Solving Problems: Intelligent Search

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The primary source of these notes are the slides of Professor Hwee Tou Ng from Singapore. **I sincerely thank him for this**.

Heuristic Search

- Problem with DFS and BFS: No way to guide the search
- Solution can be anywhere in tree.
- In the worst case all possible states will be traversed

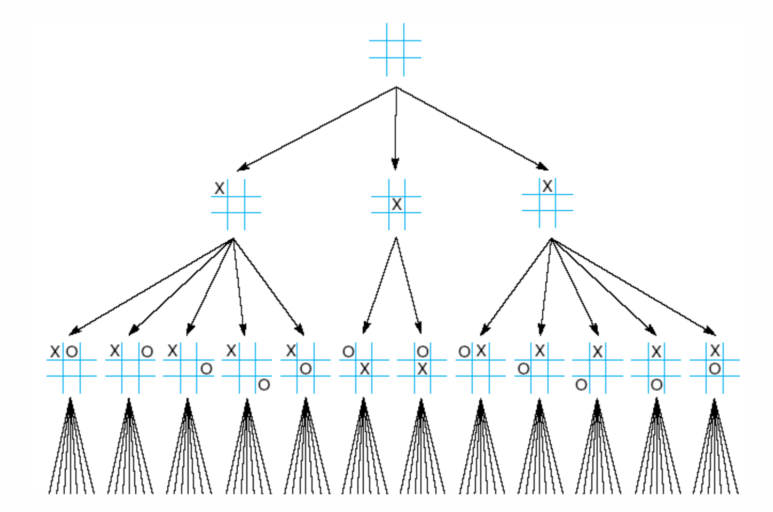
- One "solution" to this problem
  - Probe the search space
  - Where is the final state likely to be
- This of course will be problem specific
- A function is usually created that evaluates:
  - How good the current solution is
  - This function is used to help guide the search process

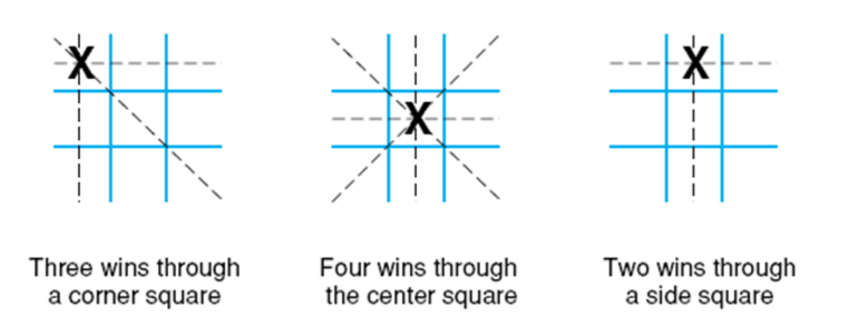
- This guided search called a Heuristic Search

A Heuristic

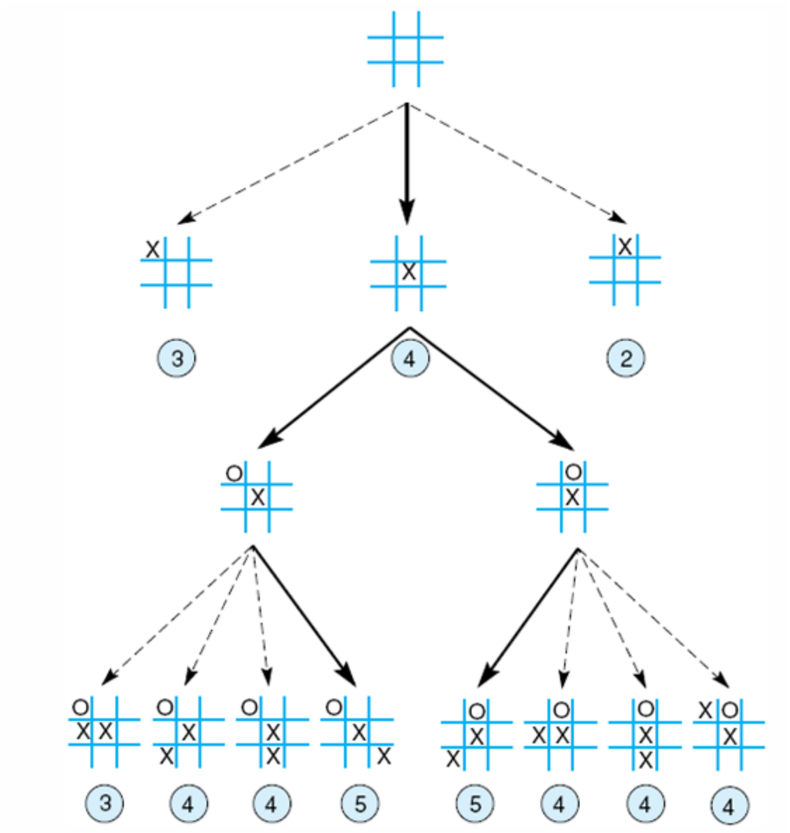
- Derived from the Greek: *heuriskein*: “to find”; “to discover”
- Has been used (and is sometimes still used) to mean:
  - “A process that may solve a given problem, but offers no guarantees of doing so” Newall, Shaw, & Simon 1963
- Heuristics can also be thought of as a “Rule of Thumb”
- Can refer to any technique that improves average-case but not necessarily worst-case performance
- Here: A function that provides an estimate of solution cost

Advantage of Heuristics



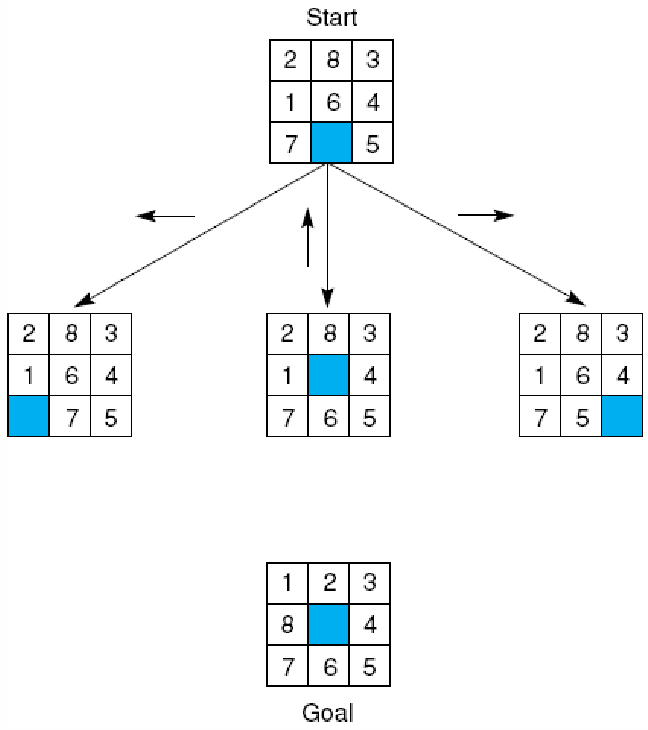


Advantage of Heuristics: Reduced State Space



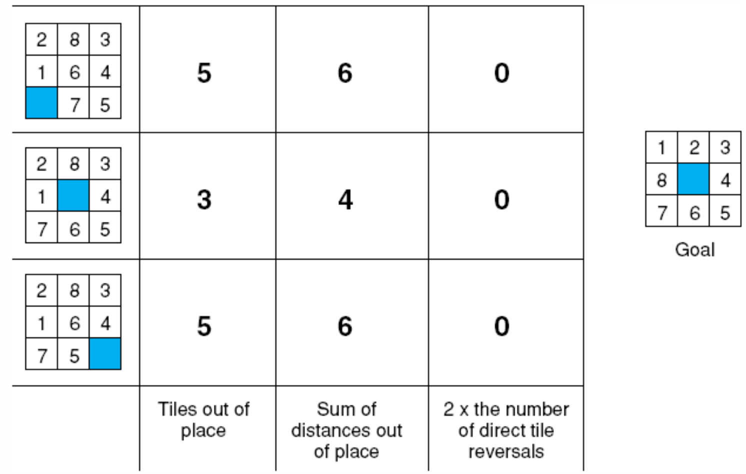
Performance of Heuristics

- Performance of several heuristics…



Possible Heuristics

- Count the tiles out of place:
  - State with fewest tiles out of place is closer to the desired goal
- Distance Summation:
  - Sum all the distance by which the tiles are out of place
  - State with the shortest distance is closer to the desired goal
- Count reversal Tiles:
  - If two tiles are next to each other, and the goal requires their position to be swapped. The heuristic takes this into account by evaluating the expression (2 * *number of direct tiles reversal*)



Best-first Search

- **Idea**: use an evaluation function *f(n)* for each node
  - Estimate of "desirability"
  - Expand most desirable unexpanded node

- **Implementation**:

  Order the nodes in fringe in decreasing order of desirability

- **Special cases**:
  - Greedy best-first search
  - A^* search

Best-first Search

- Combine BFS and DFS using a heuristic function
- Expand the branch that has the best evaluation under the heuristic function
- Similar to hill climbing (move in the best direction)
- But can go back to “discarded” branches

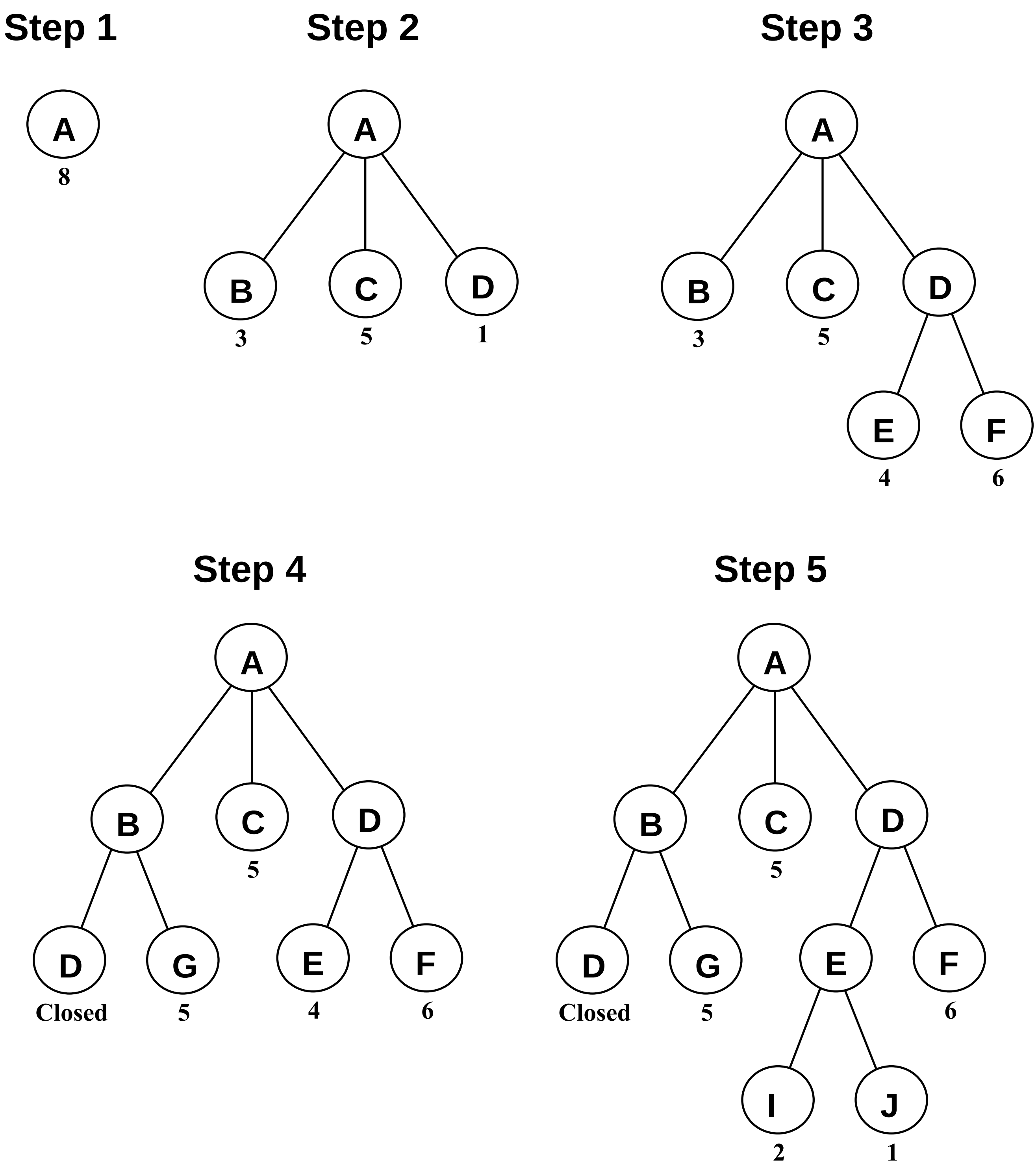
Best-first Search Algorithm

- Initialize OPEN to initial state, CLOSED to Empty list
- Until a Goal is found or no nodes left in Open do:
  - Pick the best node in OPEN
  - Generate its successors, place node in CLOSED
  - For each successor do:
    - ➢ If not previously generated (not found in OPEN or CLOSED)
      - Evaluate
      - Add to OPEN

