Quadtrees and Their Applications

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Overview

1. Quadtrees
2. Image Processing
3. Spatial Queries
   - Point Quadtrees
   - Point-Region Quadtrees
   - Quadtrees for Polygons
   - Queries
4. Compressed Quadtrees
5. Mesh Generation
   - Balanced Quadtrees
   - Quadtrees $\rightarrow$ Triangulations

Basic Idea

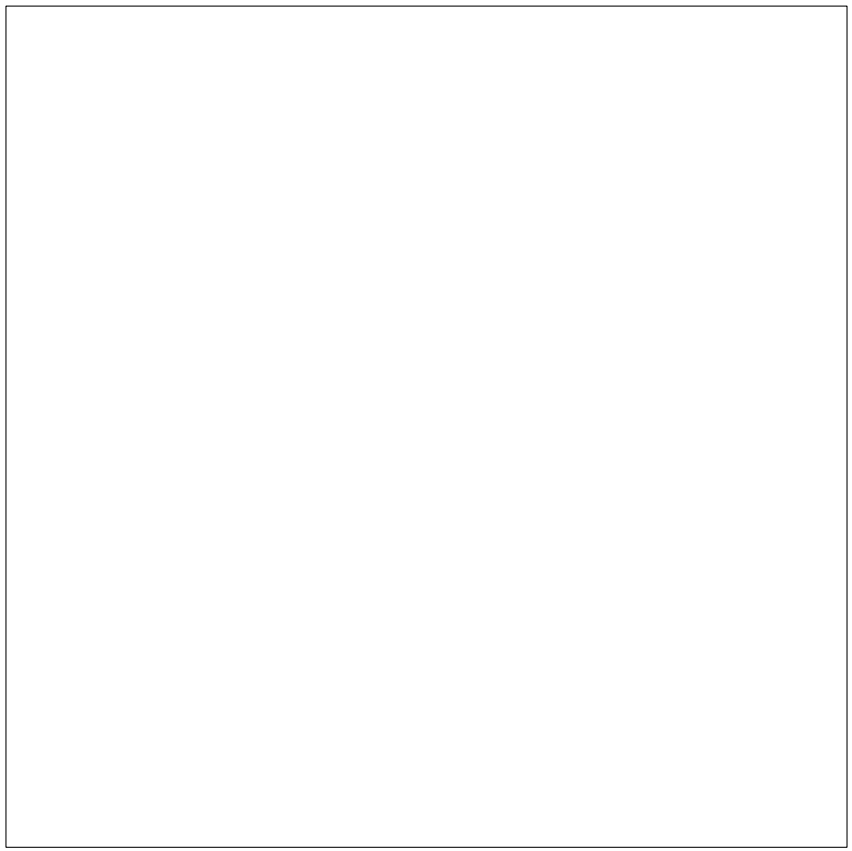

Figure 1 : The quadtree: a hierarchical spatial data structure

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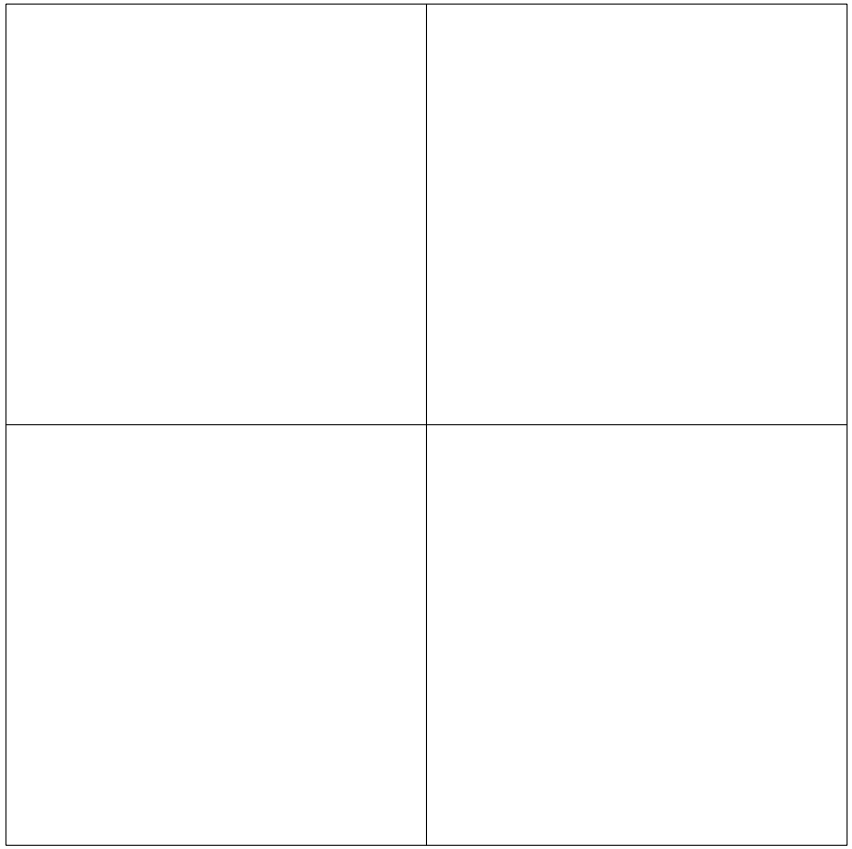

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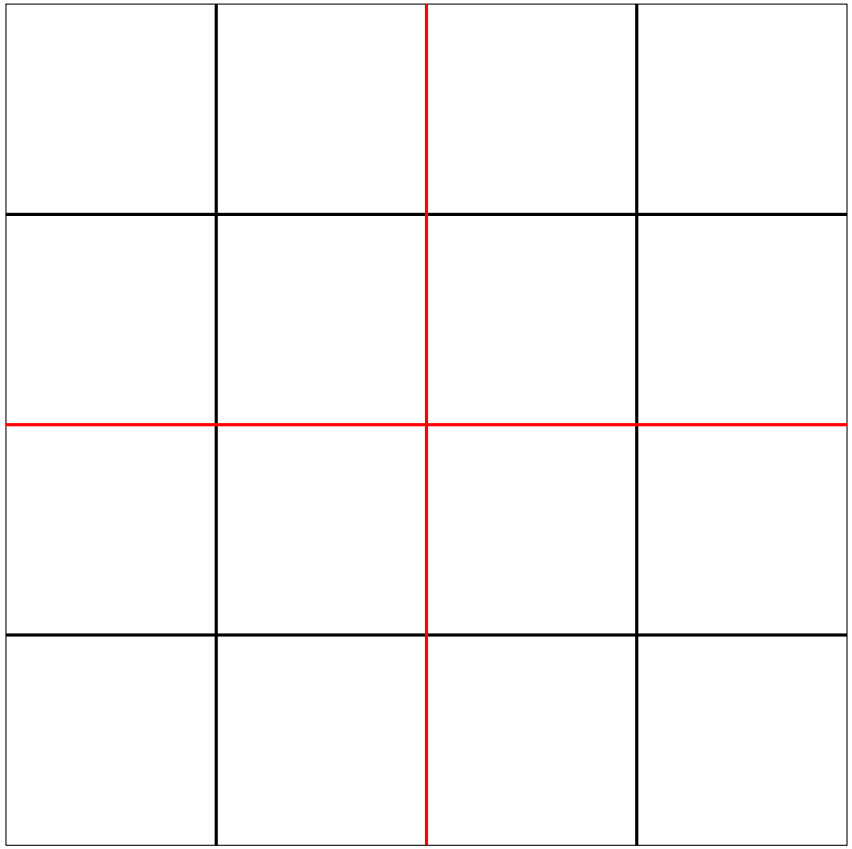

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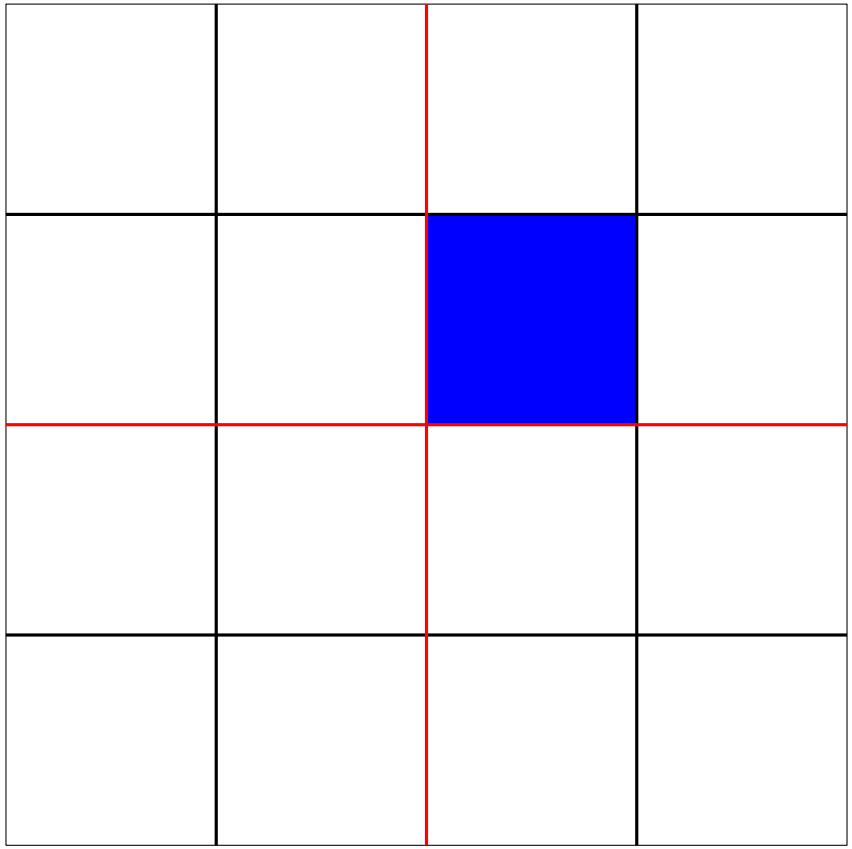

