address of b = 0x7fffc5b2b872c</td>
</tr>
<tr>
<td><code>printf(&quot;address of c = %p\n&quot;, (void *) &amp;c);</code></td>
<td>address of c = 0x7fffc5b2b8730</td>
</tr>
<tr>
<td><code>printf(&quot;address of d = %p\n&quot;, (void *) &amp;d);</code></td>
<td>address of d = 0x7fffc5b2b8720</td>
</tr>
<tr>
<td><code>}</code></td>
<td></td>
</tr>
</tbody>
</table>

We may also declare a `char` or string as **constant** … or non-modifiable. We do this by using the `const` keyword:
const char d = 'G';
const char *e = "Wow";
const char f[] = "Amazing";

If we try to modify a variable that has been declared as constant within our code, e.g., like this:

d = 'H';

then the compiler will stop us:

strings.c:40:5: error: assignment of read-only variable 'd'
d = 'H';

For strings, the use of const is like a safety measure so that we don’t modify the string accidentally. It also makes it clear in your code (when someone else is reading it) that you won’t be modifying this later in your program. You should use const whenever you have fixed Strings (e.g., error messages, text used for printf statements, or labels on a GUI, etc…).

There are some useful functions that we can use on strings. They are located in the string.h library. To use them, we must therefore put this header file at the top of our code:

#include <string.h>

strlen()

One of the most useful functions is the strlen() function which returns the number of characters in a string. It actually returns the number of characters up until it finds the null-terminating character.

char *t1 = "Mark";
printf("strlen(t1) = %d\n", strlen(t1)); // 4

We have to be careful to make sure that at least one character is available in the string. For example, notice the varying results when we call the strlen() function on somewhat empty strings as shown below:

char *t1 = "Please enter your two names: ";
char *t2 = "";
char *t3;
char t4[];
char t5[] = "";
char t6[25];
char t7[25] = "";
printf("strlen(t1) = %d\n", strlen(t1)); // 29
printf("strlen(t2) = %d\n", strlen(t2)); // 0
printf("strlen(t3) = %d\n", strlen(t3)); // segmentation fault!!
printf("strlen(t5) = %d\n", strlen(t5)); // 0
printf("strlen(t6) = %d\n", strlen(t6)); // 0
printf("strlen(t7) = %d\n", strlen(t7)); // 0
Note that `t3` does not reserve any space for characters ... so it is like an *undefined* string. Finding the length of it can cause strange results as well as a segmentation fault. Also, `t4[]` will not compile because we must specify a size for all arrays. Both `t6` and `t7` reserve enough space to hold plenty of characters but there are no characters in there upon initialization ... so the content of the array is unpredictable. We typically use `strlen()` in our loops. For example, if we want to count the spaces in a string we can do this:

```c
char *s = "Please enter your two names: ";

int count = 0;
for (int i=0; i<strlen(s); i++) {
    if (s[i] == ' ')
        count++;
}
printf("Number of spaces = \%d\n", count);
```

### `strcpy(dest, src)` & `strncpy(dest, src, n)`

The `strcpy()` function copies the string pointed to by `src` (including the terminating null byte) to the buffer pointed to by `dest`. It is important to ensure that the destination string is large enough to receive the copy otherwise we might overwrite memory locations and crash our program. Consider this code:

```c
char *s1 = "This is the original";
char *s2;
strcpy(s2, s1);
```

It generates a **segmentation fault** (i.e., program crash)! Why? Examine the second string `s2`. It is declared as a pointer to a string, but no space has been allocated for the characters. Therefore, when we try to copy over the string, it starts writing over memory locations that may contain our program or other variables. To fix this, we need to reserve space, perhaps using a character array as follows:

```c
char *s1 = "This is the original";
char s2[50];
strcpy(s2, s1);
s2[8] = ' ';
s2[9] = 'u';
s2[10] = 'n';
printf("s1 = \%s\n", s1); // displays "This is the original"
printf("s2 = \%s", s2); // displays "This is un-original"
```

A similar function is `strncpy()` which allows us to specify that we just want to copy a few characters, not all of them. Again, we need to be careful to reserve space for at least `n+1` characters (an extra one to hold the null terminator).
Be aware though, if there is no null byte among the first \( n \) bytes of the src, then the string placed in dest will not be null-terminated. Also, if the length of src is less than \( n \), then strncpy() will write additional null bytes to dest to ensure that a total of \( n \) bytes are written.

\[
\begin{align*}
\text{char } & \ast s1 = "\text{This is the original}"; \\
\text{char } & s3[20]; \\
\text{strncpy}(s3, s1, 11); \\
\text{printf}("s3 = %s\n", s3); & // displays "This is the\n" \\
& \text{s3[11]} = '\0'; \\
\text{printf}("s3 = %s\n", s3); & // displays "This is the"
\end{align*}
\]

