Quadtrees and Their Applications

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Overview



- Point-Region Quadtrees
- Quadtrees for Polygons
- Queries
- 4 Compressed Quadtrees
- 5 Mesh Generation
 - Balanced Quadtrees
 - $\bullet \ \mathsf{Quadtrees} \to \mathsf{Triangulations}$

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Image Processing Spatial Queries Compressed Quadtrees Mesh Generation

Basic Idea

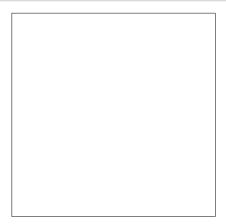


Figure 1 : The quadtree: a hierarchical spatial data structure

Image Processing Spatial Queries Compressed Quadtrees Mesh Generation

Basic Idea

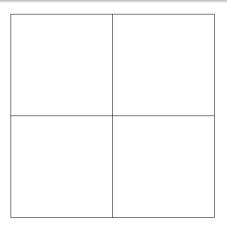


Figure 1 : The quadtree: a hierarchical spatial data structure

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Image Processing Spatial Queries Compressed Quadtrees Mesh Generation

Basic Idea

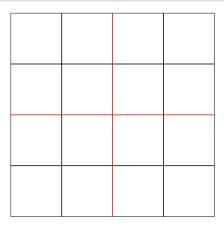


Figure 1 : The quadtree: a hierarchical spatial data structure

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Image Processing Spatial Queries Compressed Quadtrees Mesh Generation



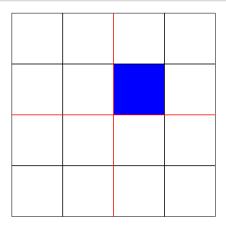


Figure 1 : The quadtree: a hierarchical spatial data structure

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Space-Filling Curves

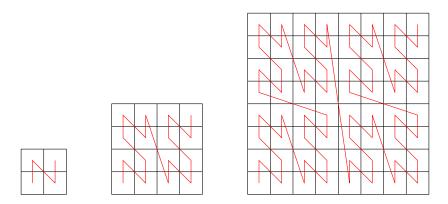


Figure 2 : Z-curves for 2x2, 4x4, and 8x8 grids [1, 3]

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Space-Filling Curves

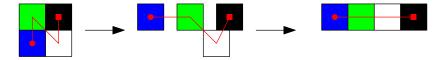


Figure 3 : Z-curve defines a total order on the cells

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Figure 4 : Converting an image into a region quadtree

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Region Quadtrees

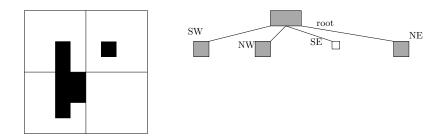


Figure 4 : Converting an image into a region quadtree

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Region Quadtrees

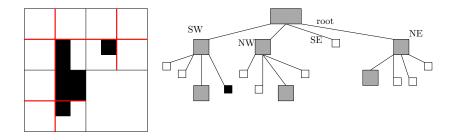


Figure 4 : Converting an image into a region quadtree

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Region Quadtrees

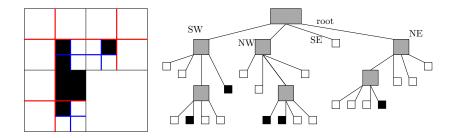
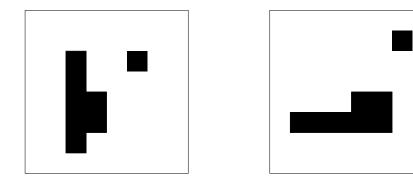


Figure 4 : Converting an image into a region quadtree

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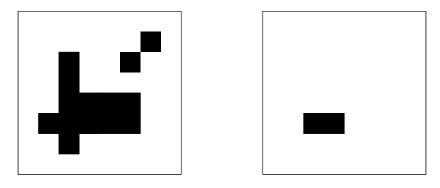
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Image Union/Intersection