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Space-Filling Curves

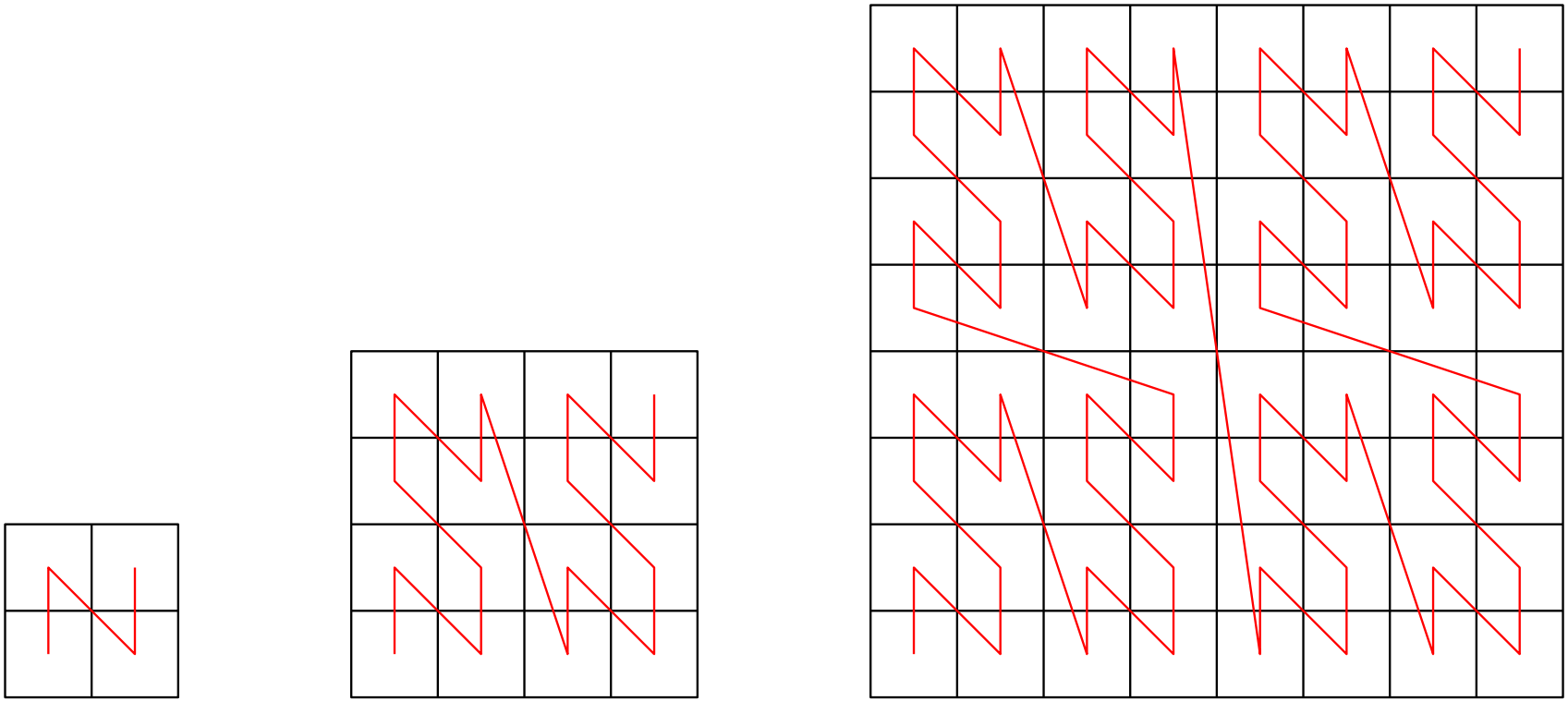


Figure 2 : Z-curves for 2x2, 4x4, and 8x8 grids [1, 3]

Space-Filling Curves

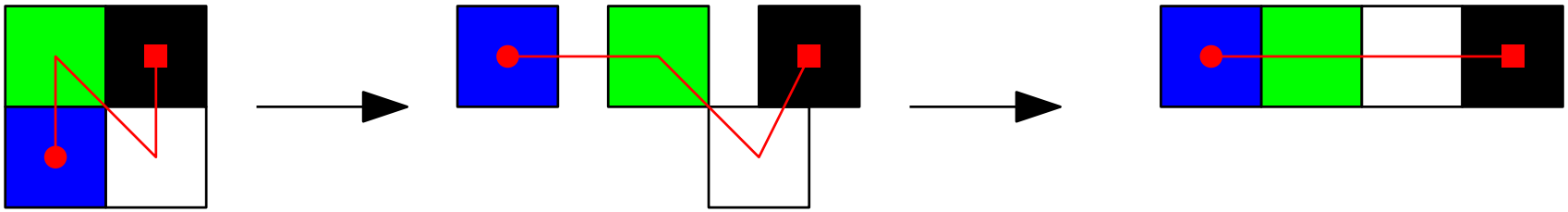
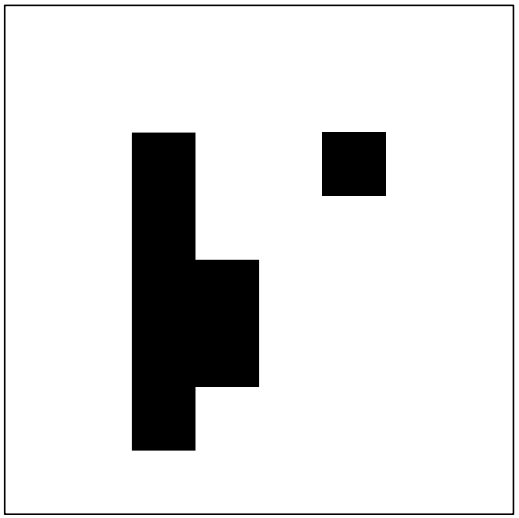


Figure 3 : Z-curve defines a total order on the cells

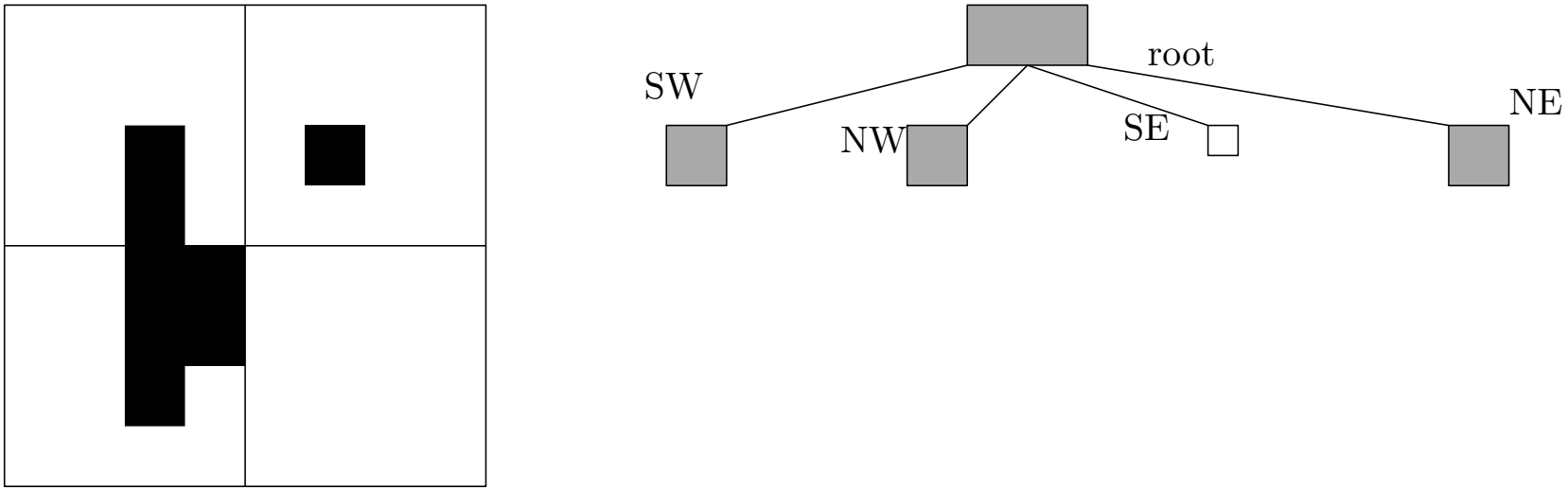
Region Quadtrees



root

Figure 4 : Converting an image into a region quadtree

Region Quadtrees



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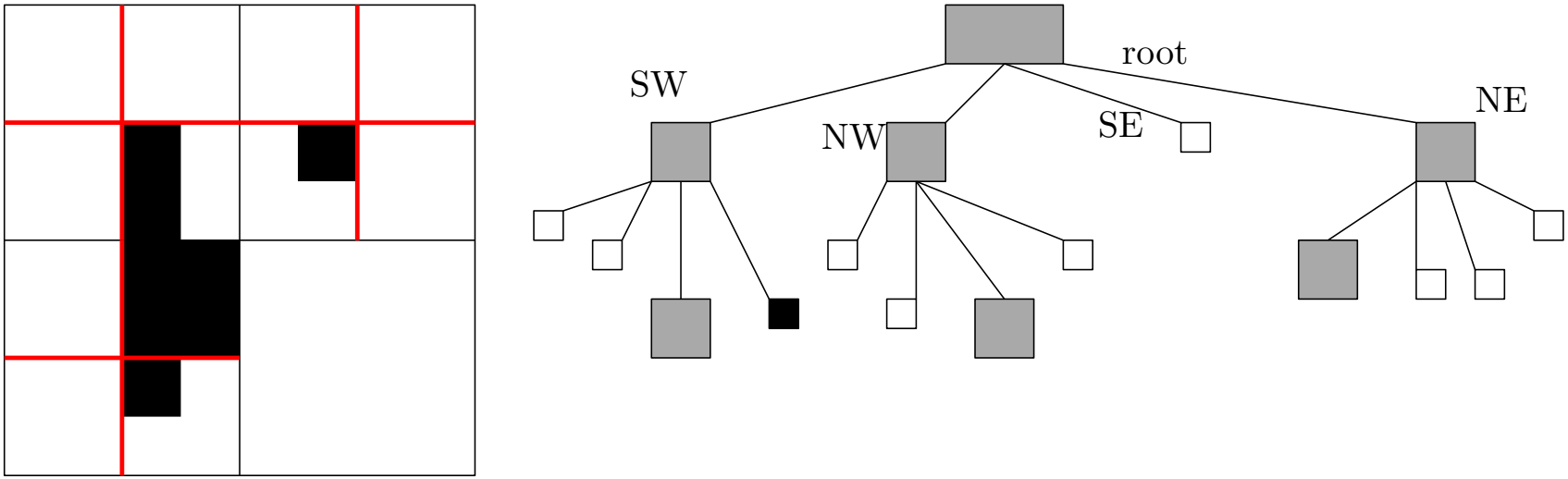