**strncpy(s1, s2) & strncpy(s1, s2, n)**

The **strncpy(s1, s2)** function compares two strings, s1 and s2, for equality. If \( s1 < s2 \) (alphabetically), then a negative value is returned. If \( s1 > s2 \) (alphabetically), then a positive value is returned. If \( s1 == s2 \) (alphabetically), then 0 is returned.

\[
\begin{align*}
\text{char } & \ast s4 = "\text{Chris}"; \\
\text{char } & \ast s5 = "\text{Jen}"; \\
\text{char } & \ast s6 = "\text{Will}"; \\
\text{printf}"\text{strcmp(Chris, Jen) = %d}\n", & \text{strcmp}(s4, s5)); // -1 \\
\text{printf}"\text{strcmp(Jen, Chris) = %d}\n", & \text{strcmp}(s5, s4)); // 1 \\
\text{printf}"\text{strcmp(Jen, Will) = %d}\n", & \text{strcmp}(s5, s6)); // -1 \\
\text{printf}"\text{strcmp(Will, Chris) = %d}\n", & \text{strcmp}(s6, s4)); // 1 \\
\text{printf}"\text{strcmp(Jen, Jen) = %d}\n", & \text{strcmp}(s5, s5)); // 0
\end{align*}
\]

Note that when the strings are not equal, the above code returned -1 and +1. However, these values are not guaranteed … only the sign is guaranteed.

The **strncpy()** function works like **strcmp()**, however it only compares the first (at most) \( n \) characters of the strings:

\[
\begin{align*}
\text{char } & \ast s7 = "\text{Max}"; \\
\text{char } & \ast s8 = "\text{Mark}"; \\
\text{char } & \ast s9 = "\text{Melvin}"; \\
\text{printf}"\text{strcmp(Max, Mark, 2) = %d}\n", & \text{strcmp}(s7, s8, 2)); // 0 \\
\text{printf}"\text{strcmp(Max, Mark, 3) = %d}\n", & \text{strcmp}(s7, s8, 3)); // 1 \\
\text{printf}"\text{strcmp(Melvin, Mark, 1) = %d}\n", & \text{strcmp}(s9, s7, 1)); // 0 \\
\text{printf}"\text{strcmp(Max, Melvin, 5) = %d}\n", & \text{strcmp}(s7, s9, 5)); // -1 \\
\text{printf}"\text{strcmp(Max, Max, 8) = %d}\n", & \text{strcmp}(s7, s7, 8)); // 0
\end{align*}
\]

**strcat(dest, src) & strncat(dest, src, n)**

The **strcat()** function appends the src string to the dest string, over-writing the terminating null byte at the end of dest, and then adds a terminating null byte. The dest string must have enough space for the result to be stored otherwise you again may overwrite valuable memory.
```c
char *n1 = "Steve";
char *n2 = "Martha";
char *n3 = "Jacob";
char result[50] = ""; // 50 is big enough for this example
strcat(result, n1);
strcat(result,", ");
strcat(result, n2);
strcat(result,", ");
strcat(result, n3);
printf("result = %s\n", result); // displays "Steve, Martha, Jacob"

Similarly, the `strcat()` appends at most \( n \) bytes from `src`. Therefore, the space that `dest` is stored in must be at least `strlen(dest) + n + 1` bytes.

```c
char *n4 = "Steve";
char *n5 = "ariel";
char *n6 = "tup";
char result2[50] = "";
strncat(result2, n4, 2);
strncat(result2, n5, 2);
strncat(result2, n6, 3);
strncat(result2, n4, 999); // only appends 5 chars since null is there
printf("result2 = %s\n", result2); // displays "StartupSteve"
```

### `sprintf()`

The `sprintf()` function works like `printf()`, except that instead of printing to the console, it prints the information into a specified string. This function will return the number of bytes in the resulting string, or a negative value if an error occurs.

It is most useful for converting numbers to strings and for combining multiple values into a string.