OPEN: Generated nodes who's children have not been evaluated yet
- » Implemented as a priority queue (heap structure)

CLOSED: Nodes that have been examined
- » Used to see if a node has been visited if searching a graph instead of a tree
- » Same as in DFS and BFS

Example of BestFS

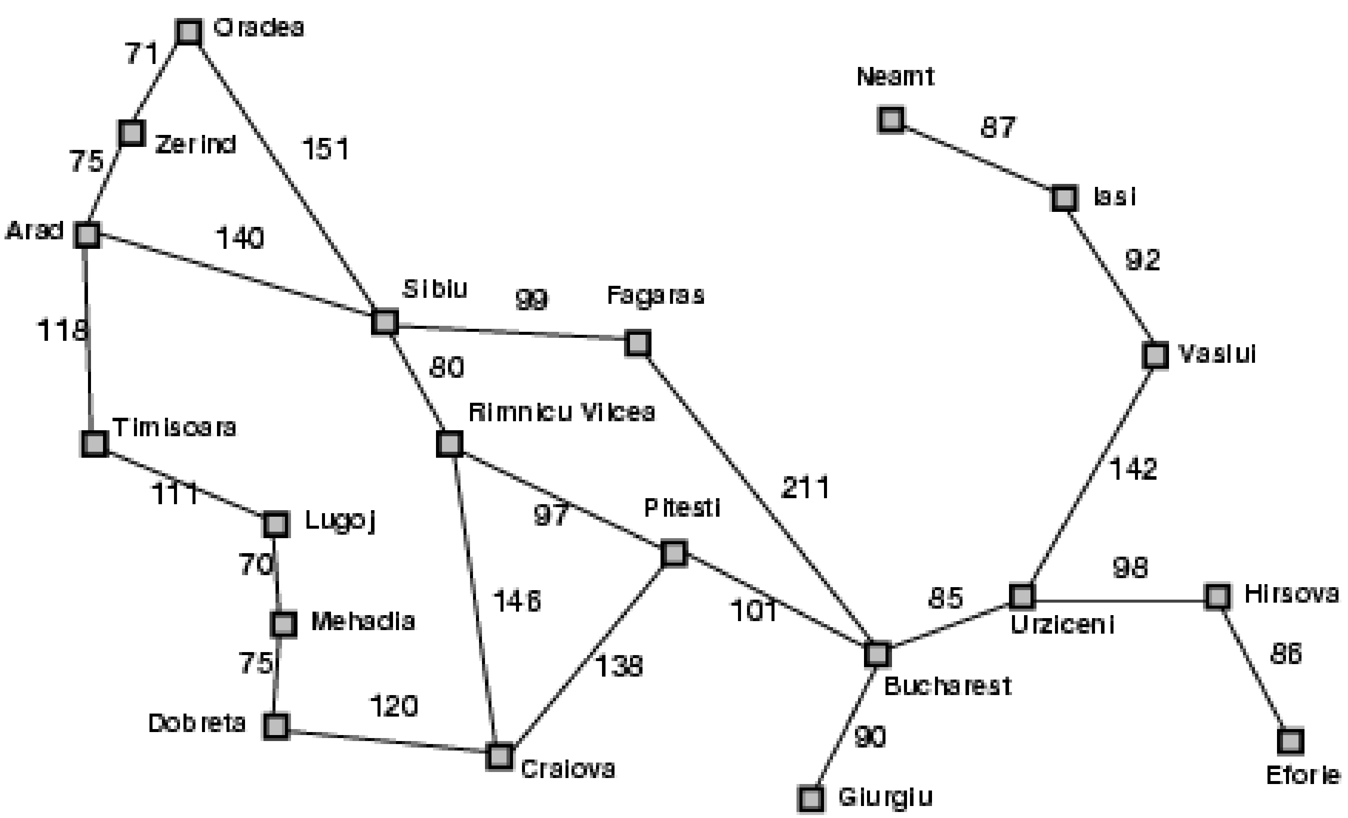


Greedy Best-first Search

- Evaluation function $f(n) = h(n)$ (heuristic)
- An estimate of cost from n to *goal*
- $h_{SLD}(n)$ = straight-line distance from n to Bucharest

- Greedy Best-first Search expands the node that appears to be closest to goal

Romania: Step Costs in Km

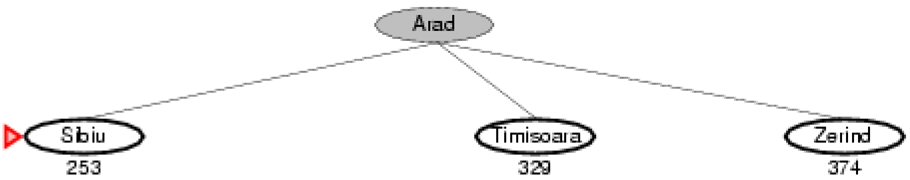


Straight–line distance to Bucharest

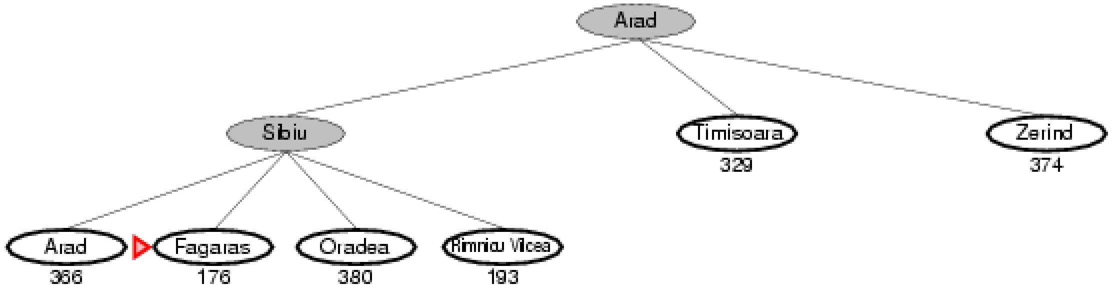
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example: Greedy Best-first Search

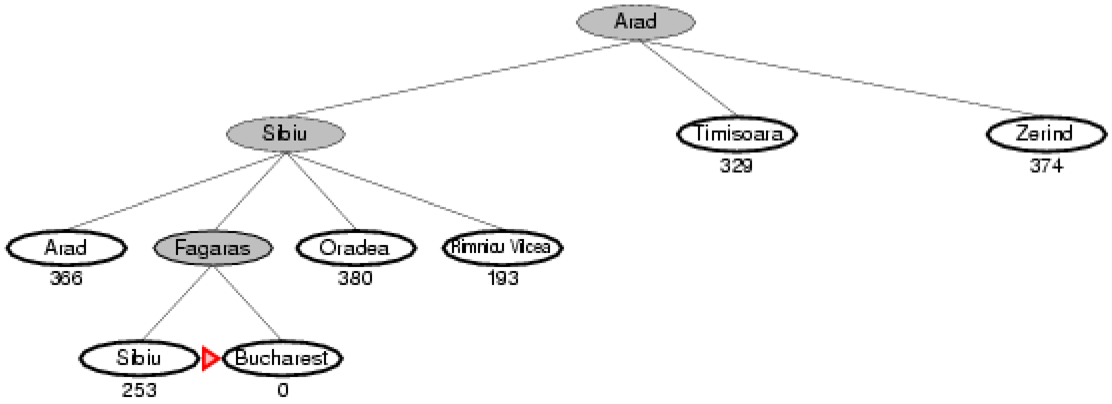
Example: Greedy Best-first Search



Example: Greedy Best-first Search



Example: Greedy Best-first Search



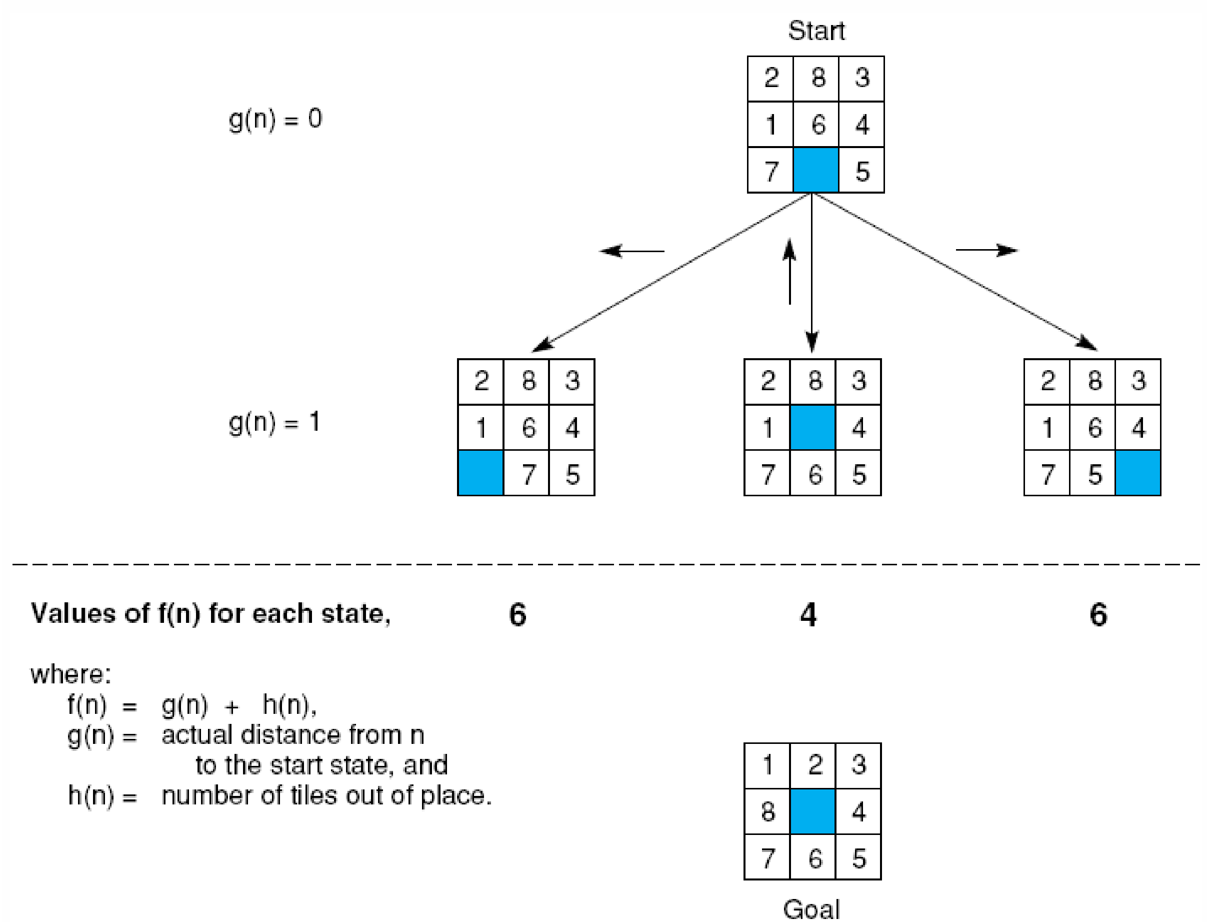
Properties: Greedy Best-first Search

- **Complete?**
  - No – can get stuck in loops
  - Iasi → Neamt → Iasi → Neamt →
- **Time?**
  - $O(b^m)$
  - But a good heuristic can a give dramatic improvement
- **Space?**
  - $O(b^m)$
  - *K*eeps all nodes in memory
- **Optimal?**
  - No