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Region Quadtrees

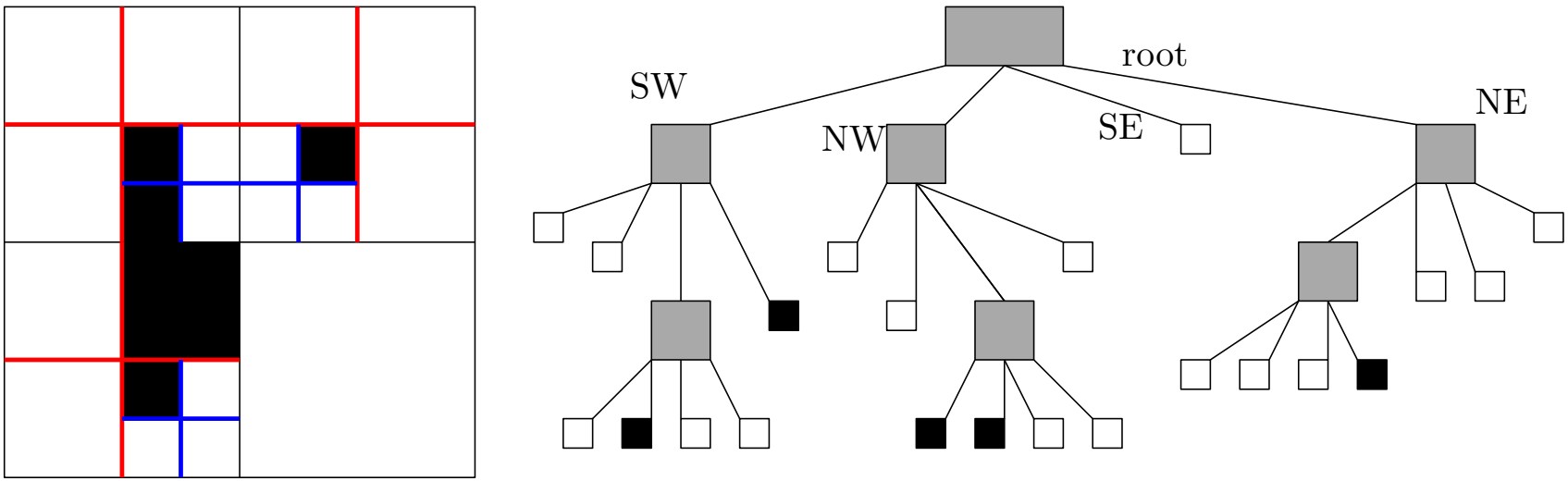

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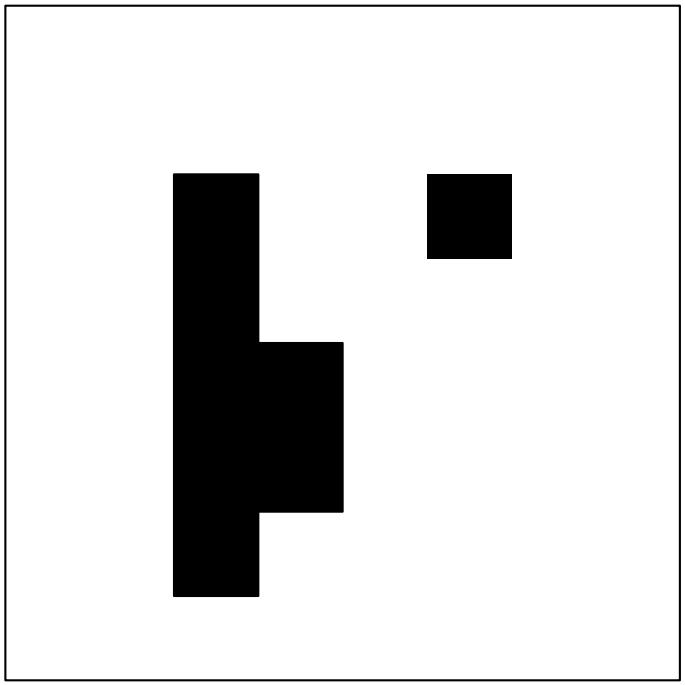
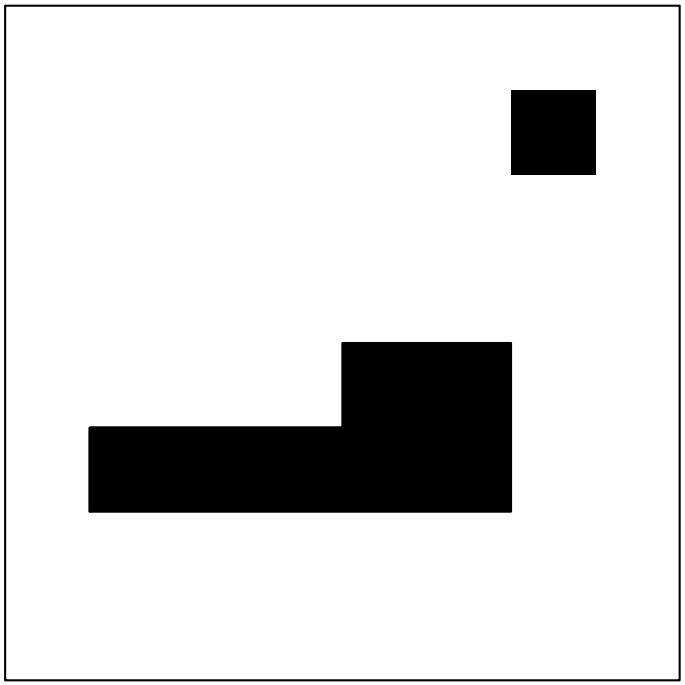
Image Union/Intersection





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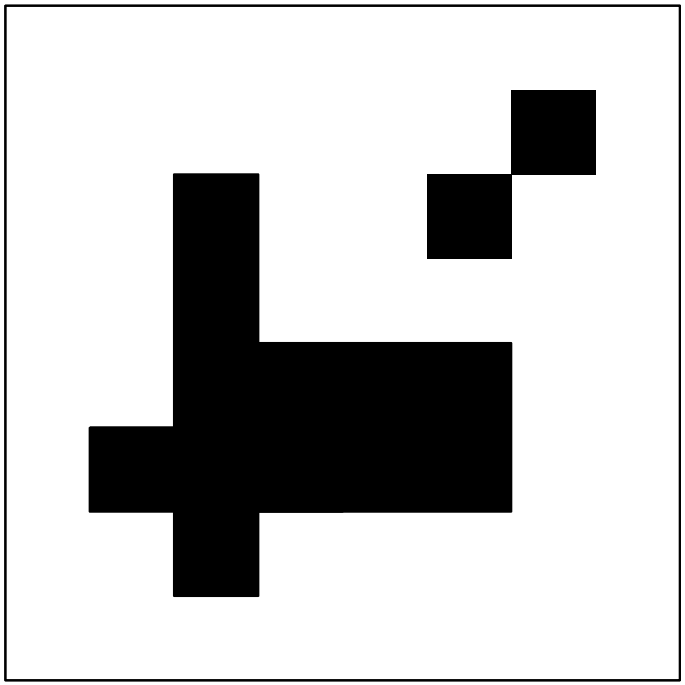
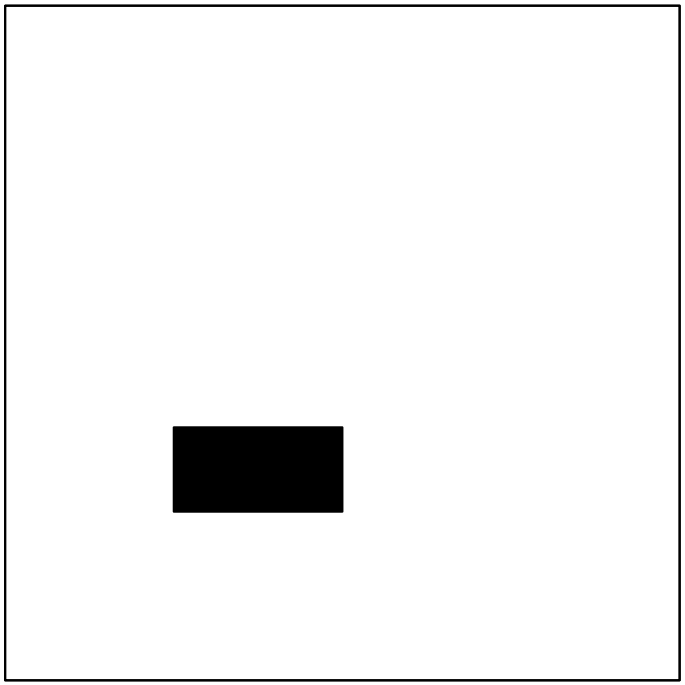
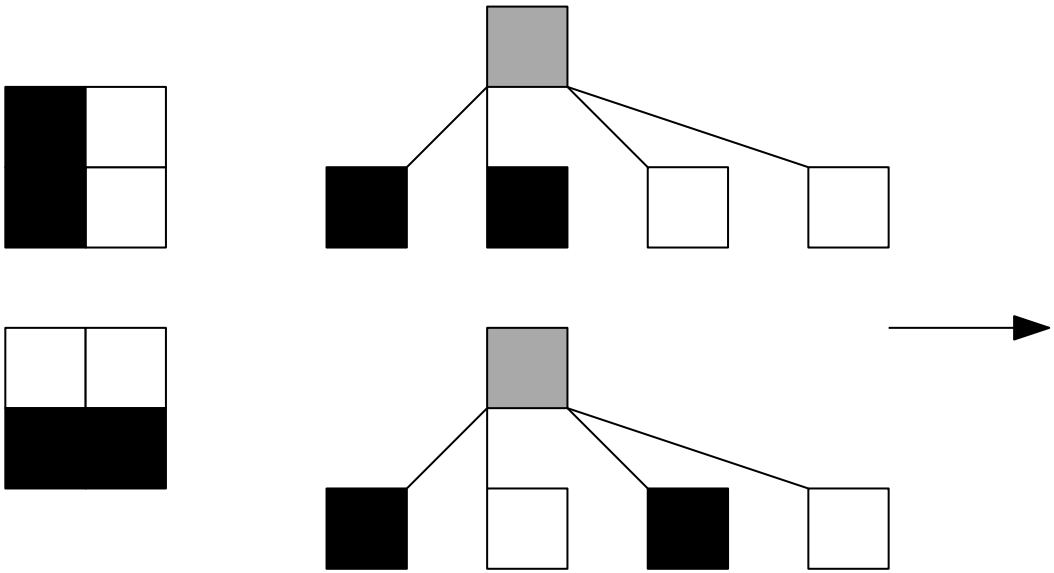


Union



Intersection

Image Union/Intersection



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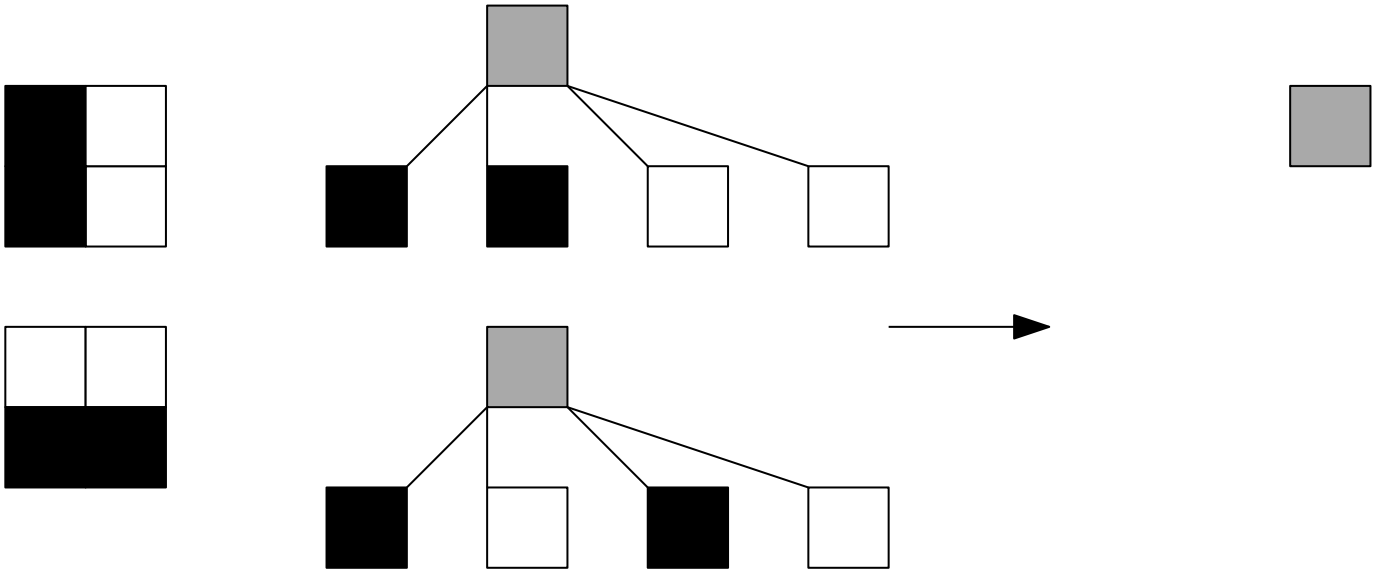

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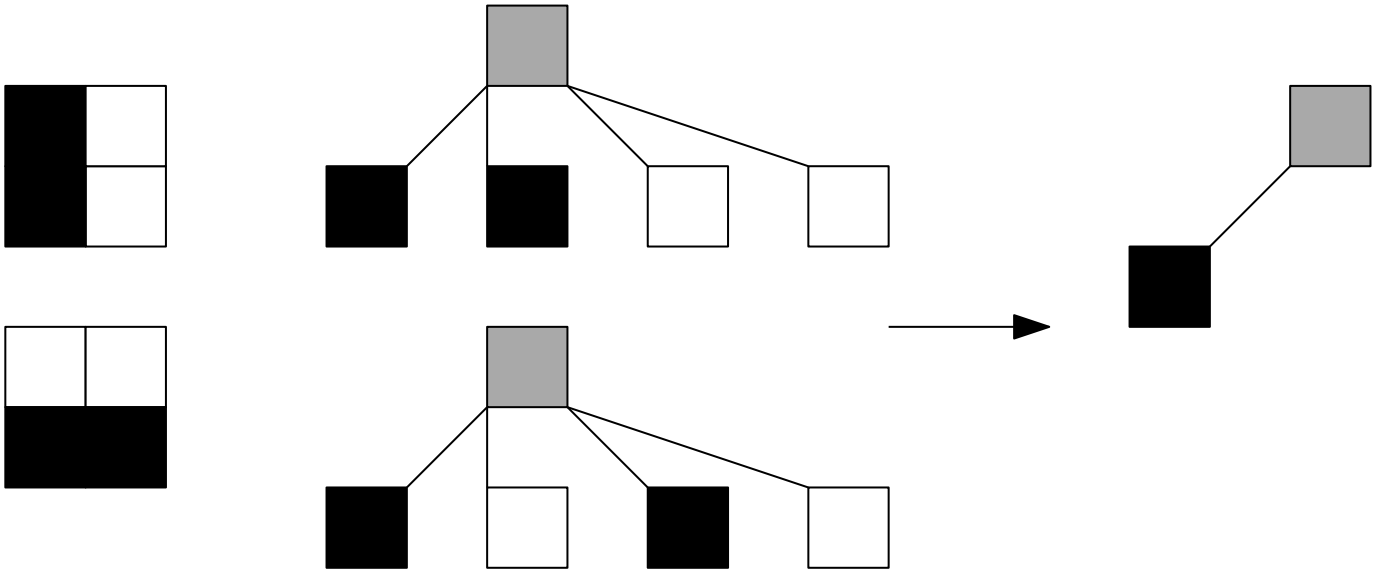