```c
char answer[50];
char *name = "Bob";
int num = 1026784;
float balance = 67.32;
int chars = sprintf(answer, "%s's account %d with $%0.2f",
                    name, num, balance);
printf("The string is "%s" with %d characters\n", answer, chars);
```

The answer is:

**The string is "Bob's account 1026784 with $67.32" with 33 characters**
2.5 Arrays

Arrays store multiple values of the same type. Every element takes up the same amount of memory. The size of the array in memory will therefore depend on the type of data stored. So, for example, the following two arrays would look different in memory:

```
cchar arr1[20] = "MARK is having fun!";
int arr2[20] = {590599, 512, 65535};
```

As we have seen, accessing the arrays is done the same way as in JAVA … by indices … which start at 0. However, unlike JAVA, C does NOT do any Array Out of Bounds checking! Therefore, you need to be much more careful when using them.

We can use the `sizeof()` function in C to determine how many bytes are required for a particular type or how many bytes a variable is using. This will help us determine how much storage space is required for our arrays. So, for example, we can determine the size of types like this:

```
printf("The size of int is %zu\n", sizeof(int)); // displays 4
printf("The size of float is %zu\n", sizeof(float)); // displays 4
printf("The size of double is %zu\n", sizeof(double)); // displays 8
printf("The size of char is %zu\n", sizeof(char)); // displays 1
```

Use `%zu` when using `sizeof()`.  

---

Use `%zu` when using `sizeof()`.  

---

Use `%zu` when using `sizeof()`.  

---

Use `%zu` when using `sizeof()`.  

---
And if we were to create some arrays of these types, we could determine how many bytes the entire array will require when stored in memory as follows:

```c
int    array1[5];
float  array2[3];
double array3[6];
char   array4[10];

printf("The size of array1 is \%zu\n", sizeof(array1)); // 5*4 = 20
printf("The size of array2 is \%zu\n", sizeof(array2)); // 3*4 = 12
printf("The size of array3 is \%zu\n", sizeof(array3)); // 6*8 = 48
printf("The size of array4 is \%zu\n\n", sizeof(array4)); // 10*1 = 10
```

In C, arrays do not have an attribute nor a function that allows us to know how many items are in the array. We could keep track of this in a separate variable, as we did in JAVA. Then we can loop through the elements using a FOR loop. However, there is another way to determine the number of elements in the array. We can use the `sizeof()` function again. Since `sizeof()` tells us how many total bytes are in the array, we just need to divide that number by the size of the type that the array is storing … as follows:

```c
for (int i=0; i<sizeof(array1)/sizeof(int); i++)
    printf("%d, ", array1[i]);
printf("\n");

for (int i=0; i<sizeof(array2)/sizeof(float); i++)
    printf("%f, ", array2[i]);
printf("\n");

for (int i=0; i<sizeof(array3)/sizeof(double); i++)
    printf("%g, ", array3[i]);
printf("\n");

for (int i=0; i<sizeof(array4)/sizeof(char); i++)
    printf("%d, ", array4[i]);
printf("\n\n");
```

Here is the result … showing the “garbage” data that is in the uninitialized arrays:

```
1276585784, 32763, 0, 0, 1, 0.000000, 0.000000, 0.000000, 0, 8.40389e-315, 6.95236e-310, 6.953e-310, 6.953e-310, 6.95236e-310, 16, 44, 67, 78, -2, 127, 0, 0,
```

We may also create and use multi-dimensional arrays in C:

```c
int array2D[3][4] = {{ 1, 2, 3, 4},
                      { 5, 6, 7, 8},
                      { 9, 10, 11, 12}};
```

Notice that we have two sets of indices. The first index indicates the row and the second one indicates the column. Therefore, `array2D[2][0]` indicates the 1st item in the 3rd row … which is 9 in the above example.
To understand fully how to access the sizes of the items, the rows and the columns, we must fully understand how to use \texttt{sizeof()}. Make sure that you understand the output from the following code:

\begin{verbatim}
printf("The size of array2D is \%zu \n", sizeof(array2D));
printf("The #elements in array2D is \%zu \n", sizeof(array2D)/sizeof(int));
printf("The #elements in each row of array2D is \%zu \n", sizeof(array2D[0])/sizeof(int));
printf("The #rows of array2D is \%zu \n", sizeof(array2D)/sizeof(array2D[0]));

for (int r=0; r<3; r++) {
    for (int c=0; c<4; c++)
        printf("%02d ", array2D[r][c]);
    printf("\n");
}
printf("\n");
\end{verbatim}

Here is the output … was it what you expected?