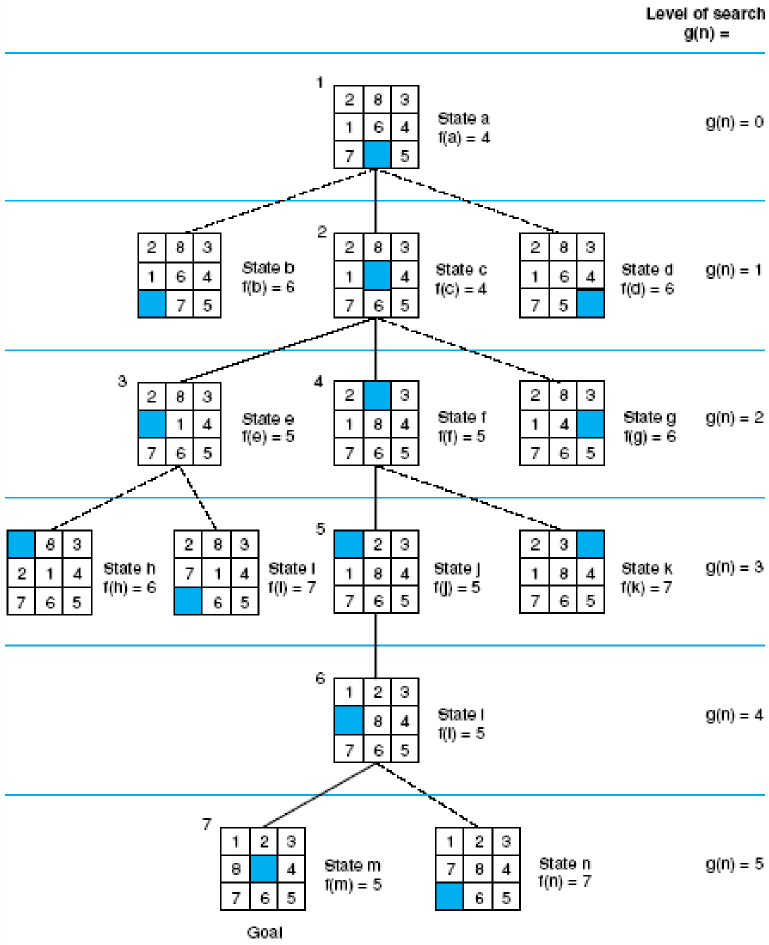
A^* Search

- A modification of the Best-first Search
- Used when searching for the Optimal path
- Idea: Avoid expanding paths that are "expensive"
- The heuristic function f(S) is broken into two parts:
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = Cost so far to reach n
  - $h(n)$ = Estimated cost from n to goal
  - $f(n)$ = Estimated total cost of path through n to goal

How A* Works


Start
2 8 3
1 6 4
7 5
g(n) = 0
g(n) = 1
2 8 3
1 6 4
7 5
2 8 3
1 4
7 6 5
2 8 3
1 6 4
7 5
Values of f(n) for each state,
6
4
6
where:
f(n) = g(n) + h(n),
g(n) = actual distance from n to the start state, and
h(n) = number of tiles out of place.
1 2 3
8 4
7 6 5
Goal

How A* Works



Level of search
g(n) =
State a
f(a) = 4
g(n) = 0
State b
f(b) = 6
State c
f(c) = 4
State d
f(d) = 6
g(n) = 1
State e
f(e) = 5
State f
f(f) = 5
State g
f(g) = 6
g(n) = 2
State h
f(h) = 6
State i
f(i) = 7
State j
f(j) = 5
State k
f(k) = 7
g(n) = 3
State l
f(l) = 5
g(n) = 4
State m
f(m) = 5
State n
f(n) = 7
g(n) = 5
Goal

A* Algorithm

- Initialize OPEN to initial state
- Until a Goal is found or no nodes left in OPEN do:
  - Pick the best node in OPEN
  - Generate its successors (recording the successors in a list);
  - Place in CLOSED
  - For each successor do:
    - If not previously generated (not found in OPEN or CLOSED)
      - Evaluate, add to OPEN , and record its parent
    - If previously generated (found in OPEN or CLOSED), and if the new path is better then the previous one
      - Change parent pointer that was recorded in the found node
    - If parent changed
      - Update the cost of getting to this node
      - Update the cost of getting to the children
        - Do this by recursively "regenerating" the successors using the list of successors that had been recorded in the found node
      - Make sure the priority queue is reordered accordingly

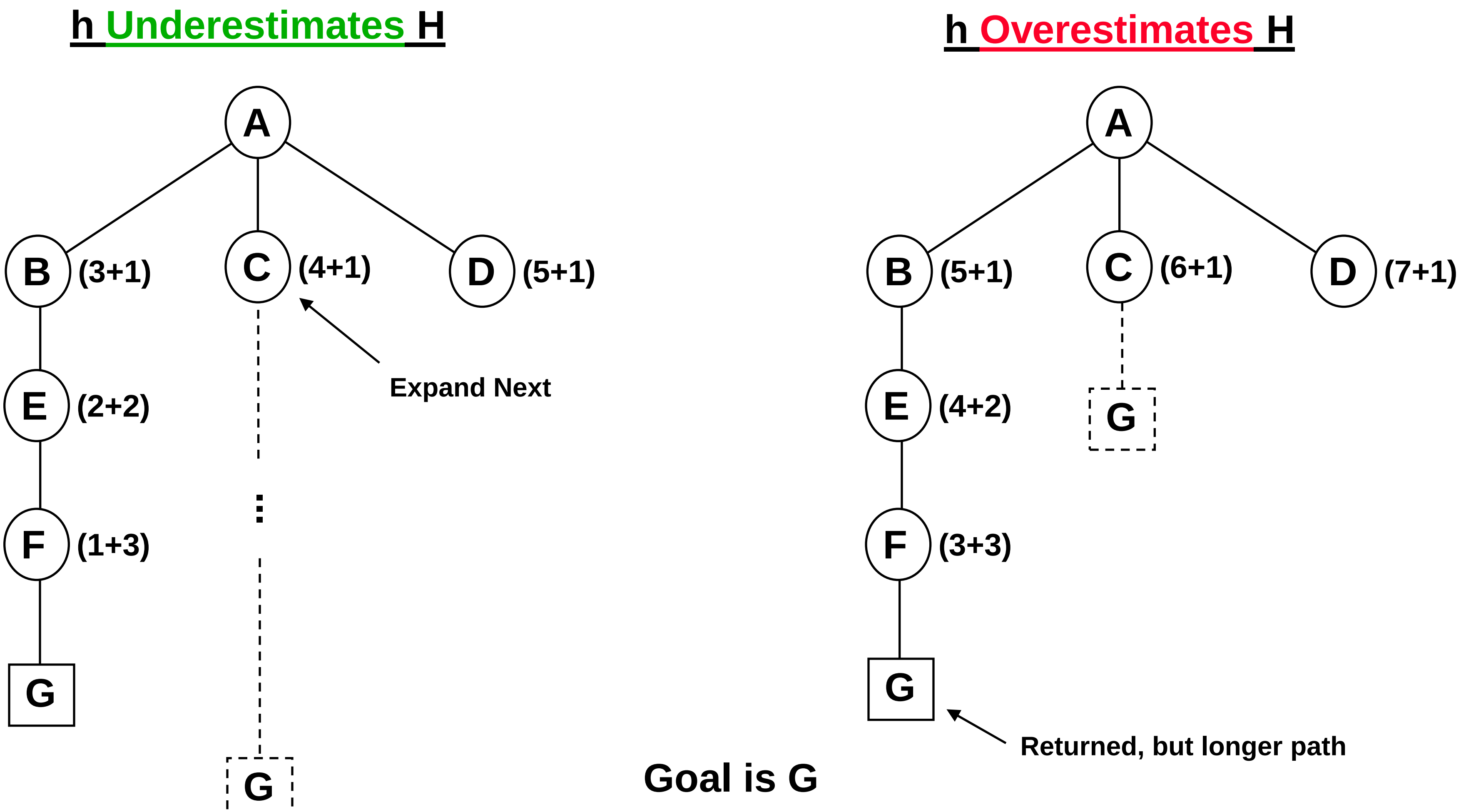
Properties of A*

- Becomes simple Best-first Search if g(S) = 0 for every S
- When a child state is formed
  - g(S) can be incremented by 1
  - Or be weighted based on the production system operator generated the state
- Is Breadth-first Search if g += 1 per generation and h=0 always

Properties of A*

- If h is the perfect estimator of the distance to the Goal (say, H)
  - A* will immediately find and traverse the optimal path to the solution
  - Will need NO backtracking
- If h never overestimates H
  - A* will find an optimal path to the solution (if it exists)
  - Problem lies in finding such an h

h Under/Over Estimates H



Goal is G

Importance of Heuristic Function

- If we have the exact Heuristic Function H
  - The search gets solved optimally

- Exact H is usually very hard to find
  - In many cases it would be a solution to an NP problem in polytime
  - Which is probably not possible to compute in less time than it would take to do the exponential sized search

- Next best: Guarantee h underestimates distance to the Sol^n.
  - A minimum path to the Goal is then guaranteed

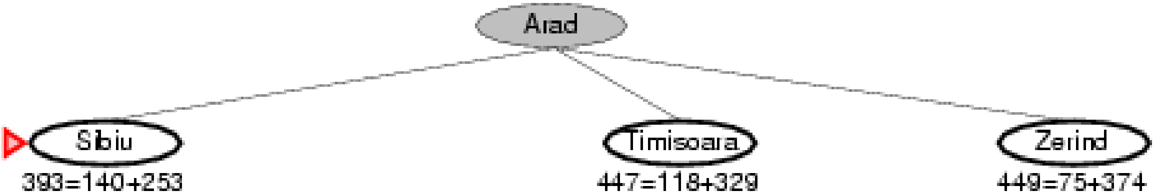
Heuristic Function *vs*. Search Time

- The better the heuristic, the less searching
  - Improves the average time complexity

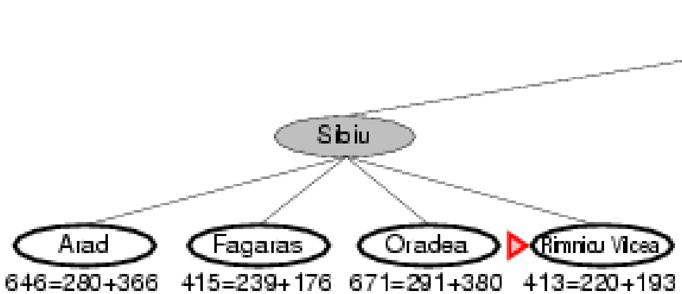
- However, to compute such a heuristic
  - Can figure out a good algorithm
  - Usually costs computation cycles
  - This could be used to process more nodes in the search
  - Trade-off between complex heuristics vs. more search done