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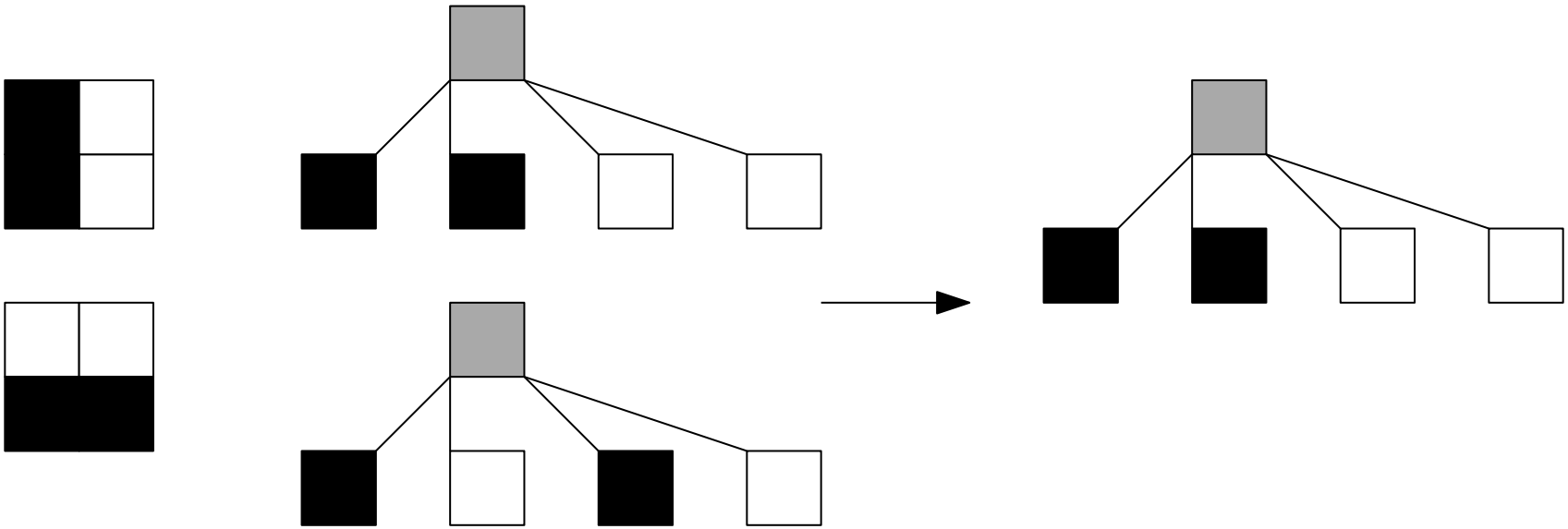
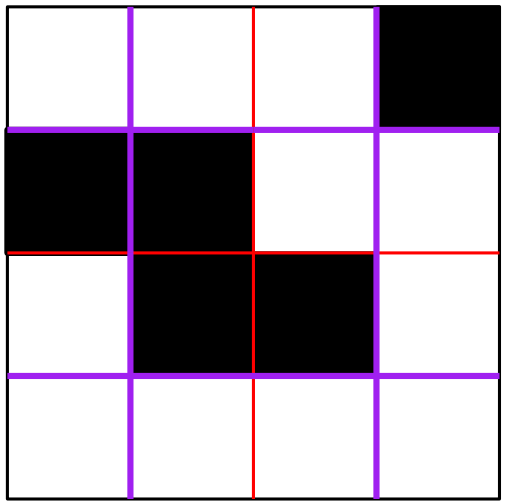
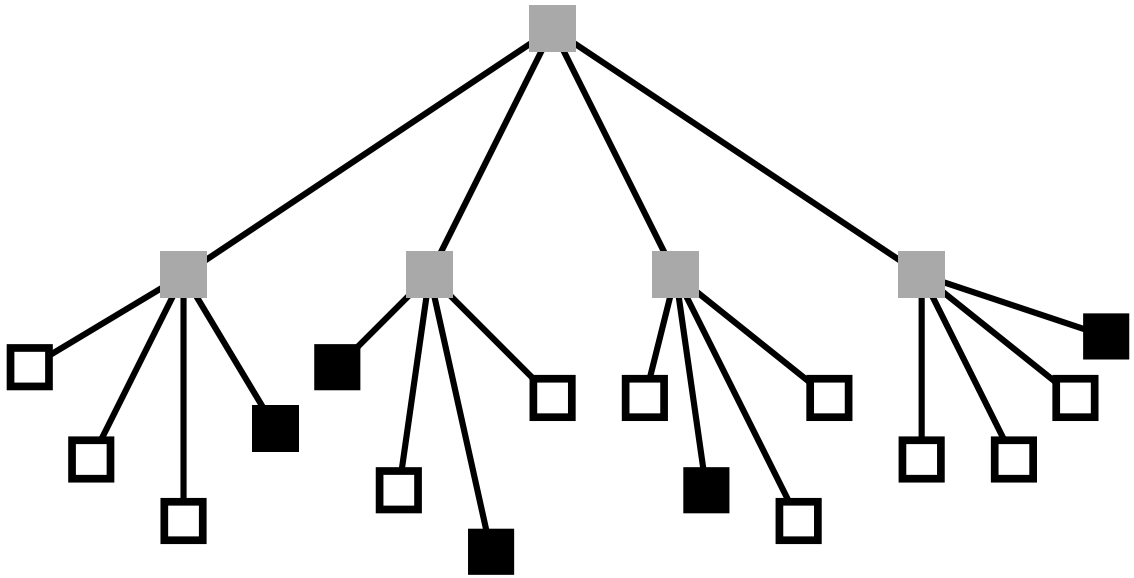


Connected Component Labeling





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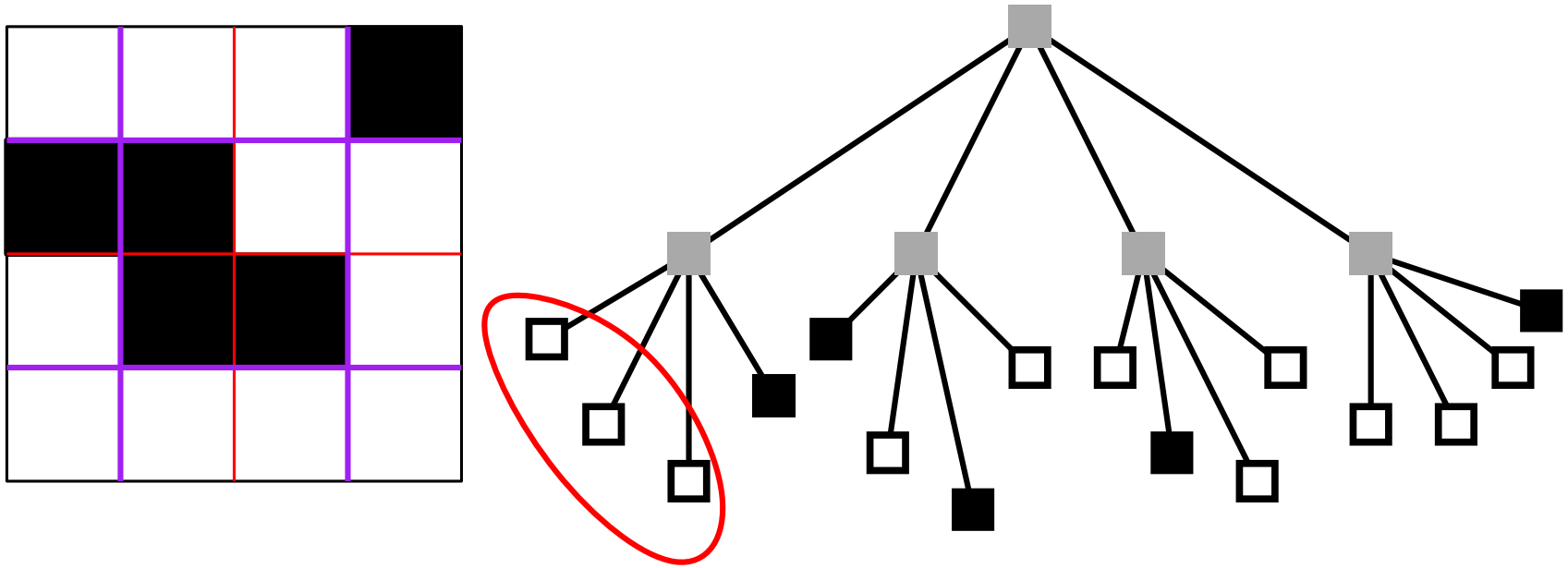

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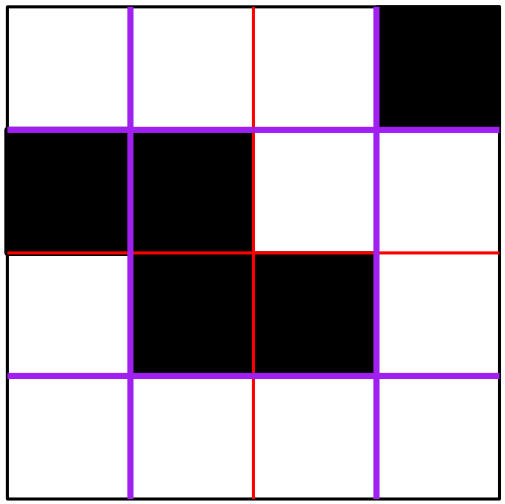

