School of Computer Science

Carleton University

Name: Problem Set: **1**
Course: **COMP 3804** Week: **1**

Problems

Consult references [1, 2].

Problem 1. *Define* O, Ω, Θ *in the context of this course.*

Problem 2. *Applying the definitions, show the following:*

1. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.
2. $1 + 2 + 3 + \cdots + n \in O(n^2)$.
3. $1 + 2 + 3 + \cdots + n \in \Omega(n^2)$.
4. $1 + 2 + 3 + \cdots + n \in \Theta(n^2)$.

Problem 3. *Show that* $\sum\limits_{k=1}^{n} k^2 \in \Theta(n^3)$

Problem 4. *Show that* $\frac{1}{3}n^2 - 3n \in O(n^2)$

Problem 5. *Let* $p(n) = \sum_{i=0}^{d} a_i n^i$ *be a polynomial of degree* d *and assume that* $a_d > 0$*. Show that* $p(n) \in O(n^k)$*, where* $k \geq d$ *is a constant. What are* c *and* n_0 *if we use the following definition of* O*-notation:* $O(g(n)) = \{f(n) : \exists\ c, n_0 > 0 \text{ such that } f(n) \leq cg(n),\ \forall n \geq n_0\}$.

Problem 6. *Let* $p(n) = \sum_{i=0}^{d} a_i n^i$ *be a polynomial of degree* d *and assume that* $a_d > 0$*. For* $k \geq d$*, show that* $\lim\limits_{n\to\infty} \frac{\sum\limits_{i=0}^{d} a_i n^i}{n^k} = a_d n^{d-k} \geq a_d > 0$ *and conclude that* $p(n) \in \Omega(n^k)$.

Problem 7. *Let* $p(n) = \sum\limits_{i=0}^{d} a_i n^i$ *be a polynomial of degree* d *and assume that* $a_d > 0$*. Show that* $p(n) \in \Theta(n^d)$*. (Hint: Using limits show that* $\lim\limits_{n\to\infty} \frac{p(n)}{n^d} = a_d$ *and* $0 < a_d < \infty$*. Thus,* $p(n) \in \Theta(n^d)$*. Recall that* $f(n) \in \Theta(g(n))$*, if* $0 < \lim\limits_{n\to\infty} \frac{f(n)}{g(n)} < \infty$*.)*

Problem 8. *Show that* $6n \log n + \sqrt{n} \log^2 n = \Theta(n \log n)$.

Problem 9. *Fibonacci numbers are defined recursively as follows:*
$F_0 = 0$*,* $F_1 = 1$*, and* $F_n = F_{n-1} + F_{n-2}$ *for any integer* $n > 1$*.*
Using induction show that $F_n \geq 2^{\frac{n}{2}}$ *for any* $n \geq 6$*.*

References

[1] S. DasGupta, C. Papadimitriou, V. Vazirani. *Introduction to Algorithms.* McGraw Hill.

[2] A. Maheshwari. *Notes on Algorithm Design*, Chapter 1, `https://people.scs.carleton.ca/~maheshwa/Notes/DAA/notes.pdf`