Example: A* Search

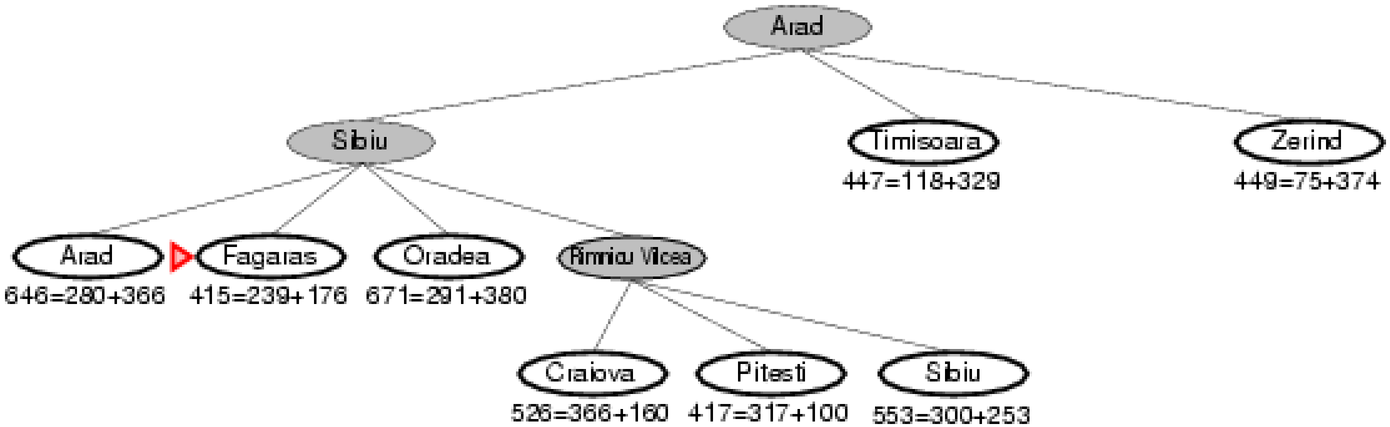
Example: A* Search



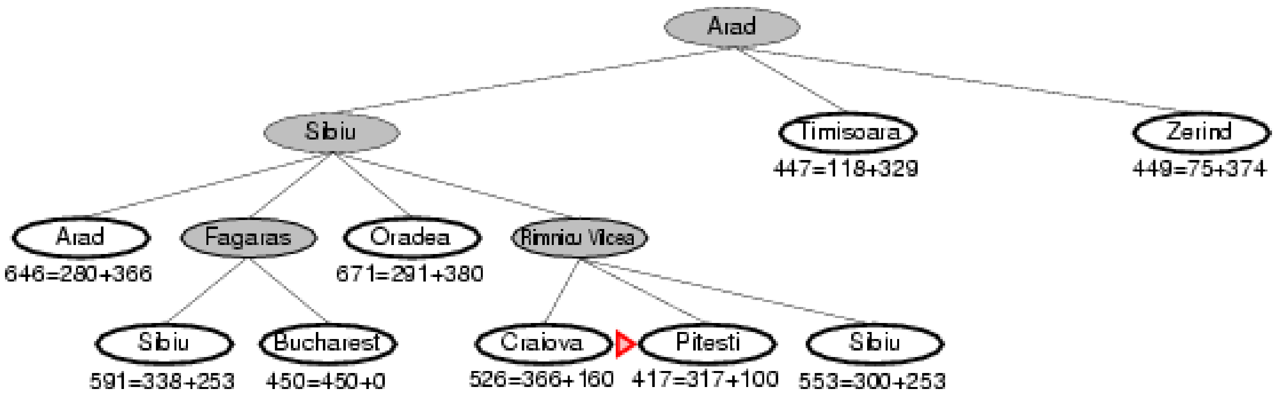
Example: A* Search



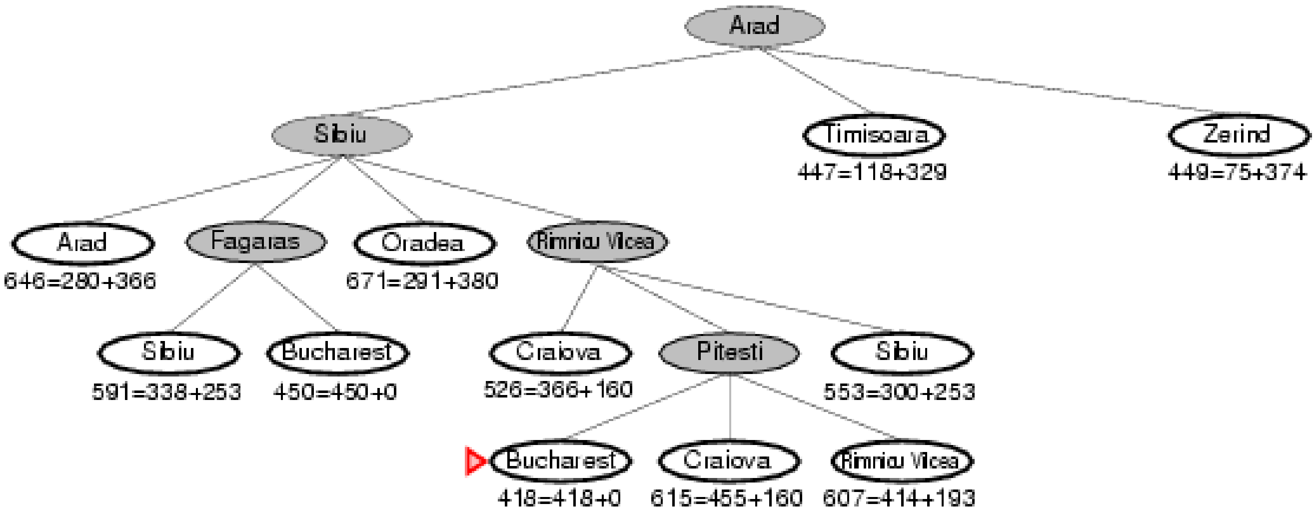
Example: A* Search



Example: A* Search



Example: A* Search



Other Example: A^* Search

- Please see the other Powerpoint in the folder...

Admissible Heuristics

- A heuristic $h(n)$ is Admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

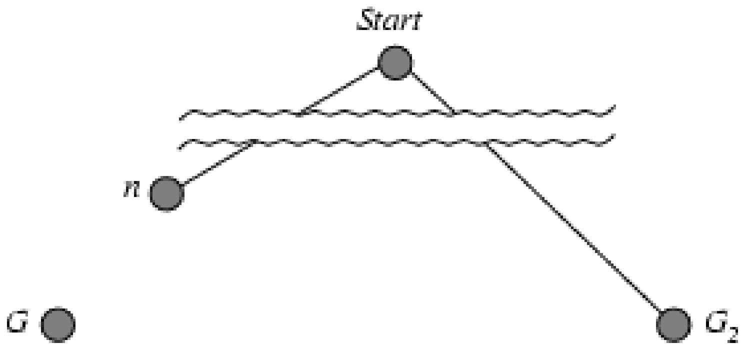
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

- Theorem: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is **optimal**

Proof: Optimality of A^*

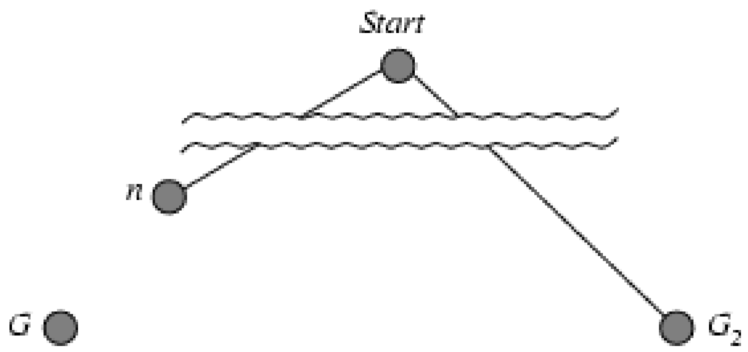
- Suppose some suboptimal goal G_2 has been generated and is in the fringe.
- Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



- $f(G_2) = g(G_2)$ Since $h(G_2) = 0$
- $g(G_2) > g(G)$ Since G_2 is suboptimal (2)
- $f(G) = g(G)$ Since $h(G) = 0$ (3)
- $f(G_2) = g(G_2) > g(G)$ (from (2)) $= f(G)$ (from (3))
- $f(G_2) > f(G)$ From above

Proof: Optimality of A^*

- Suppose some suboptimal goal G_2 has been generated and is in the *fringe*.
- Let *n* be an unexpanded node in the *fringe* such that *n* is on a shortest path to an optimal goal *G*.



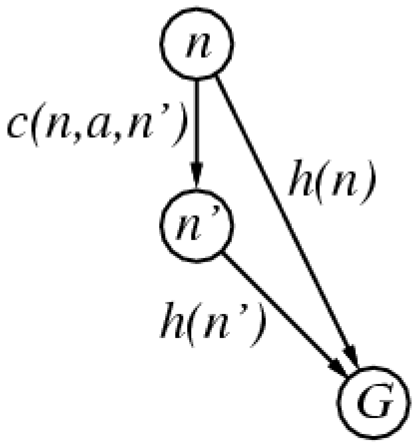
- $f(G_2)$ $> f(G)$ From above
- $h(n)$ $\leq h^*(n)$ Since h is admissible
- $g(n) + h(n)$ $\leq g(n) + h^*(n)$
- $f(n)$ $\leq f(G)$

Hence $f(G_2) > f(n)$. Thus A^* will never select G_2 for expansion

Consistent Heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \leq c(n,a,n') + h(n')$ (4)



- If *h* is consistent, we have

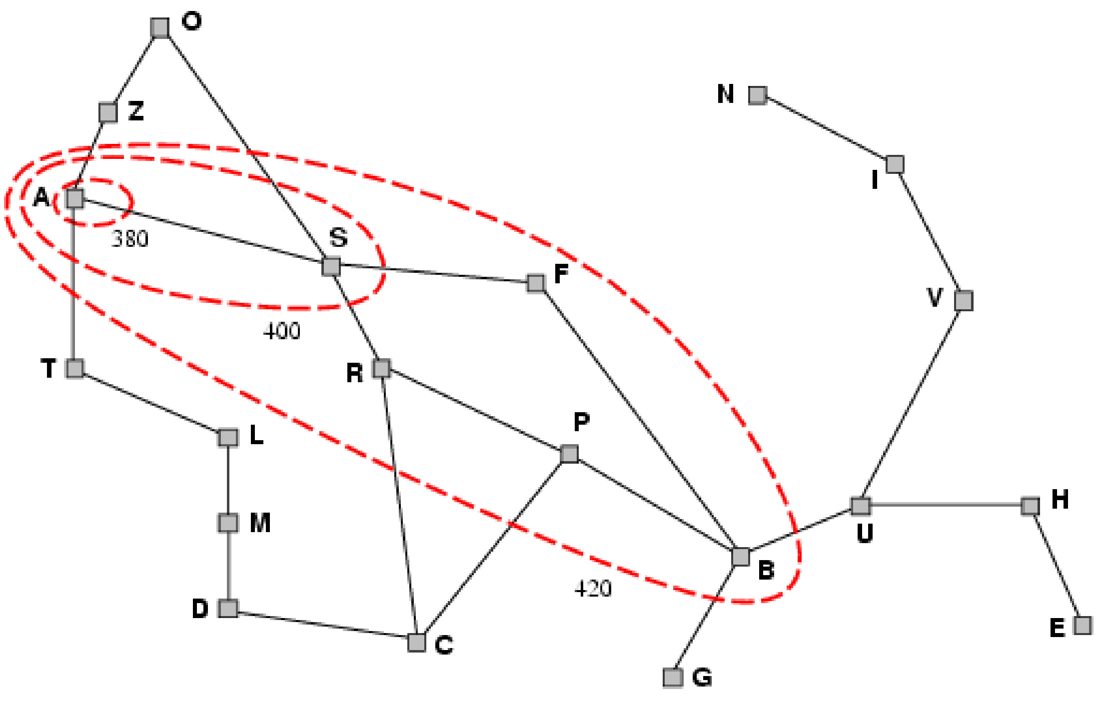
  f(n') = g(n') + h(n')
  = g(n) + c(n,a,n') + h(n') (By (4))
  ≥ g(n) + h(n)
  = f(n)

- i.e., *f(n)* is non-decreasing along any path.

- Theorem: If *h(n)* is consistent, A* using GRAPH-SEARCH is optimal.
- Essentially since: At the very end – h(G) = 0.

Optimality of A^*

- A^* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

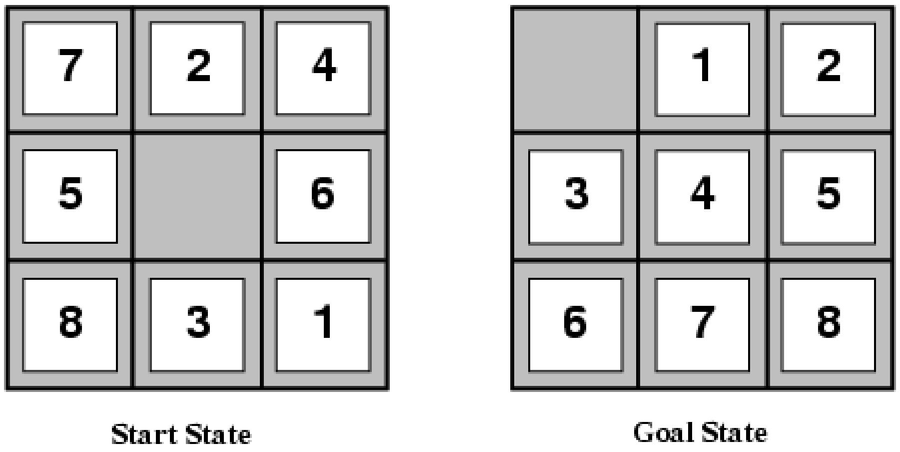
- **Complete?**
  - Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- **Time?**
  - Exponential
- **Space?**
  - Keeps all nodes in memory
- **Optimal?**
  - Yes

Admissible Heuristics

The 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., No. of squares from desired location of each tile)



- **$h_1(S)$ = ? 8**
- **$h_2(S)$ = ?** 3+1+2+2+2+3+3+2 = **18**

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed Problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Beam Search

- Same as BestFS and A* with one difference
- Instead of keeping the list OPEN unbounded in size, Beam Search fixes the size of OPEN
- OPEN only contains the best K evaluated nodes

Beam Search

- If new node considered is not better then any in OPEN, and OPEN is full, new node is not added
- If new node is to be inserted in the middle of the priority queue, and OPEN is full, drop the node at the end of OPEN (the one with the least priority)

Local Beam Search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list & repeat.