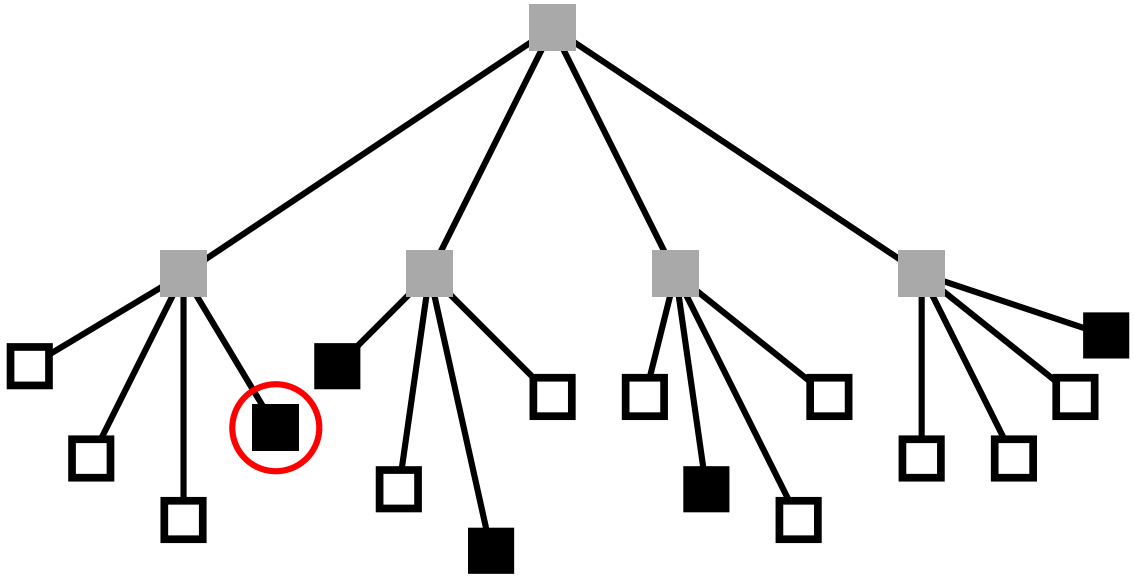

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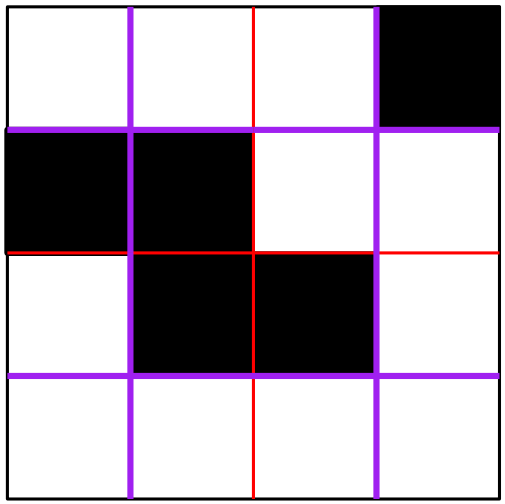

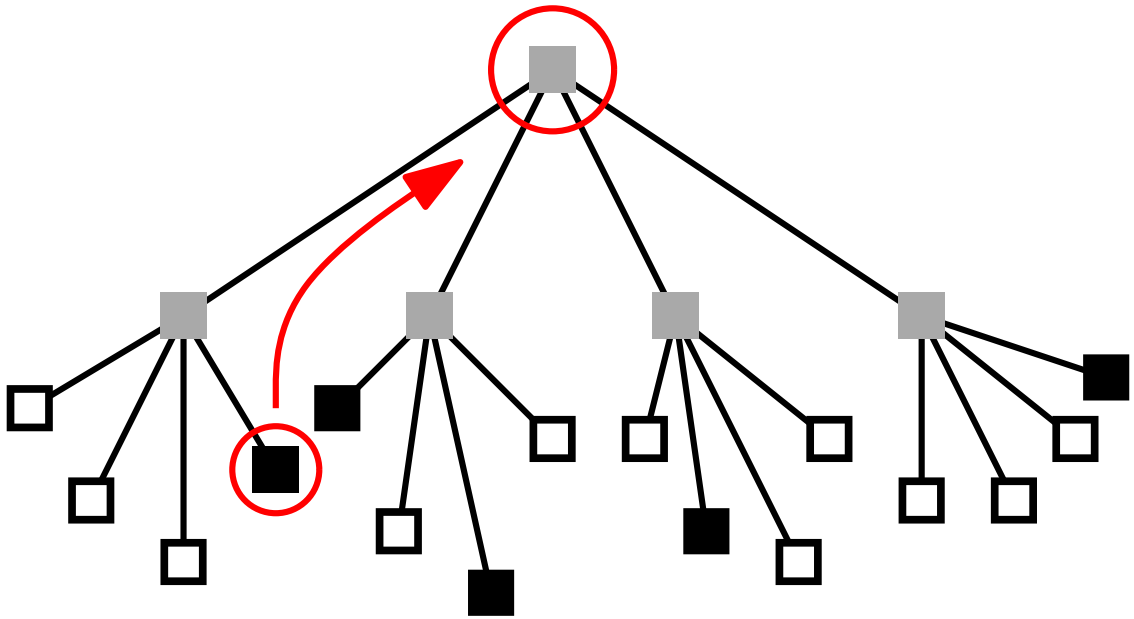

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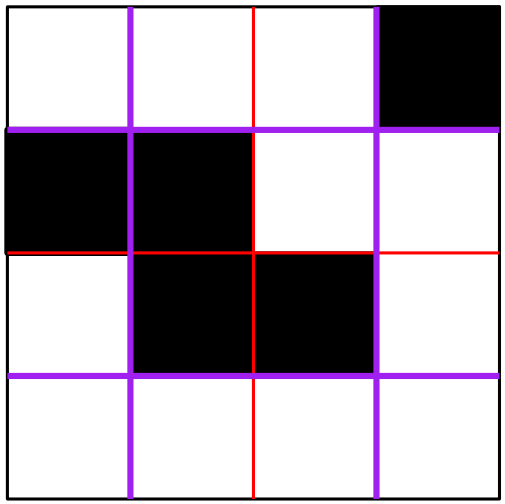

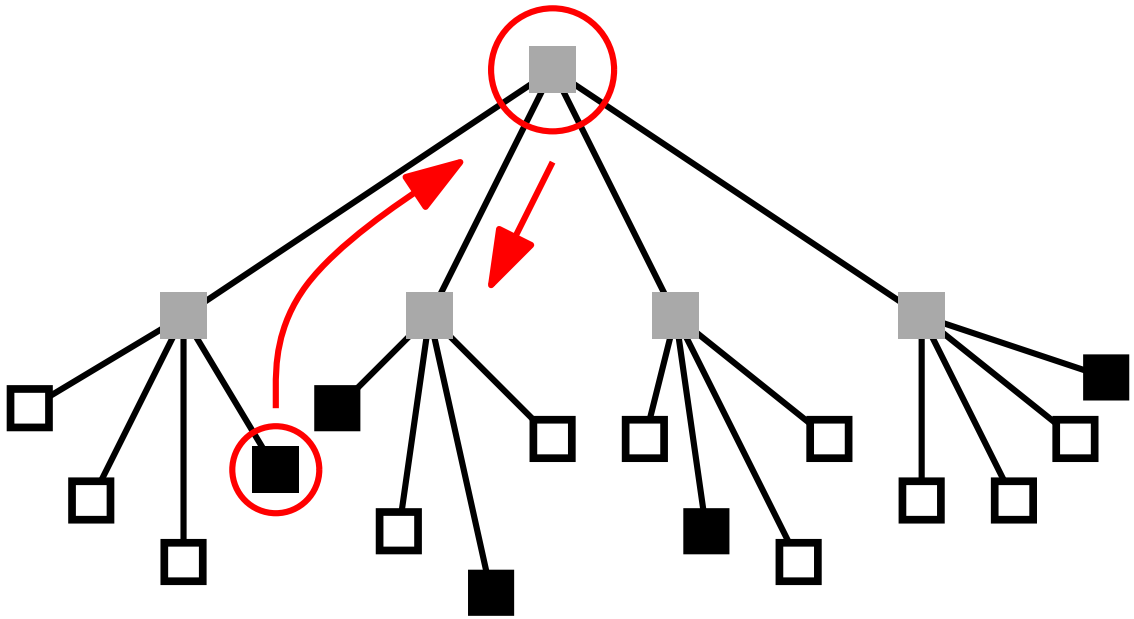

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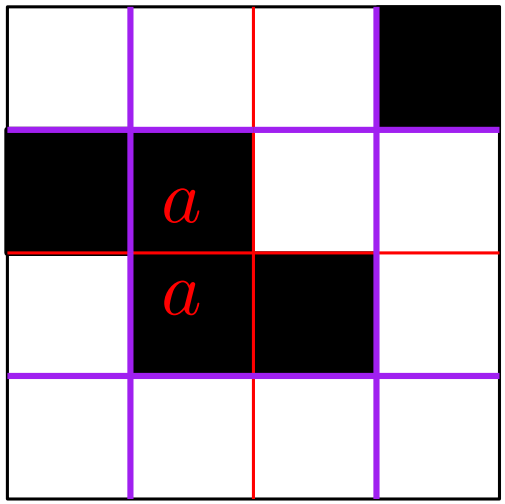

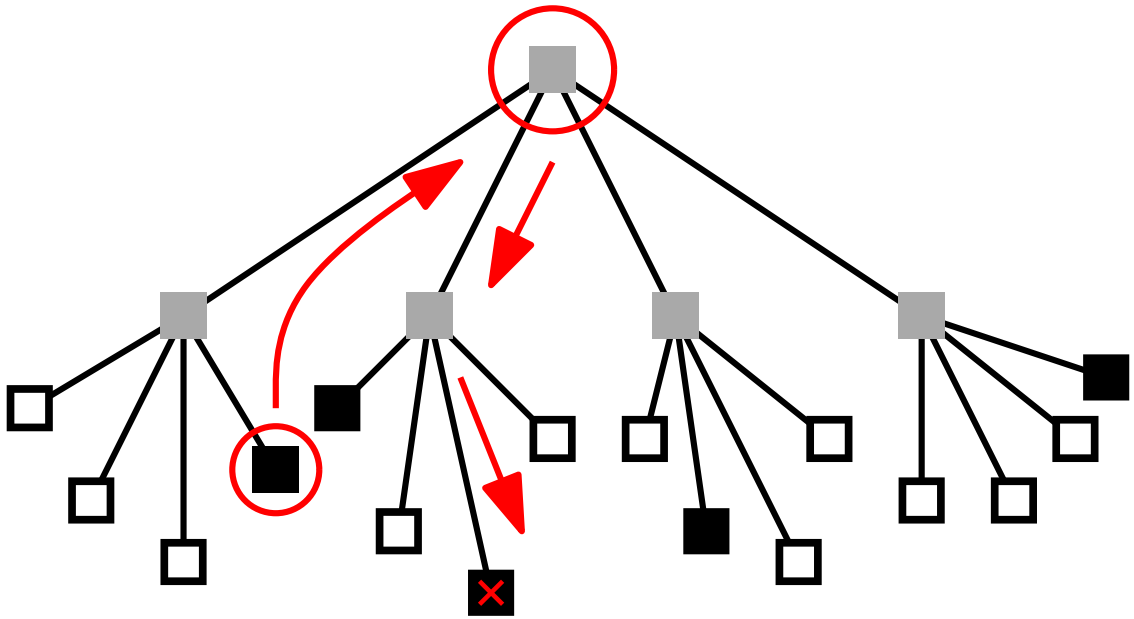

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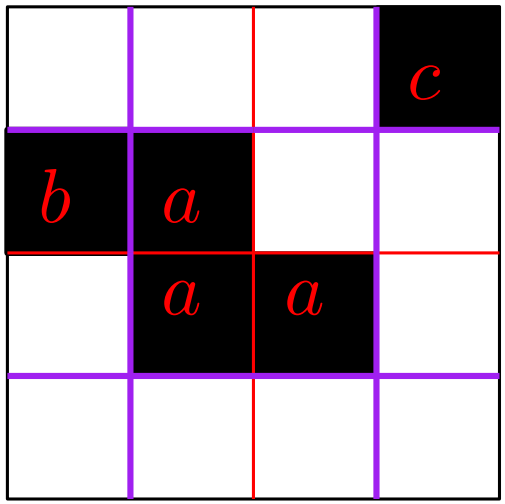

