Test 1: COMP 4804 A

Instructor: A.M.

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1 Problems - Maximum Marks 100

Provide necessary details and write clearly please. All questions carry equal marks.

1. We toss a fair coin 10 times. Compute the probability of the following events.

   (a) Obtaining at least five heads.

   (b) Obtaining same number of heads and tails.

   (c) Obtaining at least eight consecutive heads.

   (d) Obtaining at least as many tails as heads.

2. Suppose \( n \) musically inclined people go to a concert and check in their jackets at the customer service. Assume that no two jackets are the same. After the concert, customer service agent hands the jackets back to customers, but in a completely random order. Therefore a customer may go home with a completely random jacket! What is the expected number of customers who got back their own jackets? (An alternative way to think about this problem is to find the expected number of elements in a random permutation \( \pi \) of \( \{1, \ldots, n\} \) with the property that the \( i \)-th element of \( \pi \) is \( i \), i.e. \( \pi[i] = i \).)

3. Consider the following experiment. Suppose you have a large collection of identical marbles, and you have \( n \)-bins. Initially all bins are empty. In this experiment you repeatedly pick a marble from the collection, and throw it in a random bin (i.e. all bins are equally likely, irrespective of whether they already contain none or some marbles). The experiment is terminated when for the first time all bins are occupied. Determine, on the average how many marbles you need to throw?

4. Let \( A = (a_1, a_2, \ldots, a_n) \) be a sequence of \( n \) distinct numbers. We say that \( a_i \) and \( a_j \) is an inversion if \( i < j \) and \( a_i > a_j \). (Intuitively, it means that they are not in the correct order if we are interested in sorting \( A \).) Suppose that \( A \) is a random permutation of \( \{a_1, a_2, \ldots, a_n\} \). First determine the expected number of inversions in \( A \). Given this number, can you say something about the lower bound on the runtime of the bubble sorting algorithm?