```
The size of array2D is 48
The #elements in array2D is 12
The #elements in each row of array2D is 4
The #rows of array2D is 3
01 02 03 04
05 06 07 08
09 10 11 12
```

Interestingly, we can create even higher dimensional arrays. For example, an array as follows may be used to represent a cube of colored blocks:

\begin{verbatim}
int cube[4][4][4] = {
    {{1, 2, 3, 4},
     {5, 6, 7, 8},
     {9, 10, 11, 12},
     {13, 14, 15, 16}},
    {{17, 18, 19, 20},
     {21, 22, 23, 24},
     {25, 26, 27, 28},
     {29, 30, 31, 32}},
    {{33, 34, 35, 36},
     {37, 38, 39, 40},
     {41, 42, 43, 44},
     {45, 46, 47, 48}},
    {{49, 50, 51, 52},
     {53, 54, 55, 56},
     {57, 58, 59, 60},
     {61, 62, 63, 64}}};

int rows = sizeof(cube)/ sizeof(cube[0]);
int cols = sizeof(cube[0])/ sizeof(cube[0][0]);
int depth = sizeof(cube)/(rows*cols*sizeof(int));
\end{verbatim}
printf("Cube has %d rows\n", rows);
printf("Cube has %d columns\n", cols);
printf("Cube has %d layers\n", depth);

for (int r=0; r<4; r++) {
    for (int c=0; c<4; c++) {
        for (int d=0; d<4; d++)
            printf("%02d ", cube[r][c][d]);
        printf("\n");
    }
    printf("\n");
}
printf("\n");

Here is the output:

Cube has 4 rows
Cube has 4 columns
Cube has 4 layers
  01 02 03 04
  05 06 07 08
  09 10 11 12
  13 14 15 16
  17 18 19 20
  21 22 23 24
  25 26 27 28
  29 30 31 32
  33 34 35 36
  37 38 39 40
  41 42 43 44
  45 46 47 48
  49 50 51 52
  53 54 55 56
  57 58 59 60
  61 62 63 64

2.6 Custom Type Definitions: Structures and Unions

Recall that in JAVA we defined various classes to represent objects. These classes allowed us to group together a bunch of attributes (a.k.a. fields) that are either primitive types or other objects. It keeps them all together in a bundle. You can think of a structure as a bunch of small pieces of information with an elastic around it:

A structure represents multiple pieces of information that are grouped together.
Consider a person's full address, which may be represented using multiple strings as follows:

```c
char *name = "Patty O. Lantern";
char *streetNumber = "187B";
char *streetName = "Oak St.";
char *city = "Ottawa";
char *province = "ON";
char *postalCode = "K6S8P2";
```

We could display the variable values individually:

```c
printf("%s\n", name); // Patty O. Lantern
printf("%s %s\n", streetNumber, streetName); // 187B Oak St.
printf("%s, %s\n", city, province); // Ottawa, ON
printf("%s\n", postalCode); // K6S8P2
```

Recall in JAVA that we set up a `class` to define an address. In C, we do something similar, except that instead of creating a class, we create a `struct`. There are two ways of defining the struct in C. Consider the similarities and differences as shown here:

<table>
<thead>
<tr>
<th>JAVA</th>
<th>C – just struct</th>
<th>C – using typedef</th>
</tr>
</thead>
</table>
| ```java
class Address {
  String name;
  String streetNumber;
  String streetName;
  String city;
  String province;
  String postalCode;
}
``` | ```c
struct AddressType {
  char *name;
  char *streetNumber;
  char *streetName;
  char *city;
  char *province;
  char *postalCode;
};
``` | ```c
typedef struct {
  char *name;
  char *streetNumber;
  char *streetName;
  char *city;
  char *province;
  char *postalCode;
} AddressType;
``` |

Note that in C, it is common to put the word “Type” at the end of the structure name. Hence, we used `AddressType` instead of simply `Address`.

Notice also, that there is a ; character after the `struct`'s definition. That is because the `struct` keyword is just another C statement. So, we can define this struct anywhere in our code.

This is different than JAVA classes, which we usually define each in their own unique file. However, we usually define all of our structs outside of our functions ... for example ... at the top of our program before the main function ... or in a header file (more on this later).

In the second C example, we are actually defining a new `type`. This will make it easier when we declare variables and parameters. Notice the JAVA vs. C comparison when defining a variable called `addr`:

<table>
<thead>
<tr>
<th>JAVA</th>
<th>C – just struct</th>
<th>C – using typedef</th>
</tr>
</thead>
</table>
| ```java
Address addr;
``` | ```c
struct AddressType addr;
``` | ```c
AddressType addr;
``` |
As you can see, unless we create the `typedef`, we would need to use the `struct` keyword when declaring the variable's type. Otherwise, the declaration is similar to that of JAVA.

Assigning values to the structure's attributes is now easy, as it is done in a similar way to JAVA by using the dot operator:

```c
addr.name = "Patty O. Lantern";
addr.streetNumber = "187B";
addr.streetName = "Oak St."
addr.city = "Ottawa";
addr.province = "ON";
addr.postalCode = "K6S8P2";
```

```c
printf("%s\n", addr.name);
printf("%s %s\n", addr.streetNumber, addr.streetName);
printf("%s, %s\n", addr.city, addr.province);
printf("%s\n", addr.postalCode);
```

We can define other structs as well and use them inside one another, just like using objects within objects in JAVA. Consider this JAVA code:

```java
class Name {
    String first;
    String middle;
    String last;
}

class Address {
    String streetNumber;
    String streetName;
    String city;
    String province;
    String postalCode;
}

class Person {
    Name name;
    Address address;
    int age;
    char gender;
    float weight;
    boolean retired;
}
```
Here is a C program that constructs and uses this set of objects as structs within structs:

```c
#include <stdio.h>

#define TRUE 1;
#define FALSE 0;

// Structure that represents a person's full name
typedef struct {
    char *first;
    char *middle;
    char *last;
} NameType;

// Structure that represents a person's full address
typedef struct {
    char *streetNumber;
    char *streetName;
    char *city;
    char *province;
    char *postalCode;
} AddressType;

// Structure that represents a person
typedef struct {
    NameType name;
    AddressType address;
    int age;
    char gender;
    float weight;
    char retired;
} PersonType;

int main() {
    PersonType bob;

    bob.name.first = "Patty";
    bob.name.middle = "O."
    bob.name.last = "Lantern";
    bob.address.streetNumber = "187B";
    bob.address.streetName = "Oak St.";
    bob.address.city = "Ottawa";
    bob.address.province = "ON";
    bob.address.postalCode = "K6S8P2";
    bob.age = 24;
    bob.gender = 'M';
    bob.weight = 157.2;
    bob.retired = FALSE;

    printf("%s %s %s\n", bob.name.first, bob.name.middle, bob.name.last);
    printf("%d year old %s %s weighing %f pounds\n",
            bob.age,
            bob.retired ? "retired" : "non-retired",
            (bob.gender == 'M') ? "male" : "female",
            bob.weight);
```

Notice that structs are being used within other structs. Attributes are accessed the same way as in JAVA by using the dot operator.
printf("Living at: %s %s, ",
    bob.address.streetNumber,
    bob.address.streetName);

printf("%s %s ", bob.address.city, bob.address.province);
printf("%s\n", bob.address.postalCode);

    return(0);
}

Here is the output:

    Patty O. Lantern
    24 year old non-retired male weighing 157.199997 pounds
    Living at: 187B Oak St., Ottawa, ON K6S8P2

Just a few points about scope … notice that the structs are all defined before the main function. This makes them global, which means that they can be accessed from anywhere. By contrast, if we define a struct within a function, then we may ONLY use that struct within that function. Typically, we want our structs to be used throughout our entire program, so we usually put their definitions before the main function.

As with JAVA objects, we may also make arrays of structs and we can use them as parameters to functions. Consider an example in JAVA where we have objects that represent a Dive, a Performance and an Athlete in a system where athletes perform 3 dives each and each performance is judged by exactly 8 judges who provide a score each time.

<table>
<thead>
<tr>
<th>JAVA code</th>
<th>C code</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>class Dive</strong> {</td>
<td>**typedef struct {</td>
</tr>
<tr>
<td>String name;</td>
<td>char *name;</td>
</tr>
<tr>
<td>int difficulty;</td>
<td>int difficulty;</td>
</tr>
<tr>
<td>}</td>
<td>} DiveType;</td>
</tr>
</tbody>
</table>

| **class Performance** {            | **typedef struct {               |
|     Dive dive;                     |     DiveType dive;               |
|     float[] scores;               |     float scores[8];            |
| }                                   |     } PerformanceType;          |

| **class Athlete** {                | **typedef struct {               |
|     String name;                  |     char *name;                  |
|     String country;               |     char *country;               |
|     Performance[] performances;   |     PerformanceType performances[3]; |
| }                                   |     } AthleteType;               |

Remember … when creating arrays in C, even in a struct definition, we need to supply the size of the array. This is different from JAVA attribute definitions, where we just defined the array and then created the array using a constructor at a later time.
We can create arrays to hold 5 dives and 3 athletes as follows:

```c
DiveType dives[5];
AthleteType athletes[3];
```

Notice that we need to use the `struct` keyword now each time to tell the compiler that this is a struct type. Given that a `pointer` takes 8 bytes in a 64-bit system, do you understand why the following should produce the output shown?