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Image Union/Intersection



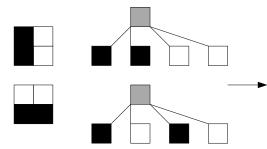
Union

Intersection

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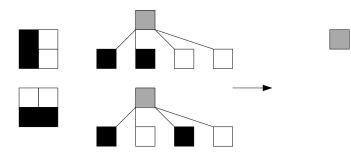
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Image Union/Intersection



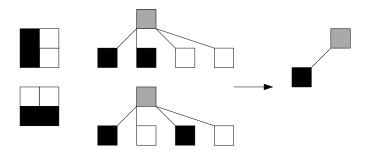
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Image Union/Intersection



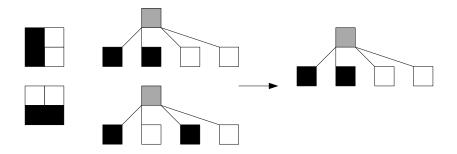
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Image Union/Intersection



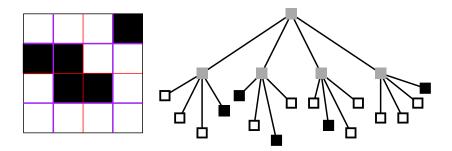
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Image Union/Intersection



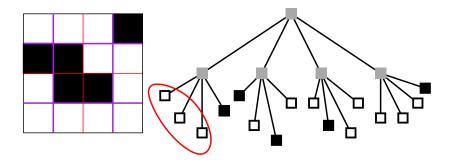
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Connected Component Labeling



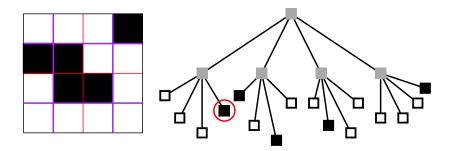
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Connected Component Labeling



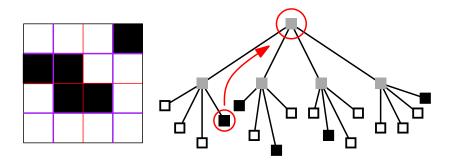
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Connected Component Labeling



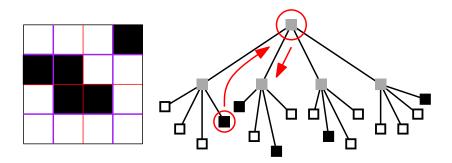
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Connected Component Labeling



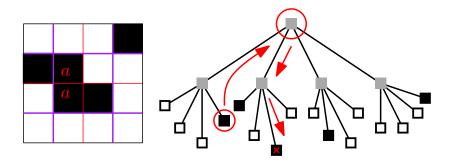
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Connected Component Labeling



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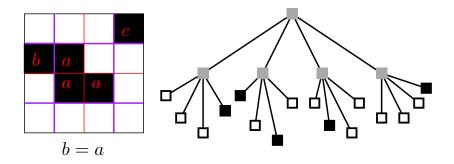
Connected Component Labeling



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Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Point Quadtrees