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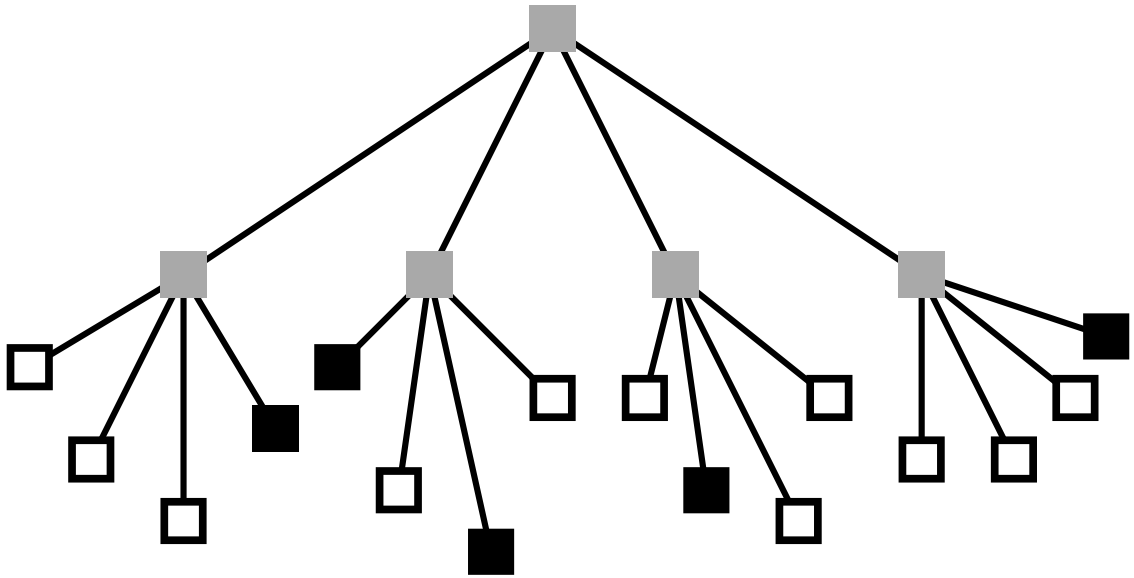


Point Quadtrees

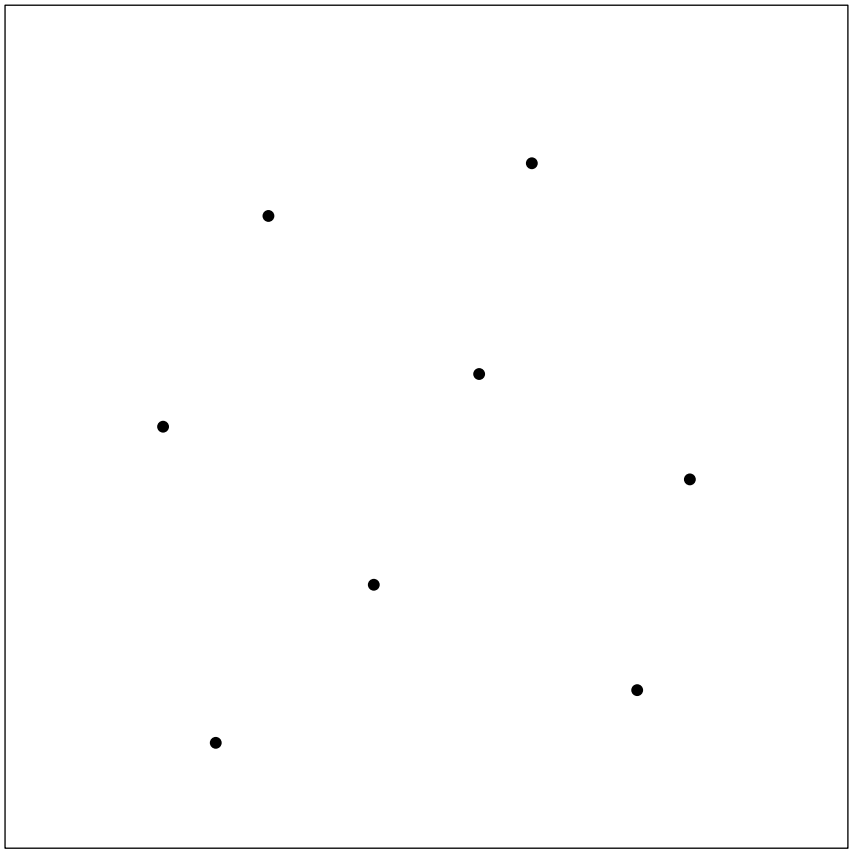
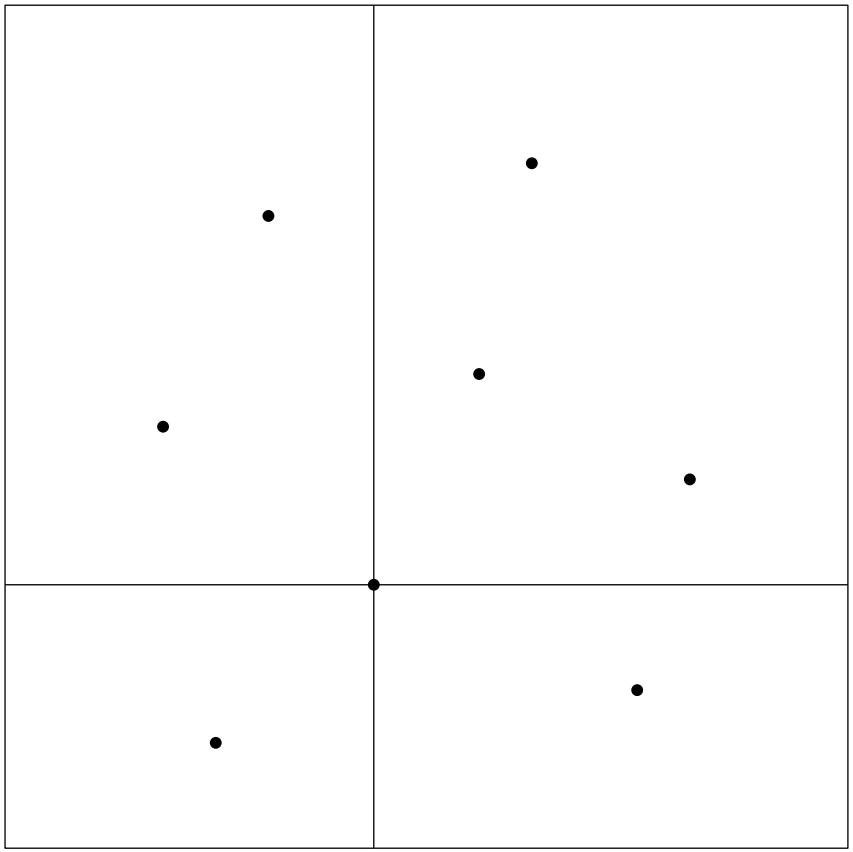


Figure 5 : Dividing up points for a point quadtree

Point Quadtrees



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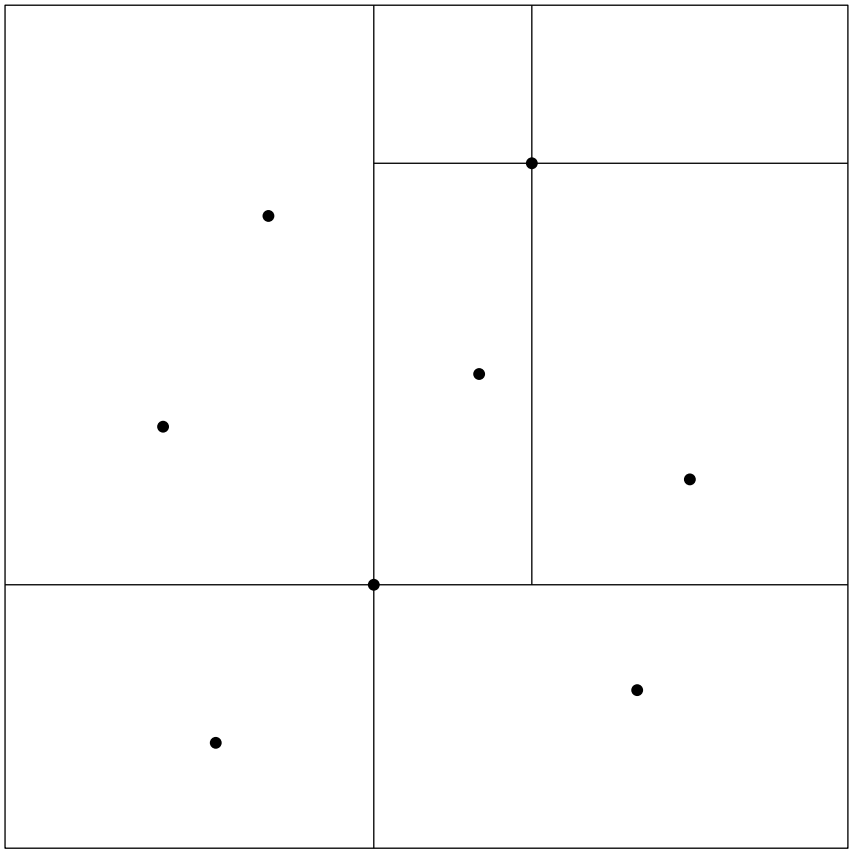

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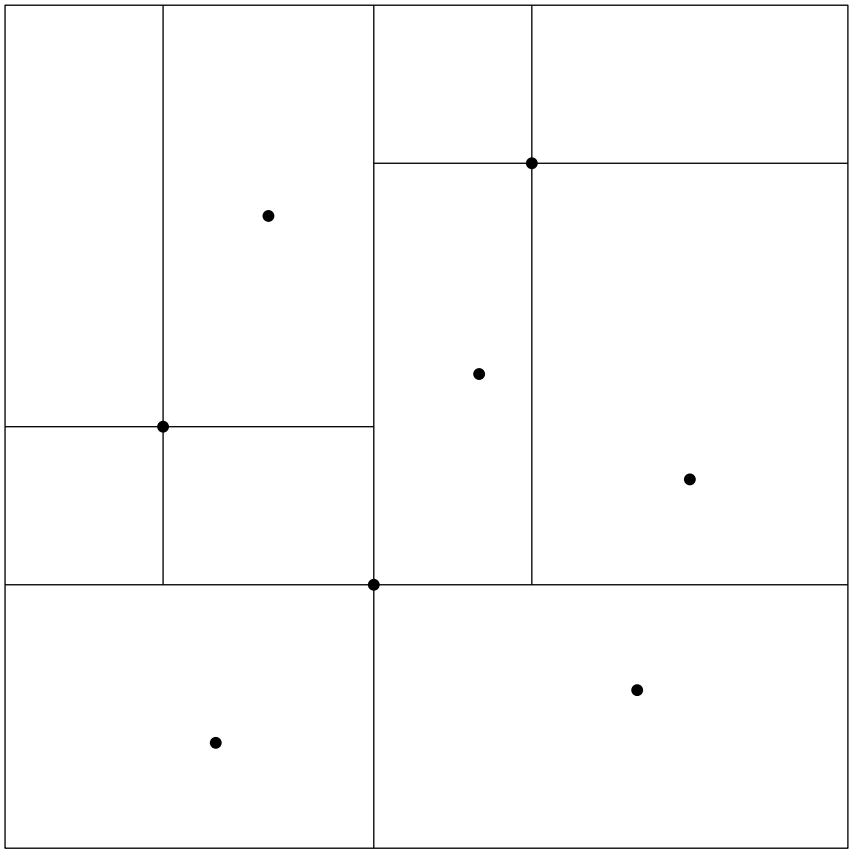