```c
printf("%zu\n", sizeof(DiveType));  // should logically print 12
printf("%zu\n", sizeof(PerformanceType));  // should logically print 44
printf("%zu\n", sizeof(AthleteType));  // should logically print 148
printf("%zu\n", sizeof(dives));  // should logically print 60
printf("%zu\n", sizeof(athletes));  // should logically print 444
```

### Data Structure Byte Alignment:

Although the above size of `DiveType` is logical, it actually is 16!! The compiler will attempt to "align" data values in similar-sized byte chunks. It sometimes will add more bytes within a struct (called `padding`) to make it a multiple of 2, 4, or 8.

Here, for example, is what you will get as an output:

```c
printf("%zu\n", sizeof(DiveType));  // prints 16
printf("%zu\n", sizeof(PerformanceType));  // prints 48
printf("%zu\n", sizeof(AthleteType));  // prints 160
printf("%zu\n", sizeof(dives));  // prints 80
printf("%zu\n", sizeof(athletes));  // prints 480
```

Although the manner in which the bytes are aligned depends on the compiler implementation, the system may tend to look at the largest of the types in the `struct` and decide to add extra padding to make it a multiple of that largest type. Some example are as follows:

<table>
<thead>
<tr>
<th>Definition</th>
<th>struct X {</th>
<th>struct X {</th>
<th>struct X {</th>
<th>struct X {</th>
<th>struct X {</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>char a;</td>
<td>char a;</td>
<td>char a;</td>
<td>char a;</td>
<td>char a;</td>
</tr>
<tr>
<td></td>
<td>char b;</td>
<td>char b;</td>
<td>short b;</td>
<td>short b;</td>
<td>int b;</td>
</tr>
<tr>
<td></td>
<td>char c;</td>
<td>char c;</td>
<td></td>
<td>char c;</td>
<td></td>
</tr>
<tr>
<td>Max type size</td>
<td>char = 1 byte</td>
<td>char = 1 byte</td>
<td>short = 2 bytes</td>
<td>short = 2 bytes</td>
<td>int = 4 bytes</td>
</tr>
<tr>
<td>Padding</td>
<td>none</td>
<td>none</td>
<td>1 extra byte</td>
<td>1 extra byte</td>
<td>3 extra bytes</td>
</tr>
<tr>
<td>sizeof(struct X)</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

The next page has a diagram showing how the structs are contained within one another. The example shows a snapshot in time after **Art** performed three dives, **Dan** performed two and **Jen** one.

Take note of the fact that the **Dives** are created just once, but then may be shared between performances. That is logical, since all athletes, in theory, could perform the exact same dive … but their performances will likely differ in terms of their individual scores. Hence, each athlete has unique performances.
Here is some code that creates an array of 5 fixed dives and an array of 3 athletes. It then generates three performances for each athlete, choosing a random dive each time. The performance is then judged via the production of 8 simulated scores. Finally, the athlete's data is displayed through use of a function that takes an `AthleteType` struct as a parameter.

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

#define NUMBER_OF_SCORES 8
#define NUMBER_OF_PERFORMANCES 3
```

- 80 -
typedef struct {   // Structure that represents Dive data
    char *name;
    int difficulty;
} DiveType;

typedef struct {   // Structure that represents Performance data
    DiveType dive;
    float scores[NUMBER_OF_SCORES];
} PerformanceType;

typedef struct {   // Structure that represents Athlete data
    char *name;
    char *country;
    PerformanceType performances[NUMBER_OF_PERFORMANCES];
} AthleteType;

DiveType dives[5];   // An array to hold the 5 types of dives
AthleteType athletes[3];   // An array to hold the 3 athletes

// Procedure that displays an athlete, his/her performances as well as
// the 8 individual scores for each performance.
void displayAthlete(AthleteType athlete) {
    printf("Athlete: %s from %s:\n", athlete.name, athlete.country);
    for (int p=0; p<NUMBER_OF_PERFORMANCES; p++) {
        printf("  Performed %s (diff: %d): [",
                athlete.performances[p].dive.name,
                athlete.performances[p].dive.difficulty);

        for (int s=0; s<NUMBER_OF_SCORES; s++) {
            printf("%0.2f", athlete.performances[p].scores[s]);
            if (s != NUMBER_OF_SCORES-1)
                printf("", " );
        }
        printf("]\n" );
    }
    printf("\n" );
}

int main() {
    // Fill in the athletes array with 3 athletes
    athletes[0].name = "Art Class";
    athletes[0].country = "Canada";
    athletes[1].name = "Dan Druff";
    athletes[1].country = "Germany";
    athletes[2].name = "Jen Tull";
    athletes[2].country = "U.S.A.";

    // Fill in the dives array with 5 dives
    dives[0].name = "reverse pike";
    dives[0].difficulty = 3;
    dives[1].name = "cannon ball";
    dives[1].difficulty = 1;
dives[2].name = "reverse triple twist";
dives[2].difficulty = 4;
dives[3].name = "forward pike";
dives[3].difficulty = 2;
dives[4].name = "inward straight twist";
dives[4].difficulty = 5;

// Assign random performances to athletes and generate random scores
srand(time(NULL));
for (int a=0; a<3; a++) {
    for (int p=0; p<NUMBER_OF_PERFORMANCES; p++) {
        // choose a random dive for this performance
        athletes[a].performances[p].dive =
            dives[(int)(rand()/(double)RAND_MAX*5)];
        for (int s=0; s<NUMBER_OF_SCORES; s++) {
            // choose a random judge's score
            athletes[a].performances[p].scores[s] =
                rand()/(double)RAND_MAX*10;
        }
    }
}

// Display the results
for (int a=0; a<3; a++)
    displayAthlete(athletes[a]);
return(0);

Here is the output (although the scores will vary randomly) ... does it make sense to you?

