COMP 3803 - Assignment 2 Solutions

Solutions written in LaTeX, diagrams drawn in `ipe`

February 13, 2015

1. **Q:** Let $\Sigma = \{a, b\}$. Write a regular expression for the set of all strings in Σ^* that start with an odd number of a's and contain at least two b's.

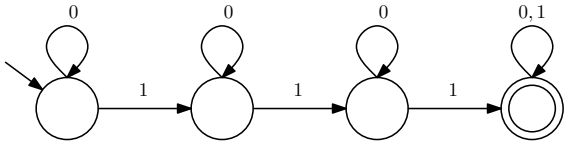
   A:

   $$a(aa)^*ba^*b\Sigma^*$$

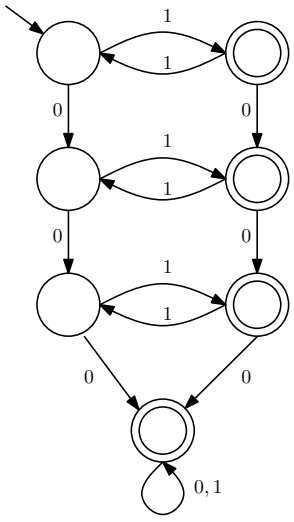
2. **Q:** For each of the following languages, construct a DFA that accepts the language. In all cases, the alphabet is $\{0, 1\}$.

   A:

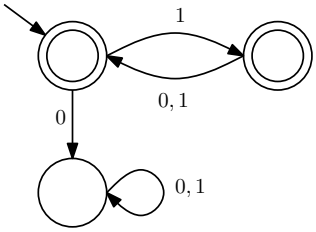
   a. $\{W : W$ contains at least three 1's$\}$.

   b. $\{W : W$ contains an odd number of 1's or more than two 0's$\}$.

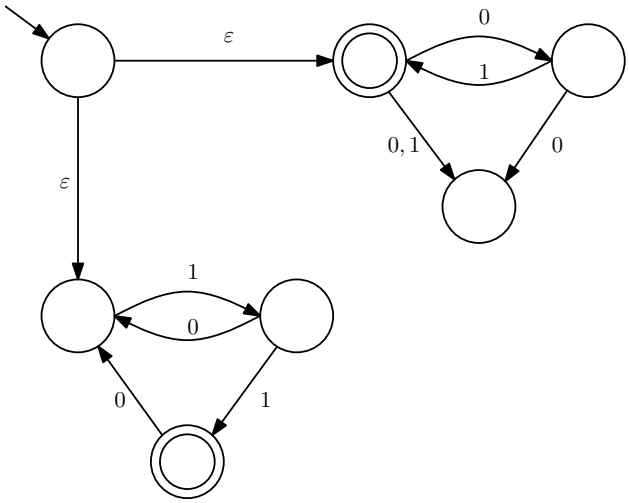
c. $\{W$: every odd position in W is $1\}$.



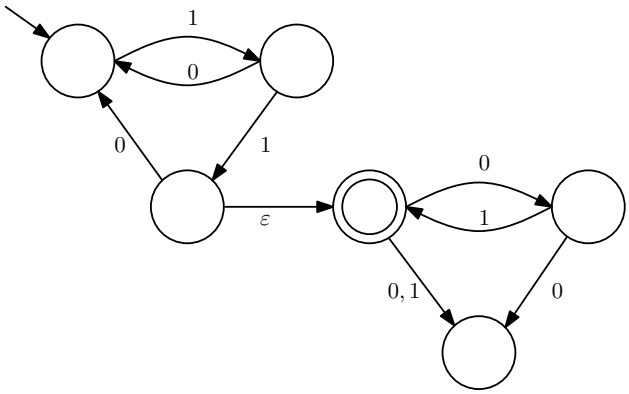
3. **Q:** For the NFA M_1 and M_2 of the assignment questionnaire, answer the following questions with some reasonable justification:

A:

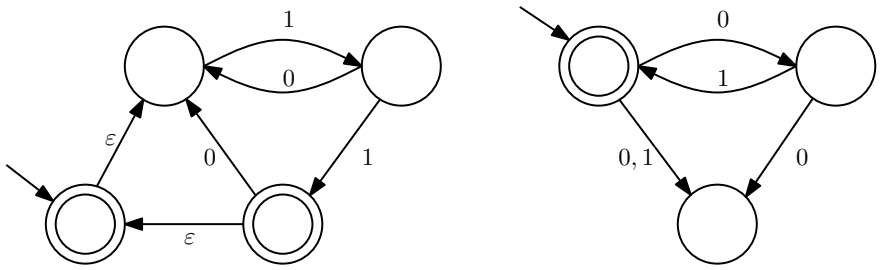
a. Construct an NFA for $L(M_1) \cup L(M_2)$:



b. Construct an NFA for $L(M_1)L(M_2)$:



c. Construct NFA for $(L(M_1))^*$ and $(L(M_2))^*$:



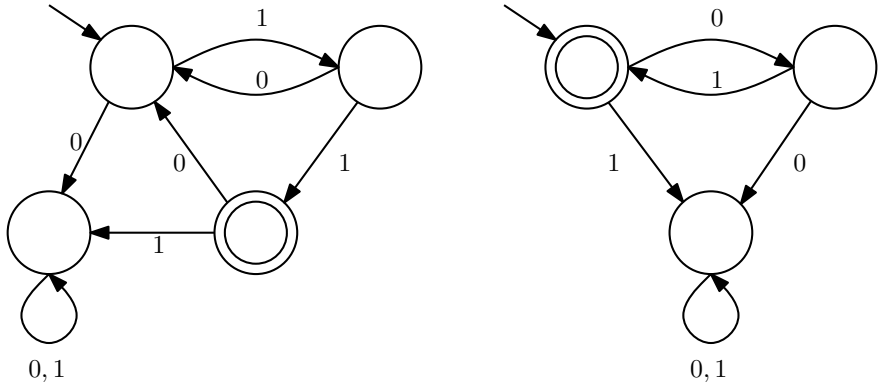
Note: M_2 already accepts $(L(M_2))^*$.

d. What is a regular expression that describes $L(M_1)$:

$$1(01)^*1(01(01)^*1)^*$$

4. **Q:** Convert both the NFA of Question 3 into equivalent DFA.

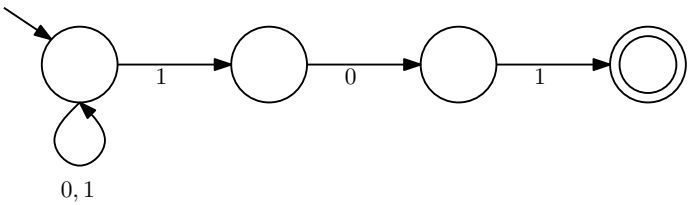
A:



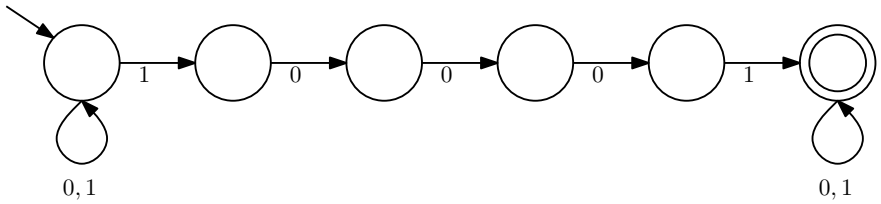
5. **Q:** For each of the following languages, construct an NFA, with the specified number of states, that accepts the language. In all cases, the alphabet is $\{0, 1\}$.