Local Search Algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

- State space = set of “complete” configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single “current” state, try to improve it

Hill Climbing Search

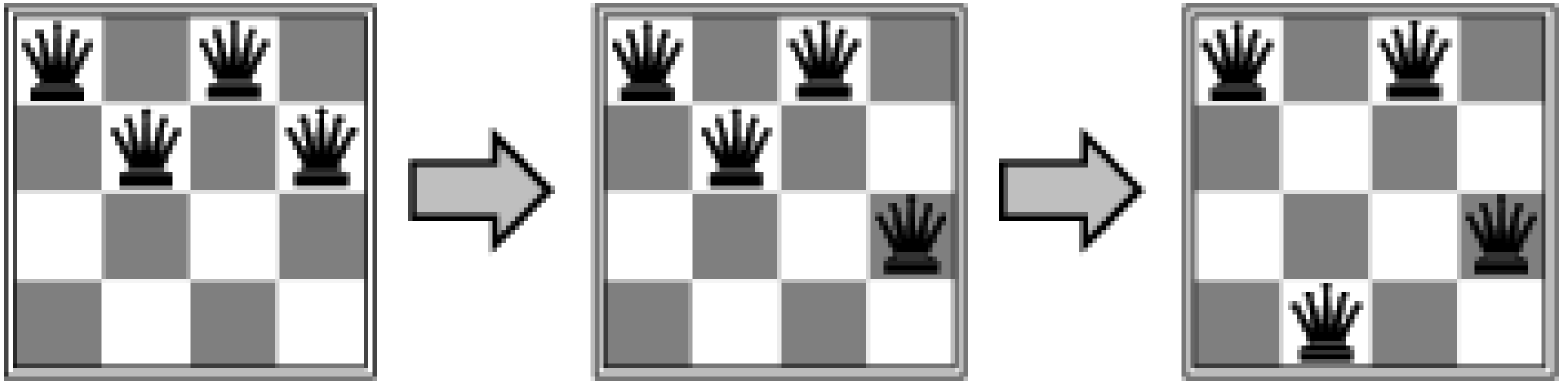
- "Like climbing Everest in thick fog with amnesia"

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum
  inputs: *problem*, a problem
  local variables: *current*, a node
          neighbor, a node

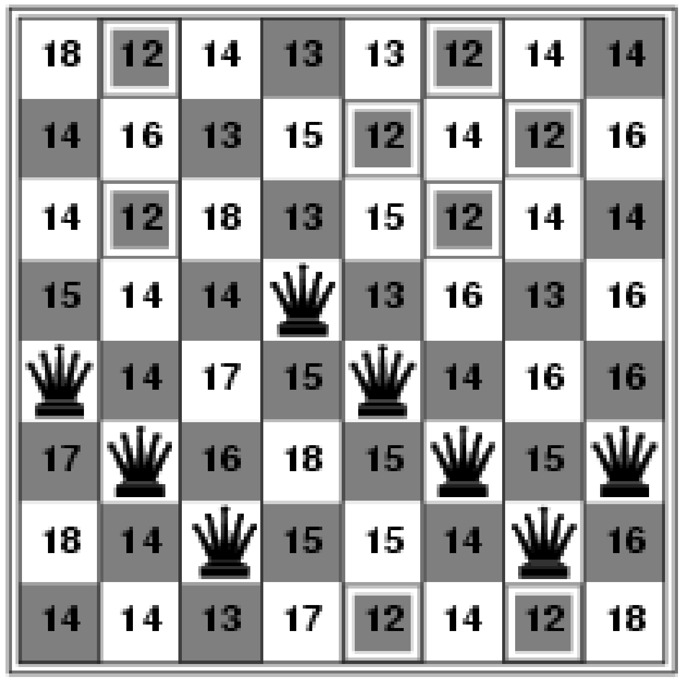
  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  loop do
    neighbor ← a highest-valued successor of *current*
    if VALUE[neighbor] ≤ VALUE[current] **then return** STATE[*current*]
    current ← *neighbor*

Example: *n*-queens

- Put n queens on an $n \times n$ board
- No two queens on the same row, column, or diagonal

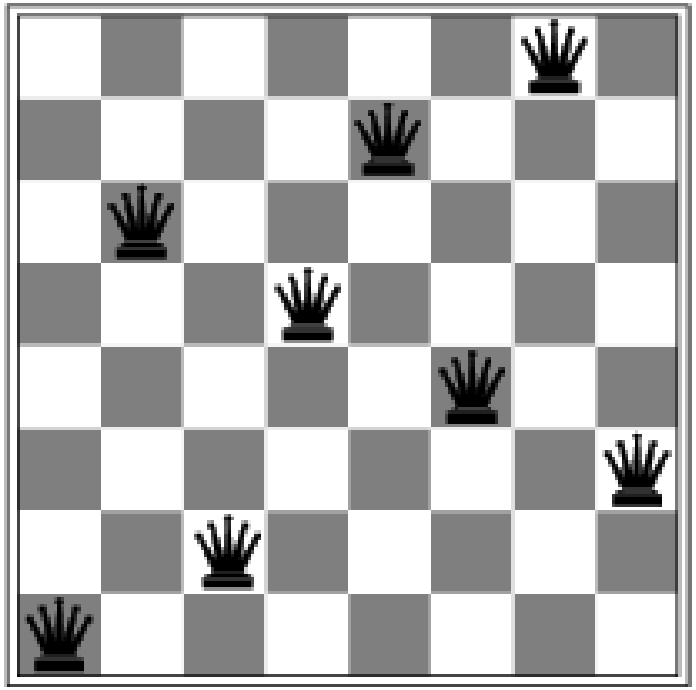


Example: *8*-queens



- *h* = No. of pairs of queens that are attacking each other, either directly or indirectly
- *h* = *17* for the above state

Hill-climbing Search: 8-queens problem



- A local minimum with $h = 1$

Hill Climbing Search

Simple-Hill-Climber (S)

- Evaluate S;   If Goal state return and quit
- Loop until a solution is found or no neighbors left
  - Look at next neighbor NN
  - Evaluate NN
    - If NN is Goal return and quit
    - If NN is better than S, S := NN
    - Reset neighbors

Hill Climbing Search

Steepest-Ascent-HC (S)

- Evaluate S;   If Goal state return and quit
- SUCC := S
- Loop until a solution is found or no neighbors left
  - For all neighbors (NN) of S
    - ➢ Evaluate NN
    - ➢ If NN is Goal then return NN and quit
    - ➢ If NN is better than SUCC then SUCC := NN
  - If SUCC is better than S then
    - ➢ S := SUCC
    - ➢ Reset neighbors

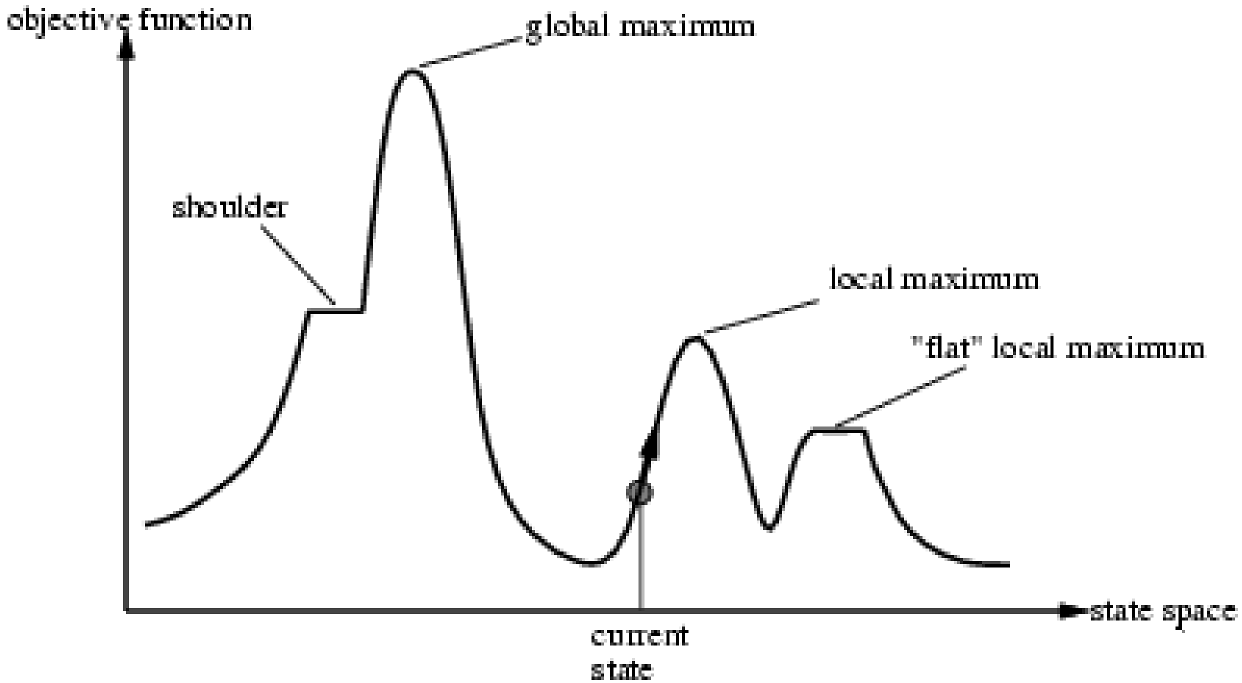
Hill Climbing Continued

Stochastic-Hill-Climber (S)

- Evaluate S;   If Goal state return and quit
- Loop until a solution is found or no neighbors left
  - Look at some random neighbor RN
  - Evaluate RN
    - If RN is Goal return and quit
    - If RN is better than S
      - S := RN
      - Reset neighbors

Hill Climbing Search

Problem: Local maxima or plateau…



Problems with Hill Climbing

- Hill Climbing will get stuck at local maxima in the space
- Can get stuck on a “plateau”

Solutions

- Backtrack to earlier node and force it to go in a new direction
- Take a big jump to somewhere else in search space
- Simulated Annealing (Will study this next)
- Genetic Algorithms

Simulated Annealing Search

- Simulate the annealing process of creating metal alloys
- Start off hot, and cool down slowly which allows the various metals to crystallize into a global uniform structure
- If cooled too fast the metals crystallize in pockets
- If cooled too slowly, a uniform crystallization but wastes time

Simulated Annealing Search

- Use this idea to try to find global minimum
- Now finding minimum instead of maximum -- but it's the same
- Wander from the hill-climbing while system still hot
- Reduce to hill climbing as system cools

Properties: Simulated Annealing

- One can prove:
  - If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching **unity**

- Widely used in VLSI layout, airline scheduling, etc

Details: Simulated Annealing

- The probability to move to a higher energy state in physics is

$$p = \frac{1}{e^{\Delta E / kT}}$$

where k is the Boltzman constant

- Similarly, in SA (when finding the minimum), the probability to move to a state with a higher (worse) heuristic is:

$$p = \frac{1}{e^{\Delta E / T(t)}}$$

where

ΔE = (value of current state) - (value of new state)

T(t) is the temperature schedule (a function of time t)

– Temperature monotonically decreases with time,

– Eventually T reaches 0 when the system becomes simple “hill descending”

SA Details When Maximizing

- The probability to move to a state with a lower (worse) heuristic function evaluation in SA is

$$p = e^{\Delta E/T}$$

where

ΔE = (value of new state) - (value of current state)

(The negation of the ΔE used when minimizing)

T(t) is the temperature schedule (a function of time t)

- Temperature monotonically decreases with time
- Eventually T reaches 0 when the system becomes simple "hill climbing"

Simulated Annealing Algorithm

Simulated-Annealing (problem, schedule)   From Russell and Norvig

Current := Initial-State(Problem)

for t := 1 to $\propto$ do

T := schedule(t)

If T = 0 then return Current

Next := a randomly selected successor of Current

ΔE := Value(Next) - Value(Current)

If $\Delta E > 0$ then

"Always go to a better solution"

Current := Next

Else

"Leave a better solution for a worse one with prob. $e^{\Delta E/T}$"

Current := Next only with probability $e^{\Delta E/T}$

SA: Meta Heuristics

- If the solution is better:
  - Always move to it
- If the solution is worse but the slope up is shallow:
  - Try it out
- If the solution is worse but the slope is steep:
  - Don’t try it out as readily (with an exponentially decreasing probability)
- As time goes on, don’t try worse solutions as frequently
  - Again with an exponentially decreasing probability

SA Effects

- **At the beginning of the process** (when T(t) is large)
  - The probability of moving to poorer states, or moving along a plateau is large.
  - So the space can be well searched
  - Local minimums can be passed over
  - Ignore steep ascents
    - This implies that you are in a deep valley, which is assumed to be good

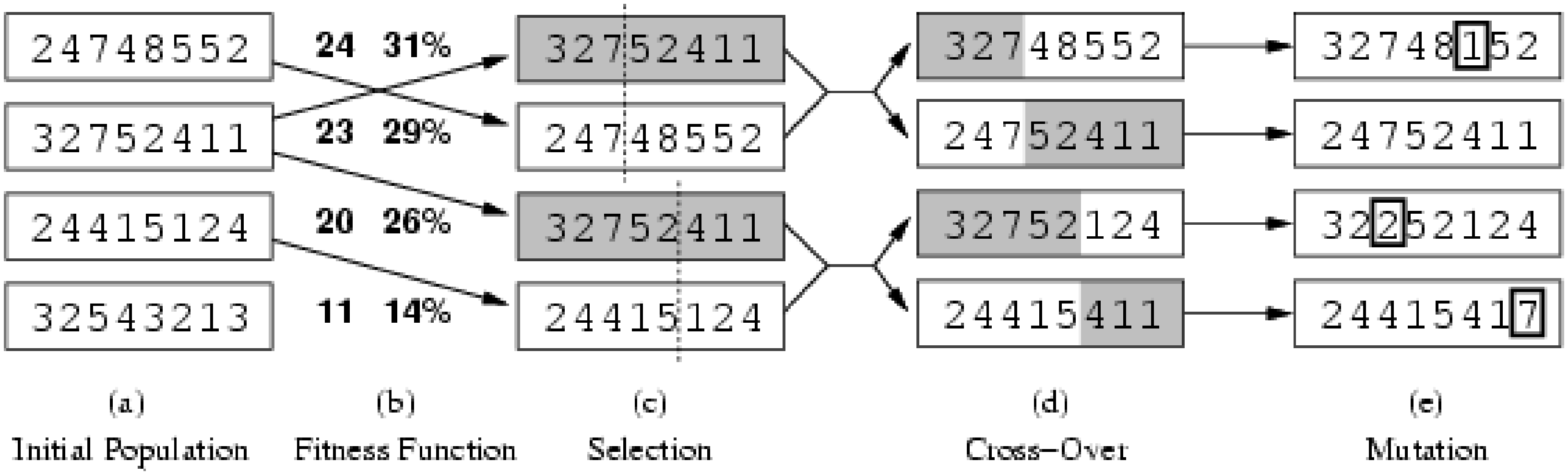
- **As time increases**
  - The search gets trapped in one valley and gets stuck as T(t) becomes small
  - The probability of getting out of the Valley is too small.

