CARLETON UNIVERSITY
SCHOOL OF COMPUTER SCIENCE
WINTER 2015
COMP. 3803
INTRODUCTION TO THEORY OF COMPUTATION
ASSIGNMENT III
DUE: FRIDAY MAR. 13, 2015 (11:30 AM)

Assignment Policy: Late assignments will **not** be accepted. You are expected to work on the assignments on your own. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- Please write your name and student number clearly. Your last name must be in Upper Case.
- The answers should be concise, clear and neat. Make sure that your TA can read your solutions.
- Please submit the solutions *in the order of the problems*, the solution to Problem 1, then to Problem 2 and so on.
- When presenting proofs, every step should be justified so as to get partial credit.
- Assignments should be stapled (or in an unsealed envelope) *with your name and student number*.

Substantial departures from the above guidelines will not be graded.

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1. Using the pumping lemma for *Regular Languages*, prove that the following languages, with the corresponding alphabets are not *Regular*:
   - $\{b^4a^Mb^{2M} \mid M \geq 0\}$
   - $\{a^Mb^Na^{M+N} \mid M, N \geq 0\}$
   - $\{a^W \mid W=n^3$, for some integer, $n \geq 0\}$

2. For the alphabet Σ = {‘(’, ‘)’}, use the pumping lemma for *Regular Languages*, to prove that the language consisting of matched parenthesis is not *Regular*. An example of a string in this language is “(()(()))”.

3. Let $\Sigma = \{0, 1\}$. Write CFGs that generate the following languages:
   - { W | W contains *no more than* three 1’s}
   - { W | W contains at *exactly* two 1’s}
   - { W | W is of even length *and* starts and ends with the same symbol}
   - $\{0^n1^n \mid n \geq 1\} \cup \{1^{2m}0^m \mid m \geq 1\}$
   - {W | W = 0X1 OR 1X0 where X is a palindrome (i.e., $X=X^R$, where X^R is the reverse of X)}

4. Let G = (V, Σ, R, S) be the context-free grammar, where V = {A, B, S}, Σ = {0, 1}, S is the start variable, and R consists of the rules

$S \rightarrow 0B|11A$

$A \rightarrow 0|0S|BAA$

$B \rightarrow 11|11S|ABB$

- By *showing a sequence of productions*, prove that 011011110 ∈ L(G).
- Can you argue the following assertion: "Every string, W in L(G) has the property that the number of 1's in W is equal to the twice number of 0's"?

5. Convert the following CFGs (where Σ = {a, b}) to the Chomsky Normal Form:
   (a) $S \rightarrow SS$; $S \rightarrow abbS$; $S \rightarrow Sbba$; $S \rightarrow \varepsilon$.
   (b) $S \rightarrow aSbb$; $S \rightarrow bbSa$; $S \rightarrow \varepsilon$.

6. If L_1 is the language generated by the grammar in Question 5 (a), and L_2 is the language generated by the grammar in Question 5 (b) both of them *not* in the Chomsky Normal Form, create the grammars that generate:
   - $L_1 \cup L_2$
   - $L_1 \cdot L_2$, where '·' is the concantenation operator.
   - L_1^*

The closure properties of CFLs are very simple and are explained in the notes.

7. Find a CFG generating $L = \{1^k 0^n 1^n\, 0^m 1^m \mid m, n, k \geq 0\}$. Give straightforward arguments to show that your answer is right. You need not formally prove that your arguments are right.

8. Write the Context Free Grammar that generates the language accepted by the DFA of Question 8 in Assignment II.