A:

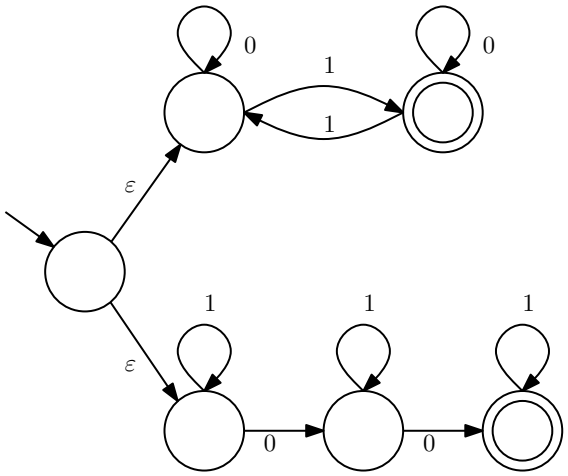
a. The language $\{W : W \text{ ends with } 101\}$. The NFA must have four states:



b. The language $\{W : W$ contains the substring 10001$\}$. The NFA must have six states:



c. The language $\{W : W$ contains an odd number of 1's or exactly two 0's$\}$. The NFA must have six states:



6. **Q:** Give regular expressions describing the following languages in which the alphabet Σ is $\{0, 1\}$:

A:

a. $\{W : W$ has length at least 3, and its second symbol is 1$\}$:

$$\Sigma 1 \Sigma \Sigma^*$$

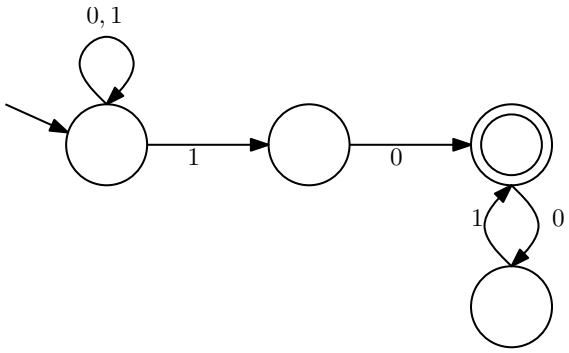
b. $\{W :$ Every odd position of W is a 0$\}$:

$$(0\Sigma)^*(0 \cup \varepsilon)$$

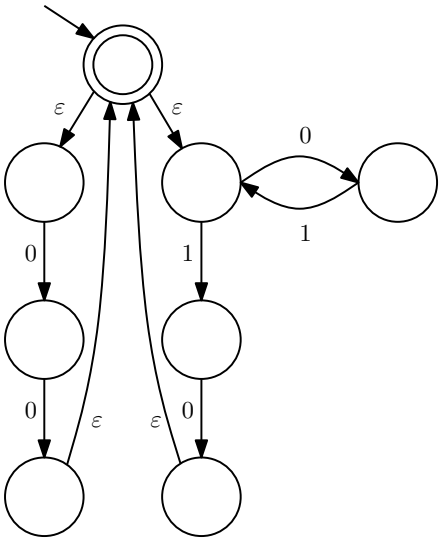
7. **Q:** Develop the NFA (recognizer) for each of the following regular languages:

   A:

   a. $(0 \cup 1)^*10(01)^*$:



   b. $((01)^*10 \cup (00))^*$:



8. **Q:** What is the regular expression accepted by the DFA presented in the questionnaire?

   A: We will determine the expression using the technique demonstrated in class and described in the course notes. We have the following system of equations:

$$L_1 = \varepsilon \cup bL_1 \cup aL_2$$
$$L_2 = aL_2 \cup bL_3$$
$$L_3 = bL_3 \cup aL_1$$

   From this, we see that $L_3 = b^*aL_1$. Substituting this expression into L_2, we have:

$$L_2 = aL_2 \cup bb^*aL_1$$

so $L_2 = a^*bb^*aL_1$. Substituting into L_1, finally:

$$L_1 = \varepsilon \cup (b \cup aa^*bb^*a)L_1$$

So we obtain:

$$L_1 = (b \cup aa^*bb^*a)^*$$

Since 1 is the starting state, then L_1 represents the regular expression of the language accepted by the given DFA. □