- **At this time**
  - SA becomes “hill descending”
  - Descends to the bottom of that valley - hopefully the global minimum

Genetic Algorithms

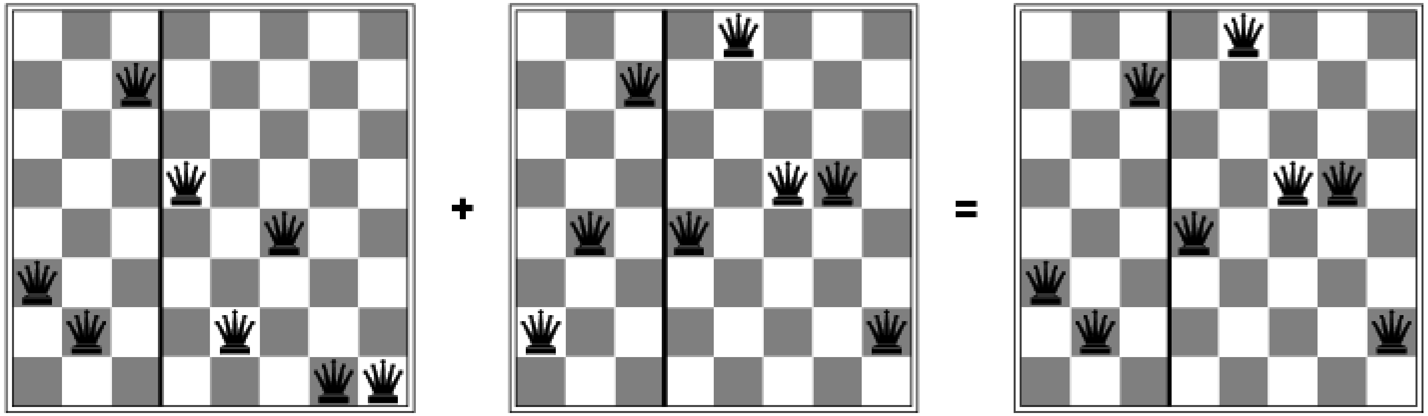
- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic Algorithms



- Fitness function: Number of non-attacking pairs of queens
  (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic Algorithms

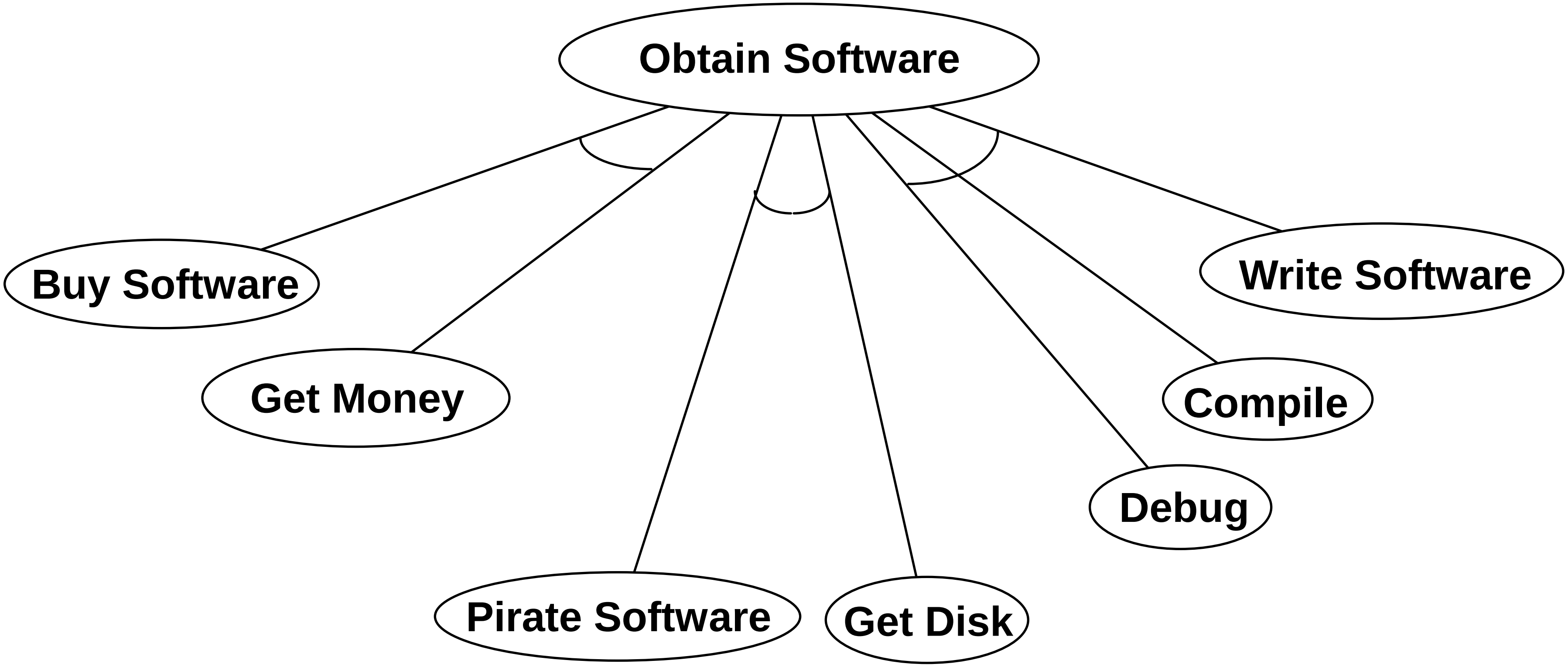


OR Graphs vs. AND-OR Graphs

- In the previous search techniques, Solution can be found down any path independent of any other path
- This is called an OR graph
- However, there may be sub-goals that must all be solved for a solution to be found
  - Each sub-goal is its own sub-tree
  - All sub-trees must have its own end state found if the path is to be considered satisfied
- This is called an AND-OR graph

Example of an AND-OR Graph

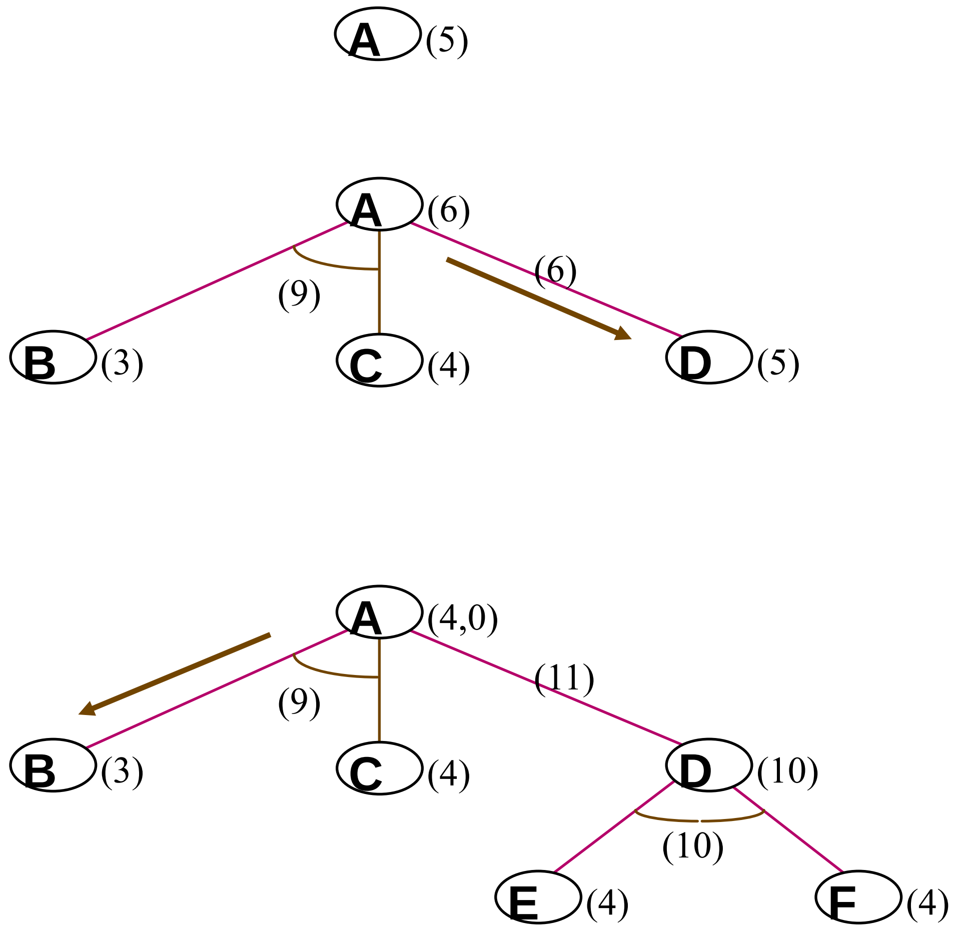
- Getting software to accomplish a task



Problem Reduction Algorithm

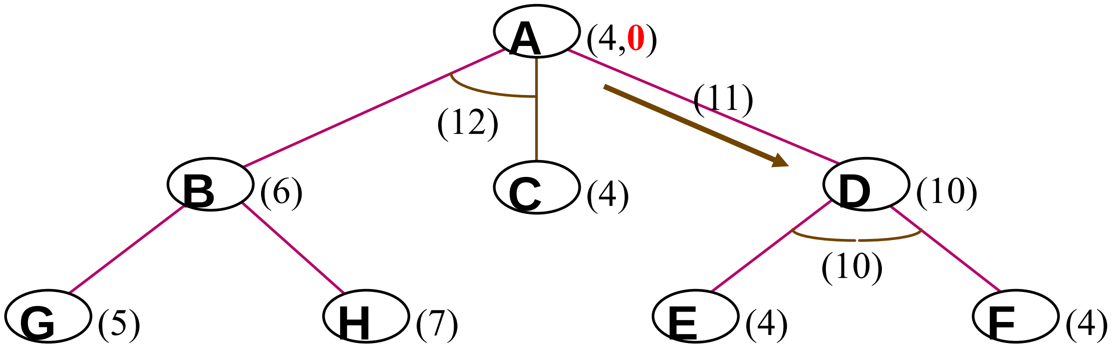
- Initialize the graph to the starting node
- Until the starting node is labeled SOLVED or its cost > FUTILITY do:
  - Start at initial node and traverse best path
    - Accumulate set of nodes on path not expanded or labeled SOLVED
  - Pick an unexpanded node and expand
    - If no successors, node cost = FUTILITY
    - Add successors to graph after computing the heuristic f for each
    - If f = 0 for any node mark node as SOLVED
  - Propagate change back through path
    - If child is an OR child and is SOLVED mark parent as SOLVED
    - If AND children are all solved, mark parent as SOLVED
    - Change the estimate of f as determined by children
    - As we back up the tree, change current best path associated with each node (on the original best path) if updated f values warrant it

Example of Problem Reduction (AO*)

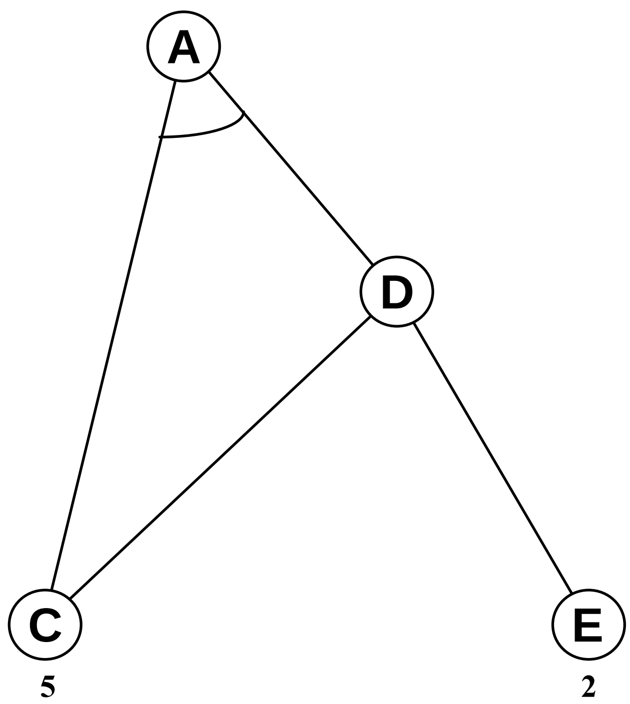


When you calculate costs, remember to use the cost PLUS the depth

Example of Problem Reduction (AO*)



Interacting Sub-goals



Branch and Bound

- If we know that current path (branch) is already worse than some other known path:
  - Stop Expanding It (Bound).