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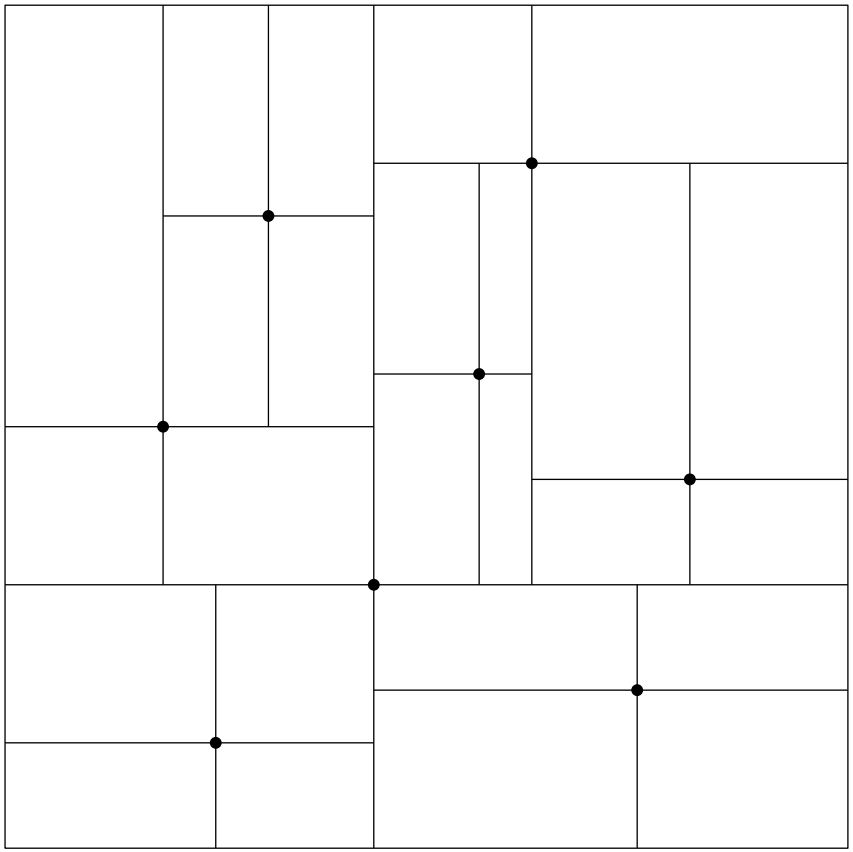
Point Quadtrees



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Point-Region Quadtrees

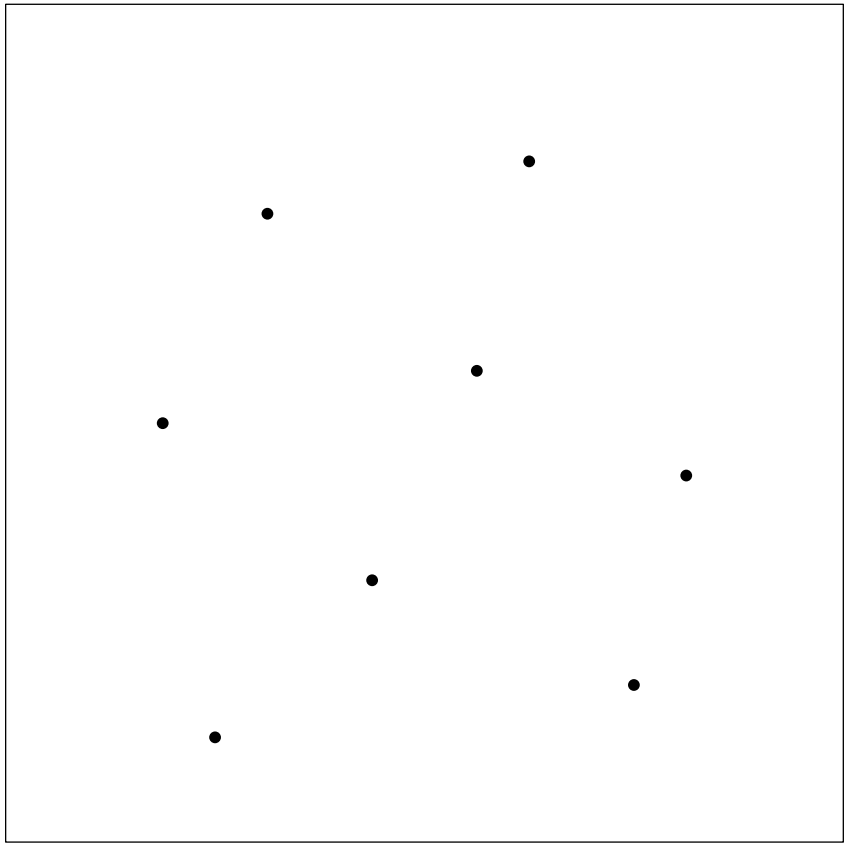
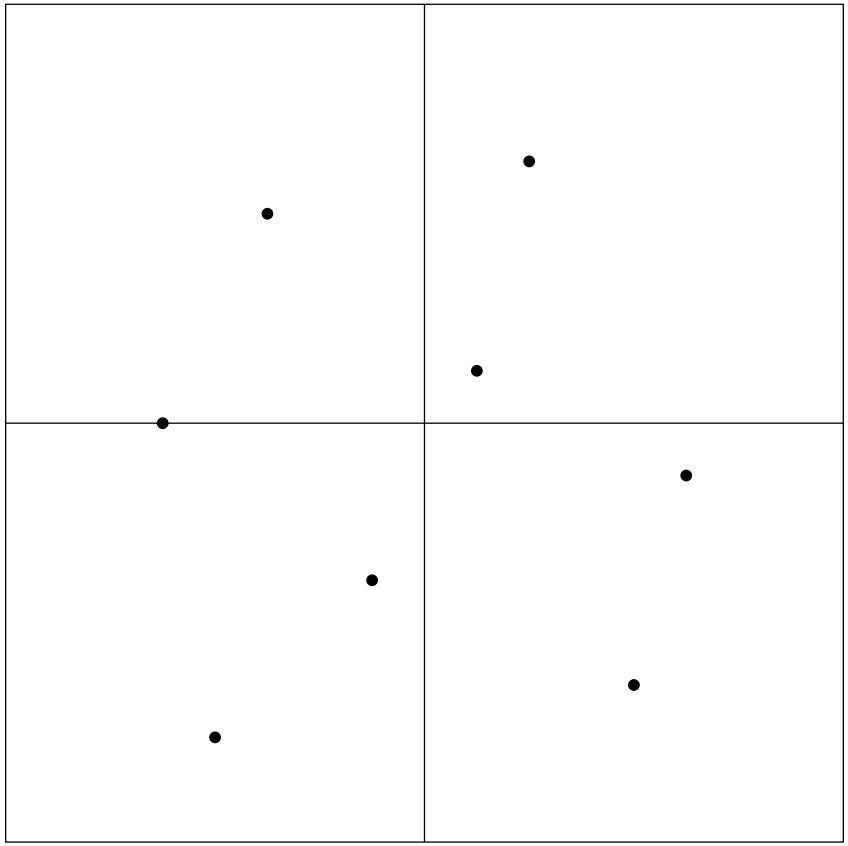


Figure 6 : Dividing up points for a Point-Region (PR) quadtree

Point-Region Quadtrees



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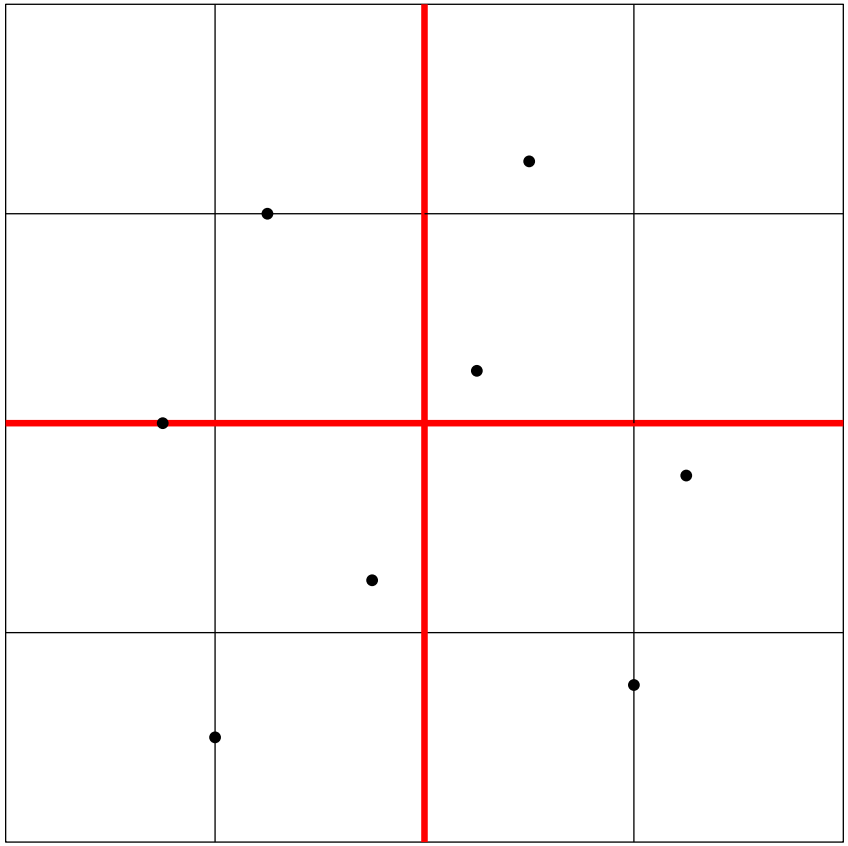

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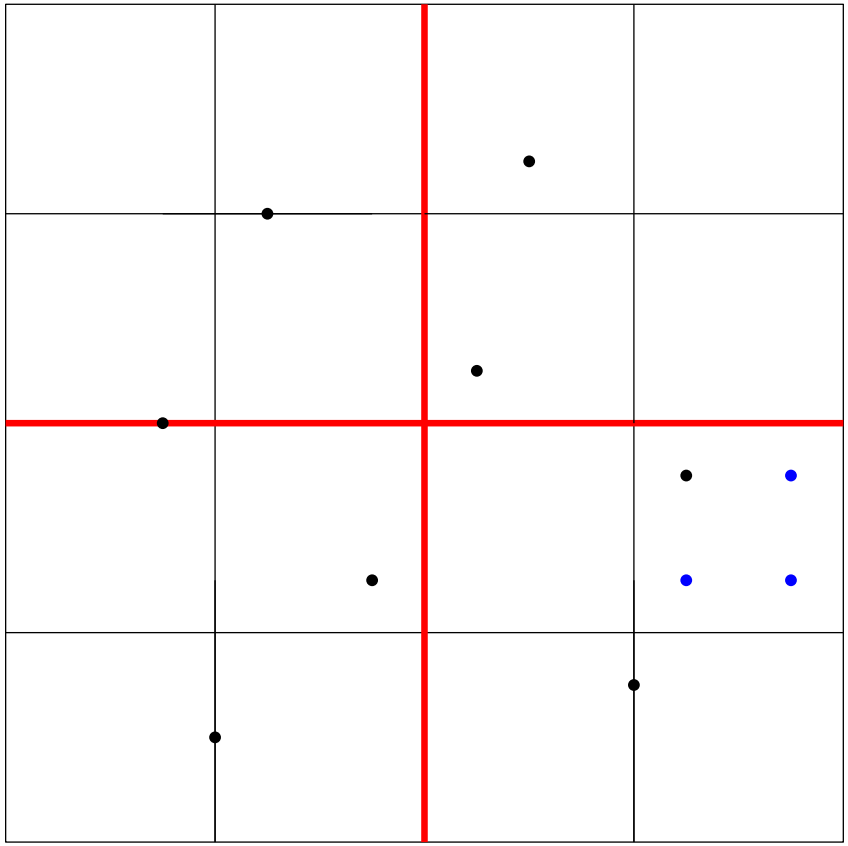