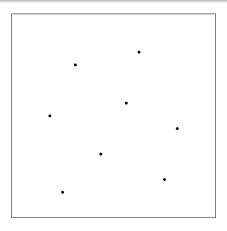


Figure 5 : Dividing up points for a point quadtree

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Point Quadtrees

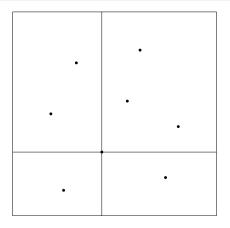


Figure 5 : Dividing up points for a point quadtree

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Point Quadtrees

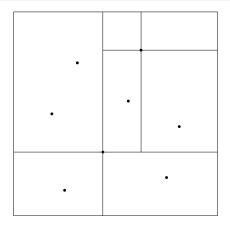


Figure 5 : Dividing up points for a point quadtree

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Point Quadtrees

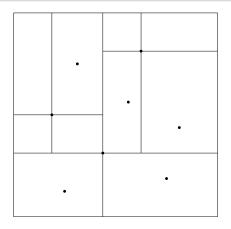


Figure 5 : Dividing up points for a point quadtree

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Point Quadtrees

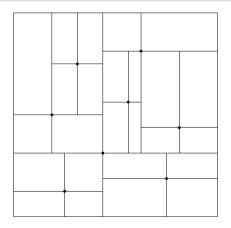


Figure 5 : Dividing up points for a point quadtree

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Point-Region Quadtrees

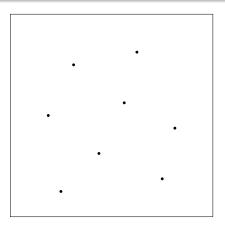


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Point-Region Quadtrees

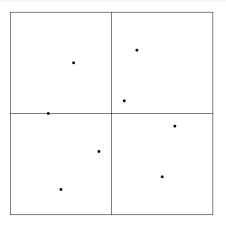


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Point-Region Quadtrees

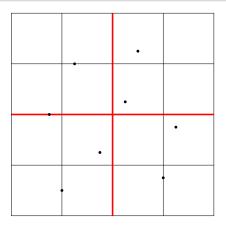


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

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Point-Region Quadtrees

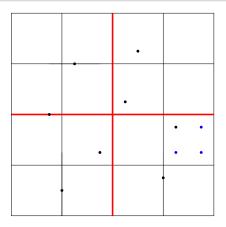


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

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Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Point-Region Quadtrees

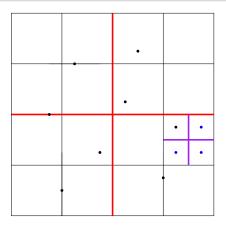
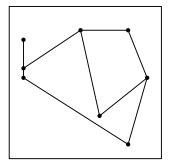


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

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Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Edge Quadtrees



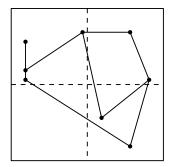
 \rightarrow Subdivide until a single line segment per square (or a corner)

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Figure 7 : Example of capturing linear features in an edge quadtree [7]

Point Quadtrees Point-Region Quadtrees Quadtrees for Polygons Queries

Edge Quadtrees

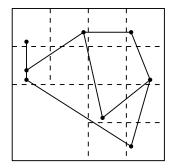


 \rightarrow Subdivide until a single line segment per square (or a corner)

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Edge Quadtrees

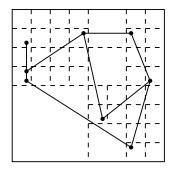


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Edge Quadtrees

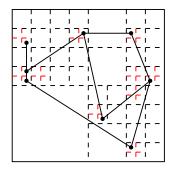


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Edge Quadtrees

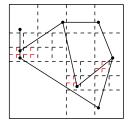


 \rightarrow Subdivide until a single line segment per square (or a corner)

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Polygonal Map (PM) Quadtrees



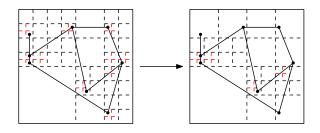
 \rightarrow Subdivide until a single line segment per square... \rightarrow ...unless the segments meet at a vertex in the block

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Figure 8 : Example of capturing features in a PM quadtree [7]

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Polygonal Map (PM) Quadtrees



 \rightarrow Subdivide until a single line segment per square... \rightarrow ...unless the

segments meet at a vertex in the block

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Figure 8 : Example of capturing features in a PM quadtree [7]

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Point Location

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Range Query

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Spherical Region Query

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kNN Query

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Compressed Quadtrees

This section from [2] and [3]

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Dynamic Compressed Quadtrees

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Balanced Quadtrees Quadtrees \rightarrow Triangulations

Mesh Generation

This section comes from chapter 14 of [4] and chapter 12 of [2]

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Balanced Quadtrees

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Building the Balanced Quadtrees

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Balanced Quadtrees Quadtrees \rightarrow Triangulations

$\mathsf{Quadtrees} \to \mathsf{Triangulations}$

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Quadtrees and octrees

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[6] Samet (1988)

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Balanced Quadtrees Quadtrees \rightarrow Triangulations

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