- Have already encountered Branch and Bound:
  - A* stops expanding a branch if its heuristic value h becomes larger than some other branch

Constraint Satisfaction Problems and Branch and Bound

- Problems where there are natural constraints on the system (fixed resources, impossibility conditions, etc.)

- Constraints: Handled by Branch and Bound technique
  - Branch out in your normal search pattern
  - Stop expanding a branch if it fails a constraint (backtracking may occur when that happens)

- Trivial example: Missionaries and Cannibals
  - Do not continue to search along a branch if the Cannibals have just eaten some (or all) of the Missionaries

Games vs. Search Problems

- "Unpredictable" opponent
  - Specifying a move for every possible opponent reply
- Time limits
  - Unlikely to find goal, must approximate

Mini-Max Search

- Search to find the correct move in a two player game
- Since 1950's: Has been the foundational scheme
- The optimal solution:
  - Exponential algorithm
  - Generate all possible paths
  - Only play those that lead to a winning final position

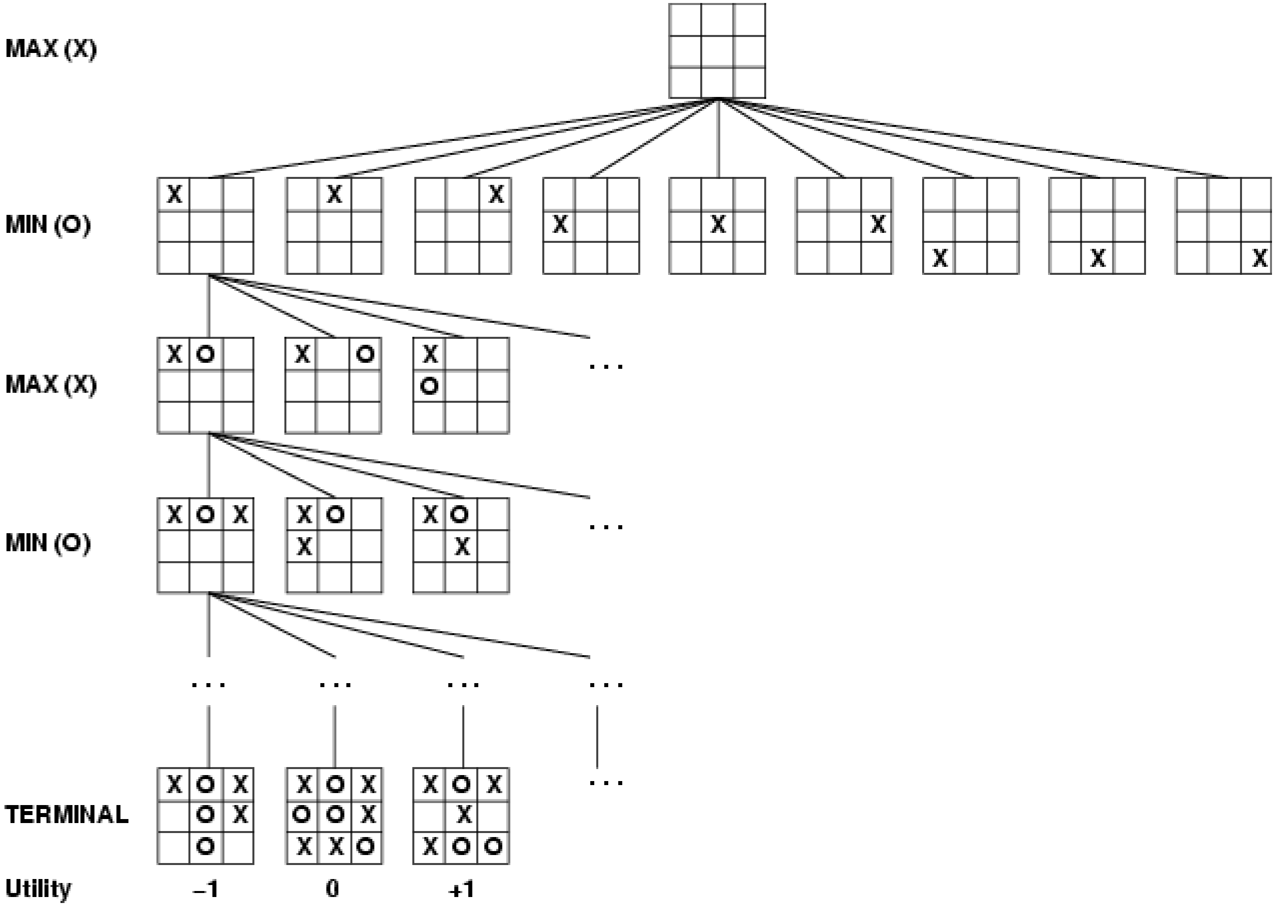
- Realistic alternative to the Optimal

- Use finite depth look-ahead with a heuristic function for evaluating how good a given game state is

Mini-Max

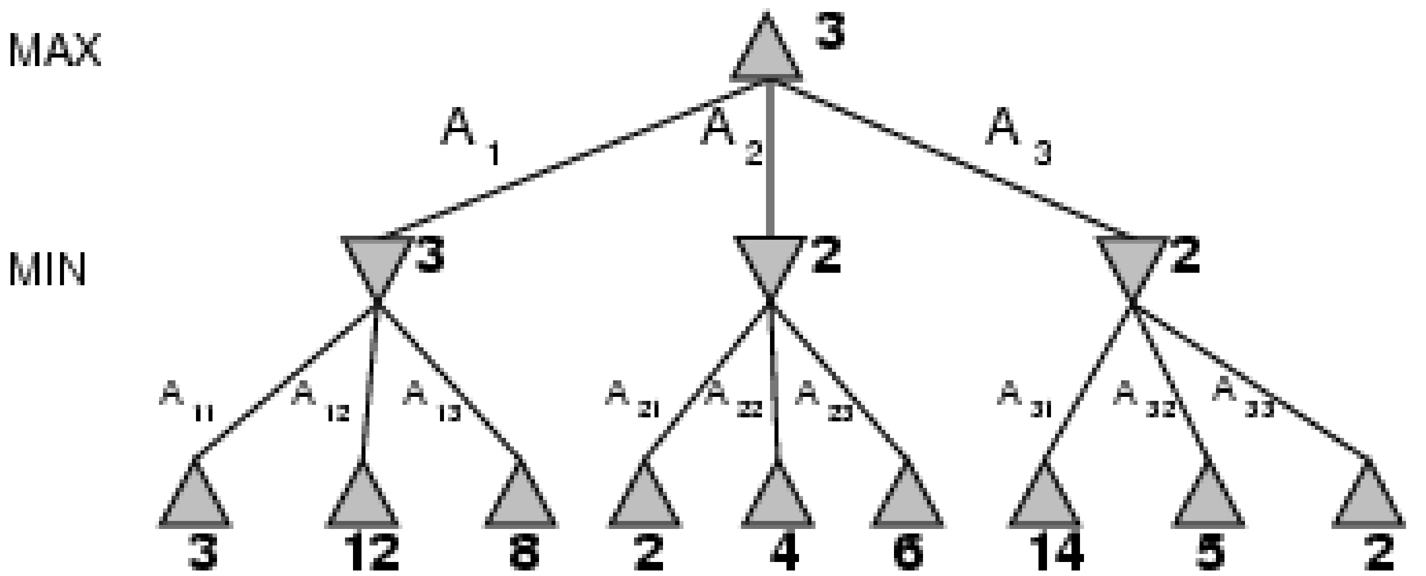
- Extend Tree down to a given search depth
- Top of tree is the Computer's move
  - Wants move to ultimately be one step closer to a winning position
  - Wants move that maximizes own chance of winning
- Next move is Opponent's
  - Opponent assumed to perform a move that his best
  - Wants move that minimizes Computer's chance of winning

Game tree
2-player, Deterministic, Turns



Mini-Max

- **Perfect** play for deterministic games
- **Idea**: Choose move to position with highest Mini-Max value
  = Best achievable payoff against best play
- Example: 2-ply game:



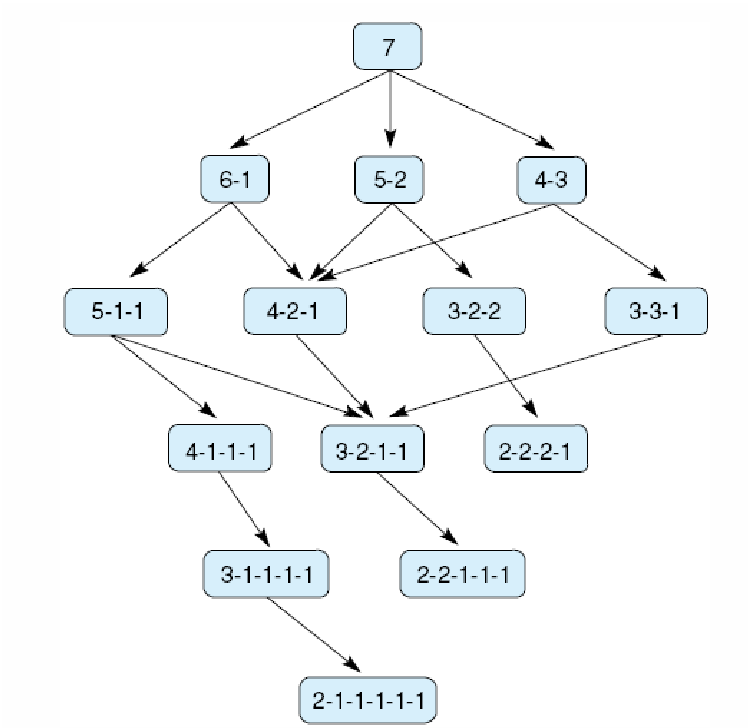
Mini-Max for Nim

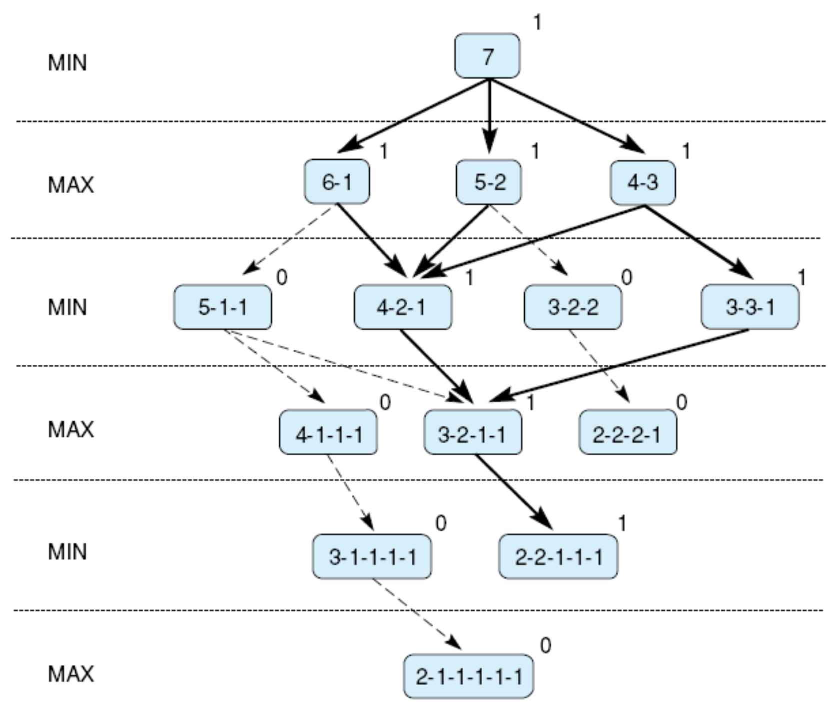
- Game of Nim
  - Two players start with a pile of tokens
  - Legal move: Split (any) existing pile into two non-empty differently sized piles
  - Game ends when no pile can be unevenly split
  - Player who cannot make his move loses the game
- Search strategy
  - Existing heuristic search methods not needed
  - Search the *whole* tree