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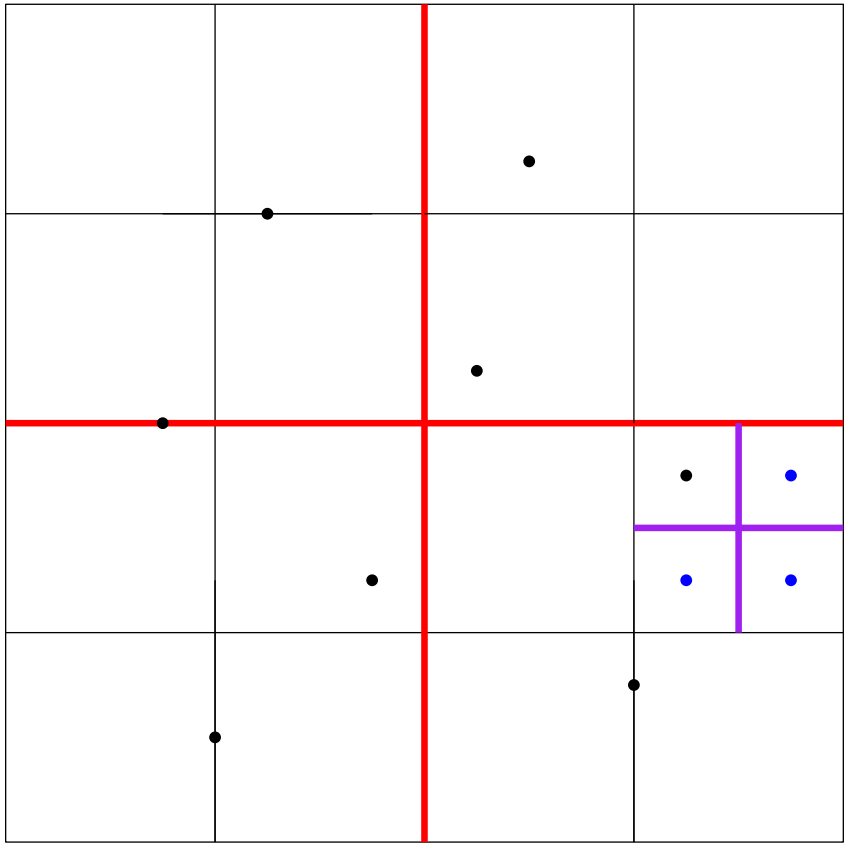

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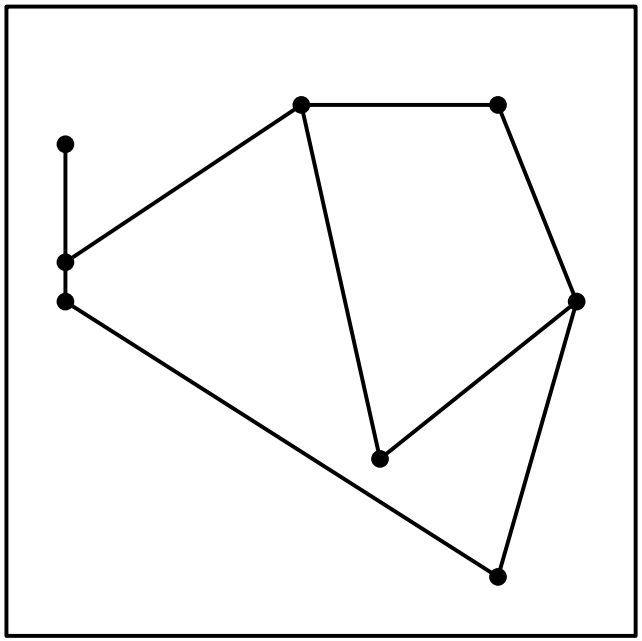
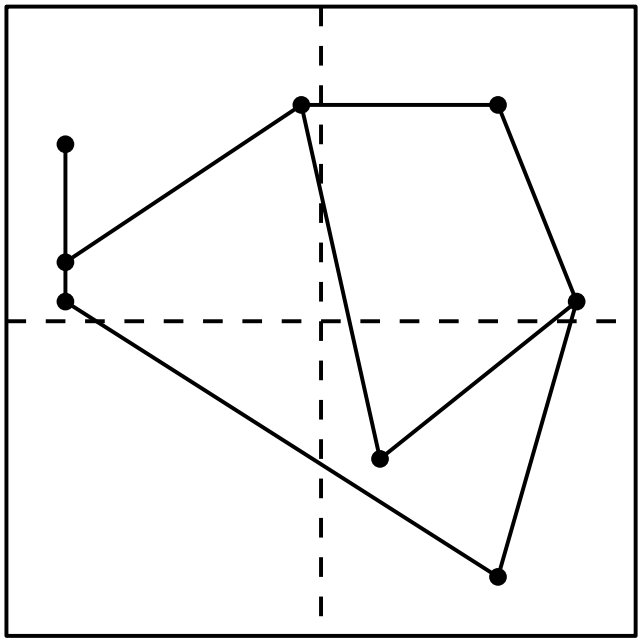
Edge Quadtrees



→ Subdivide until a single line segment per square (or a corner)

Figure 7 : Example of capturing linear features in an edge quadtree [7]

Edge Quadtrees



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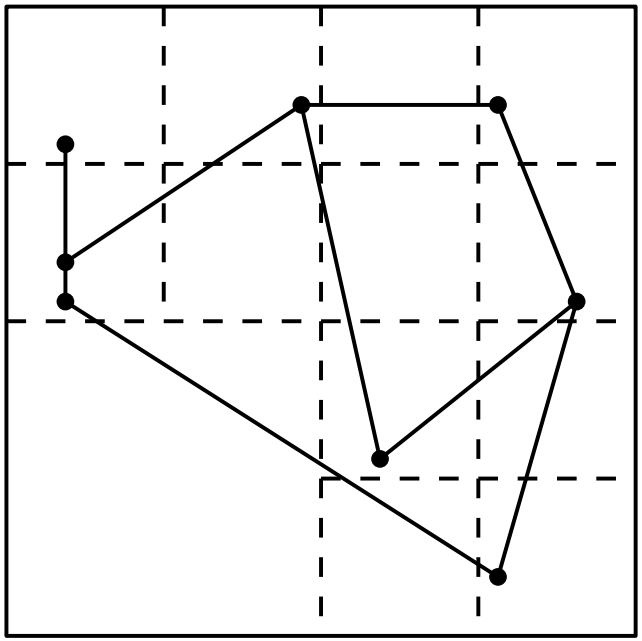

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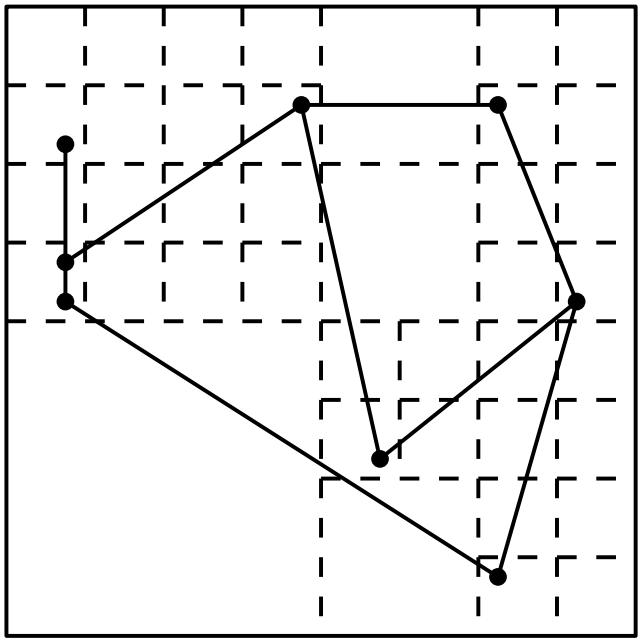

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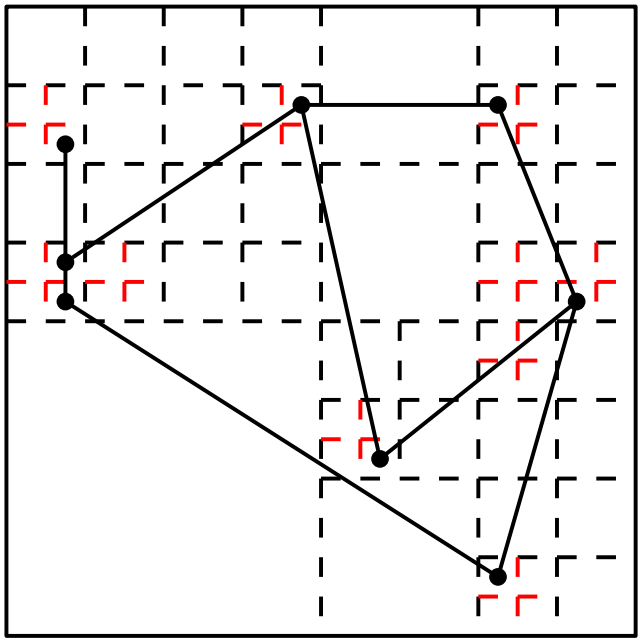

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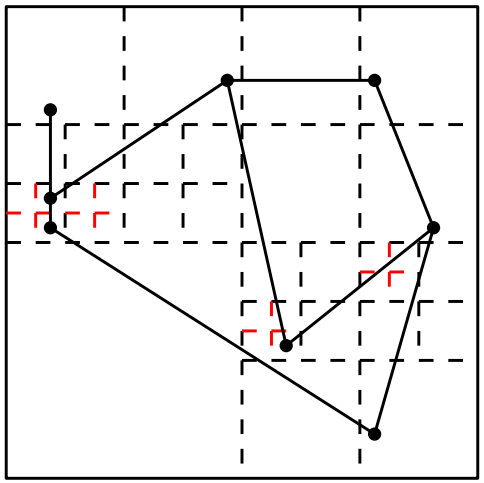
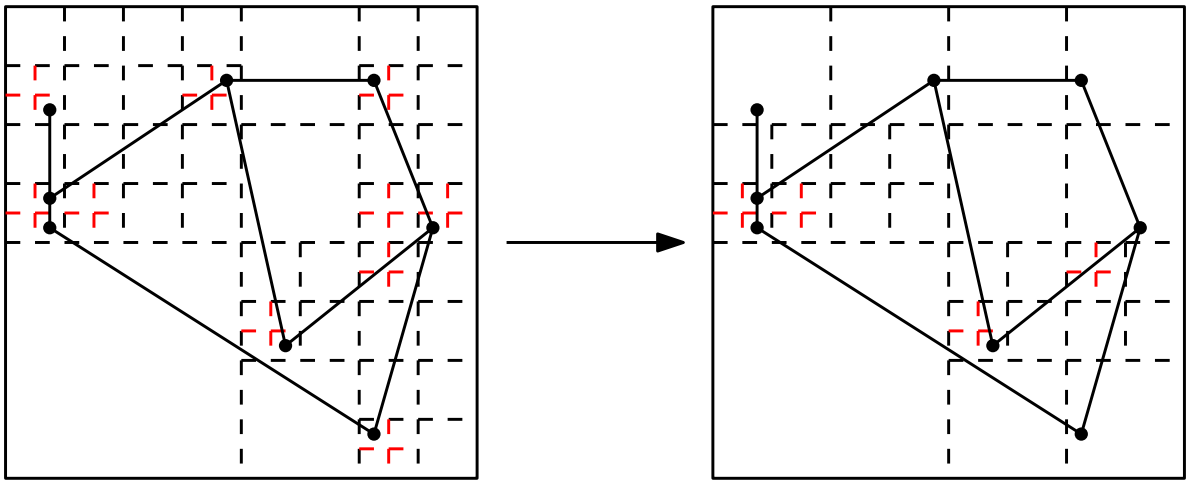
Polygonal Map (PM) Quadtrees



→ Subdivide until a single line segment per square...
→ ...unless the segments meet at a vertex in the block

Figure 8 : Example of capturing features in a PM quadtree [7]

Polygonal Map (PM) Quadtrees



→ Subdivide until a single line segment per square...
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Figure 8 : Example of capturing features in a PM quadtree [7]

Point Location

Range Query

Spherical Region Query

kNN Query

Compressed Quadtrees

This section from [2] and [3]

Dynamic Compressed Quadtrees

Mesh Generation

This section comes from chapter 14 of [4] and chapter 12 of [2]

Balanced Quadtrees

Building the Balanced Quadtrees

Quadtrees → Triangulations

References I

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Quadtrees and octrees

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