**Athlete: Art Class from Canada:**
Performed **inward straight twist** (diff: 5) : [0.1, 6.7, 6.7, 2.1, 1.4, 3.2, 3.2, 3.4]
Performed **reverse pike** (diff: 3) : [5.4, 6.1, 0.6, 7.7, 4.7, 5.6, 9.0, 9.5]
Performed **reverse pike** (diff: 3) : [0.6, 7.8, 1.4, 2.9, 4.7, 2.6, 0.5, 3.3]

**Athlete: Dan Druff from Germany:**
Performed **inward straight twist** (diff: 5) : [4.8, 1.0, 6.2, 4.5, 1.1, 2.9, 1.2, 3.2]
Performed **reverse triple twist** (diff: 4) : [4.4, 6.4, 7.7, 5.5, 1.8, 3.8, 6.1, 9.4]
Performed **inward straight twist** (diff: 5) : [1.7, 8.5, 8.0, 2.4, 9.1, 5.8, 3.9, 3.2]

**Athlete: Jen Tull from U.S.A.:**
Performed **reverse pike** (diff: 3) : [6.4, 2.5, 3.7, 4.7, 7.4, 4.7, 0.9, 1.9]
Performed **reverse triple twist** (diff: 4) : [3.8, 3.1, 9.0, 8.1, 7.5, 5.4, 5.8, 3.0]
Performed **forward pike** (diff: 2) : [9.6, 9.1, 6.6, 8.1, 0.8, 5.0, 6.1, 3.2]

The **typedef** keyword actually allows you to have multiple names (i.e., aliases) for a type. So, for example, you can do this:

```c
typedef AthleteType Athlete;
typedef int StudentNumber;
typedef char Letter;
```
This would allow you to use the types **AthleteType** and **Athlete** interchangeably.

```c
Athlete athletes[20]; // instead of AthleteType athletes[20];
Athlete jen;        // instead of AthleteType jen;
```

And you could use **char** and **Letter** interchangeably as well as **int** and **StudentNumber**:

```c
typedef struct {
    char     *name;
    StudentNumber  studentID; // instead of int studentID;
    Letter    gender;        // instead of char gender;
} StudentType;
```

But there is a need to be cautious with these aliases. The more you make, the harder it is for someone else to know what type a variable is actually storing. Imagine that we have these variables being used in our code:

```c
Name n;
Age a;
Status s;
```

Just looking at this code, we cannot be sure what each of these types are. **Name**, for example, may be a simple **char** * string or it could be a **struct** with multiple attributes. **Age** could be an **int** or perhaps a **float** where we want more accurate ages. And **Status** ... well ... we have no clue what that is. We would need to go looking through all our C code (which may be spread out over many files) in order to hunt down the particular **typedef** definition. So, we should try to minimize the number of aliases that we create so as to make life easier for our fellow programmers who will read and maintain our code at some time in the future.

When creating objects in JAVA, there is a tendency to be wasteful because we don’t usually worry about memory usage and memory allocation/deallocation. The same could be true in C. However, we are normally more space-usage-conscious when programming in the lower-level C language. This can be important in embedded systems where memory is scarce.

One way to be space-efficient, is to allow variables of different types to share the same chunk of memory.

**A union is a value that may have any of several representations or formats within the same position in memory.**

The union itself does not have a specific **type** to it. Rather, it kind of “takes on the role” of one of its member types dynamically. It is similar to the idea of having a chunk of memory changing its type at various times throughout a program’s execution. The main advantage of using unions is to be able to access pieces (or components) of a type value. In C, defining a union is as easy as replacing the word **struct** by **union**.

Consider this **union**:
typedef union {
    int    i;
    float  f;
    char   c[5];
    double d;
} Data;

How much memory space does this union take up? If it were a struct instead of a union, it would require \((4 + 4 + 5 + 8 + 3)\) bytes (padding) = 24 bytes. However, when a union is used, since all the members of the union overlap one another in memory, it only takes up \(\text{MAX}(4, 4, 5, 8) = 8\) bytes.

```c
printf("sizeof(Data): %zu\n", sizeof(Data)); // returns 8
```

The downside, is that we only get to use one member at a time:

```c
Data    data;

data.i = 678;
printf("data.i is: %d\n", data.i); // 678
data.f = 3.1415;
printf("data.f is: %f\n", data.f); // 3.141500
strcpy(data.c, "Hi!");
printf("data.c is: %s\n", data.c); // Hi!

printf("data.i has been overwritten: %d\n", data.i); // 2189640
printf("data.f has been overwritten: %f\n\n", data.f); // 0.000000
```

Notice that once we give a value to any member of the union, all other union member’s data is overwritten, and will therefore be invalid.
There are, however, some uses of **unions** in which we don’t care about the memory being overwritten. Consider this example which is useful for determining the bytes that make up an integer:

```c
typedef union {
    unsigned int value;
    unsigned char bytes[4];
} DecomposableInteger;
```

```c
DecomposableInteger number;
number.value = 584340;
printf("%d, %d, %d, %d\n", number.bytes[0], number.bytes[1],
number.bytes[2], number.bytes[3]);
```

The code allows you to treat the number either as an **unsigned int**, or as an array of 4 **bytes**. So, in this example, we put a large number (i.e., 584340) into the union. Then, we extract it as the 4 bytes that are used to store the number. The resulting output is:

```
148, 234, 8, 0
```

Which is correct, since \(148 \times 2^0 + 234 \times 2^8 + 8 \times 2^{16} + 0 \times 2^{32} = 148 + 59,904 + 524,288 = 584340\)

In this example, the least significant byte is first in the sequence of bytes. This ordering is called **Little Endian Byte Order**. The least significant byte (the "little end") of the data is placed at the byte with the lowest address. The rest of the data is placed in order in the next three bytes in memory. A different ordering is the **Big Endian Byte Order**. In that case, the most significant byte (the "big end") of the data is placed at the byte with the lowest address.

Endianness is a property of the CPU, not of the operating system. If you want to determine the ordering of the bytes on your machine, you can type `lscpu` into the terminal window:

```
student@COMPBase:~$ lscpu
Architecture:        x86_64
CPU op-mode(s):      32-bit, 64-bit
Byte Order:          Little Endian
CPU(s):              1
On-line CPU(s) list: 0
Thread(s) per core:  1
Core(s) per socket:  1
Socket(s):           1
NUMA node(s):        1
Vendor ID:           GenuineIntel
CPU family:          6
Model:               30
Model name:          Intel(R) Core(TM) i7 CPU 870 @ 2.93GHz
Stepping:            5
CPU MHz:             2926.002
BogoMIPS:            5852.00
Hypervisor vendor:   KVM
Virtualization type: full
```
Here is another example that lets us store a phone number string in format (xxx)xxx-xxxx and then allows us to extract the individual pieces:

```c
#include <stdio.h>
#include <string.h>

typedef union {
    char whole[14]; // enough to store (613)220-2600\0
    struct {
        char openPar;
        char area[3];
        char closePar;
        char prefix[3];
        char dash;
        char lineNum[4];
        char null;
    } parts;
} PhoneNumber;

int main() {
    PhoneNumber myNumber;

    strcpy(myNumber.whole, "(613)520-2600");
    printf("original num: %s\n", myNumber.whole);

    strcpy(myNumber.parts.area, "416");
    strcpy(myNumber.parts.prefix, "555");
    strcpy(myNumber.parts.lineNum, "8888");
    myNumber.parts.null = 0;

    printf("whole num: %s\n", myNumber.whole); // corrupted now
    printf("areaCode: %s\n", myNumber.parts.area);
    printf("prefix: %s\n", myNumber.parts.prefix);
    printf("lineNumber: %s\n", myNumber.parts.lineNum);
    return 0;
}
```
The output is as follows:

- original num: (613) 520-2600
- whole num: (416)
- areaCode: 416
- prefix: 555
- lineNumber: 8888

Notice that once we alter the original number, we cannot display it using the whole field. That is because when we wrote the individual parts.area, parts.prefix and parts.lineNum, each time it added a "0" null terminating character:

- after writing whole: \((6|1|3)|520|-2600\)
- after writing area: \((4|1|6|0)|520|-2600\)
- after writing prefix: \((4|1|6|0)|5|5|5|0|2|6|0|0|\)
- after writing lineNumber: \((4|1|6|0)|5|5|5|0|8|8|8|8|\)

In general, unions should be used sparingly, as it complicates what is happening in your program and could make things genuinely confusing for the typical programmer when many unions are used since at any instant of time, we may not be sure which part of the union was written last. Therefore, we may make wrong assumptions and end up with wrong data.