Mini-Max for Nim

- Label nodes as MIN or MAX, alternating for each level
- Define utility function (payoff function).
- Do full search on tree
  - Expand all nodes until game is over for each branch
- Label leaves according to outcome
- Propagate result up the tree with:
  - M(n) = max(child nodes) for a MAX node
  - m(n) = min(child nodes) for a MIN node
- Best next move for MAX is the one leading to the child with the highest value (and vice versa for MIN)

Mini-Max for Nim





Mini-Max Algorithm

- Operator: The same as “move” to be made
- Utility: The value of the heuristic at that juncture
- **EVAL**: Computes this heuristic value
- Cutoff: Either Game is *Done* or Search *Deep Enough*
- Successors: Possible moves at the next level
- Max and Min algorithms are almost identical
- **MINIMAX-DECISION**: The actual decision that is made

Mini-Max Algorithm

```
function MINIMAX-DECISION(game) returns an operator
    for each op in OPERATORS[game] do
        VALUE[op] := MIN-VALUE(APPLY(op, game), game)
    end
    return the op with the highest VALUE[op]

function MAX-VALUE(state, game) returns a utility value
    if CUTOFF-TEST(state) then return EVAL(state)
    value := - ∞
    for each s in SUCCESSORS(state) do
        value := MAX(value, MIN-VALUE(s, game))
    end
    return value

function MIN-VALUE(state, game) returns a utility value
    if CUTOFF-TEST(state) then return EVAL(state)
    value :=  ∞
    for each s in SUCCESSORS(state) do
        value := MIN(value, MAX-VALUE(s, game))
    end
    return value
```

Problems with Mini-Max

- Horizon Effect: Finite Depth; Can't see beyond
  - Exponential increase in tree size, only very limited depth feasible
  - Solution: Quiescence search (a state of quietness or inactivity)
    - » Start at the leaf nodes of the main search
    - » Try to solve this problem
    - » Is there something "obvious" we are missing?
    - » One option is good but all other options look bad???
  - In Chess: Quiescence searches usually include all capture moves
    - » Tactical exchanges don't mess up the evaluation (PXB; QXB)
    - » Quiescence searches: Look for moves which destabilize the evaluation function
    - » If there is such a move: The position is not quiescent

Problems with Mini-Max

- May want to use look up tables
  - For end games
  - Opening moves (called Book Moves)

Properties of Mini-Max

- **Complete?**
  - Yes (if tree is finite)
- **Optimal?**
  - Yes (against an optimal opponent)
- **Time complexity?**
  - $O(b^m)$
- **Space complexity?**
  - O(bm) (depth-first exploration)
- Chess: $b \approx 35$, $m \approx 100$ for “reasonable” games
  - Exact solution completely infeasible
  - Shannon: Search space as large as 10^{42}

Branch and Bound: The α-β Algorithm

- Branch and Bound:
  - If current path (branch) is worse then some other *known* path:
  - Stop expanding it (bound).

- Alpha-Beta:
  - A branch and bound technique for Mini-Max search
  - Know that the level above won't choose your branch
    - » Because you have already found a value along one of your sub-branches that is too good
    - » Stop looking at other sub-branches that haven't been looked at yet

The α-β Algorithm

- Instead of maintaining a single mini-max value
  - The α-β pruning algorithm, maintains two: α, β

- Together:
  - Provide a bound on the possible values of the mini-max tree

- At any given point, α: minimum the player can expect
- At any given point, β: maximum the
- Guarantee: *I can always get between α and β*

The α-β Algorithm

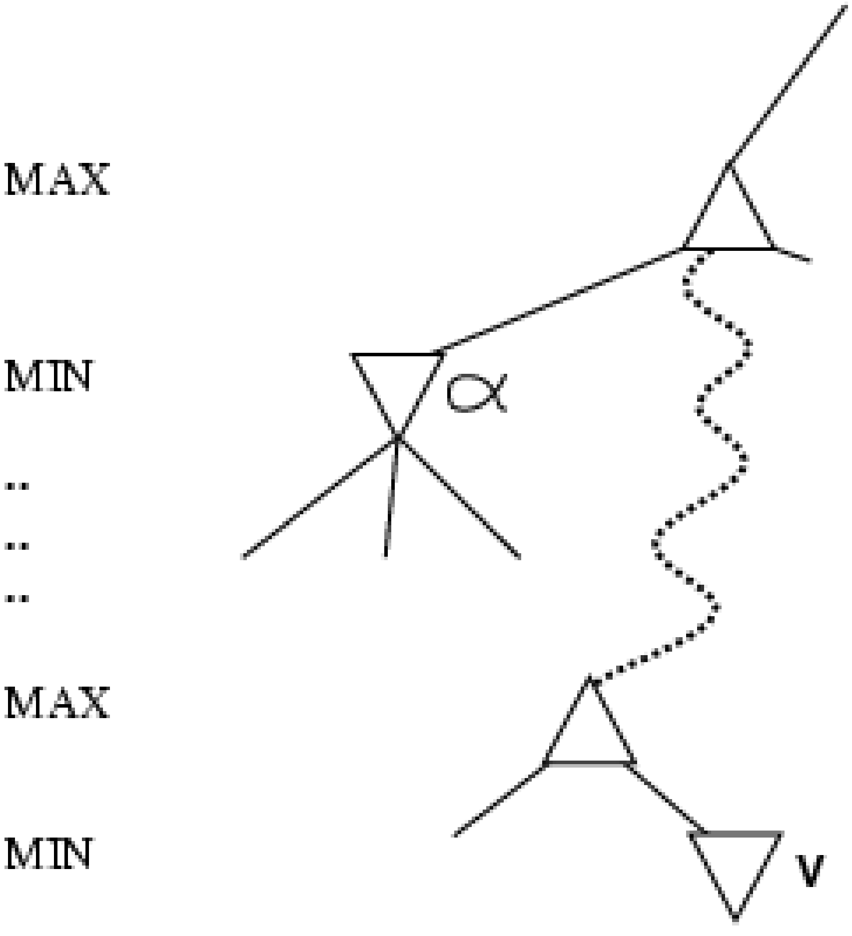
- If ever ($\beta <= \alpha$)*:* Bound is reversed or range of 0
  - *B*etter options exist for the player at other pre-explored nodes
- As α is the minimum value we know we can get
  - This node cannot be the mini-max value of the tree.
  - No point in exploring any more of this node's children
- Potentially save considerable computation time
- Fantastic when large branching factor/depth

Properties of α-β

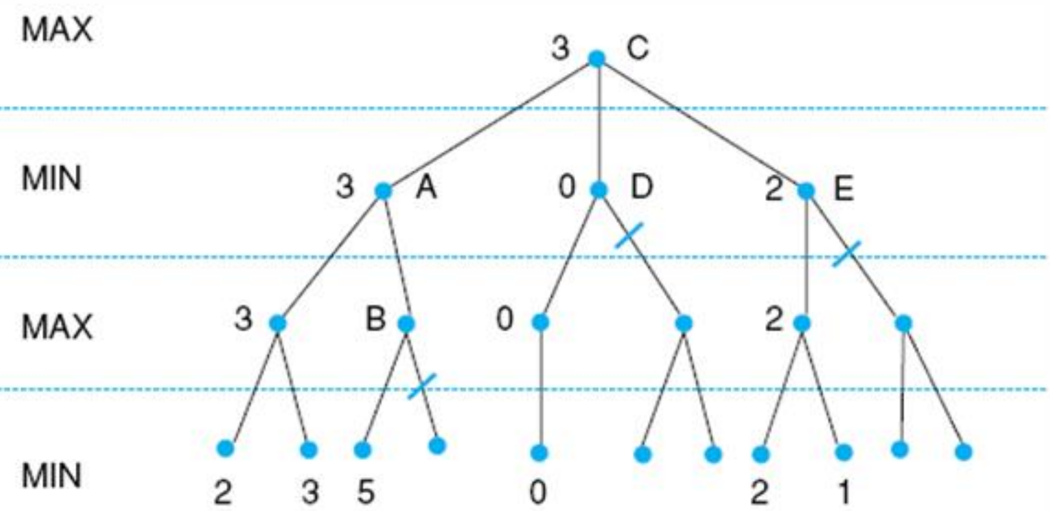
- Pruning does not affect final result (The Mini-max soln.)
- Good move ordering improves pruning effectiveness
- With "perfect ordering" time complexity = $O(b^{m/2})$
  - Doubles depth of search
- α-β Search
  - A simple example of the value of reasoning
  - Which computations are really relevant

Why it is called α-β

- α: Value of the best choice found so far at any choice point along the path for *max*
- If *v* is worse than α
  - *max* will avoid it
  - prune that branch
- Define β similarly for *min*



Effects of α-β



A has $\beta = 3$ (A will be no larger than 3)

B is β pruned, since $5 > 3$

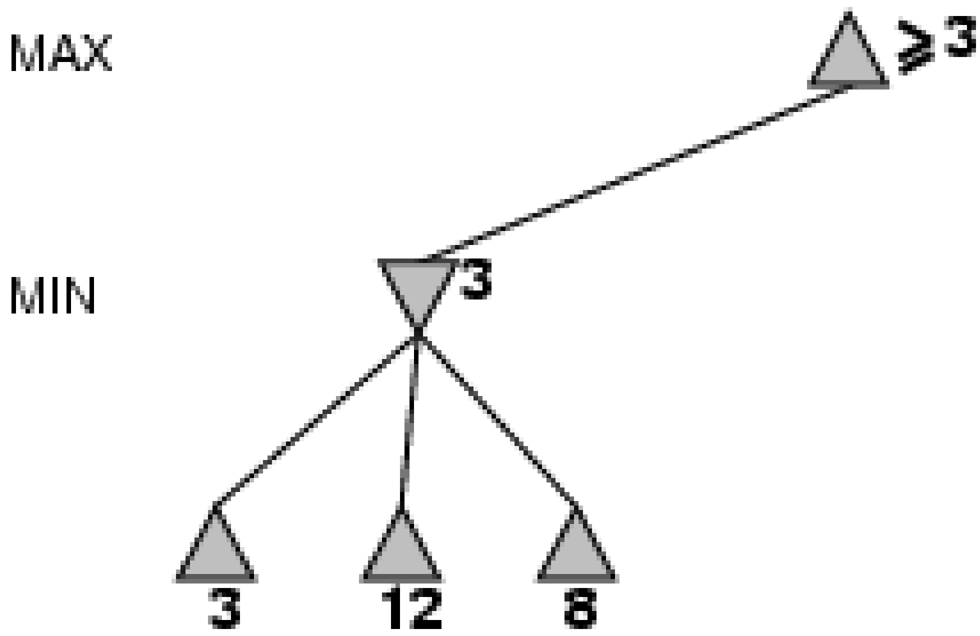
C has $\alpha = 3$ (C will be no smaller than 3)

D is α pruned, since $0 < 3$

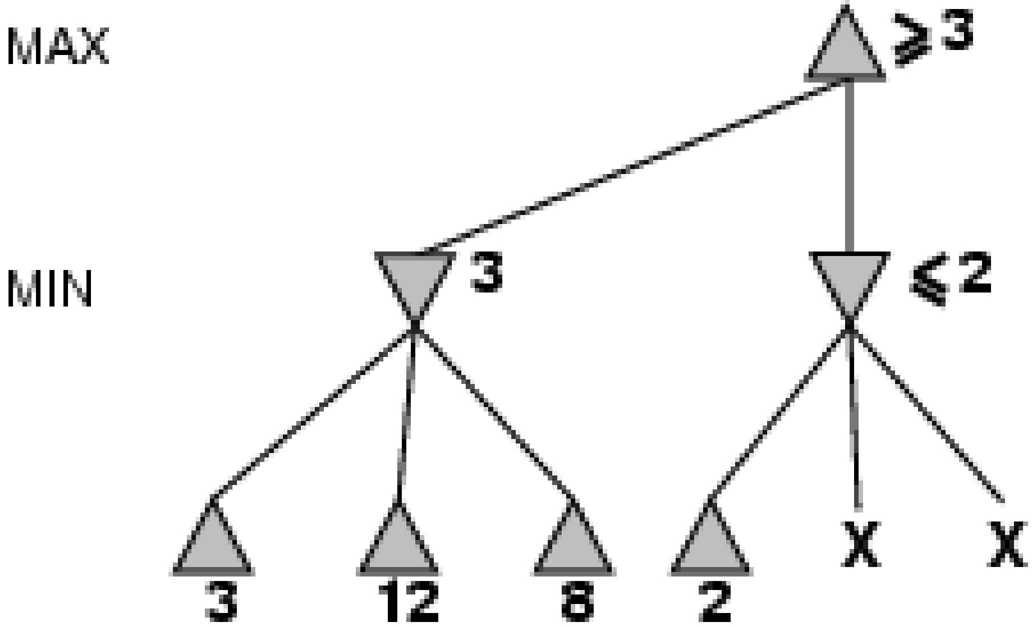
E is α pruned, since $2 < 3$

C is 3

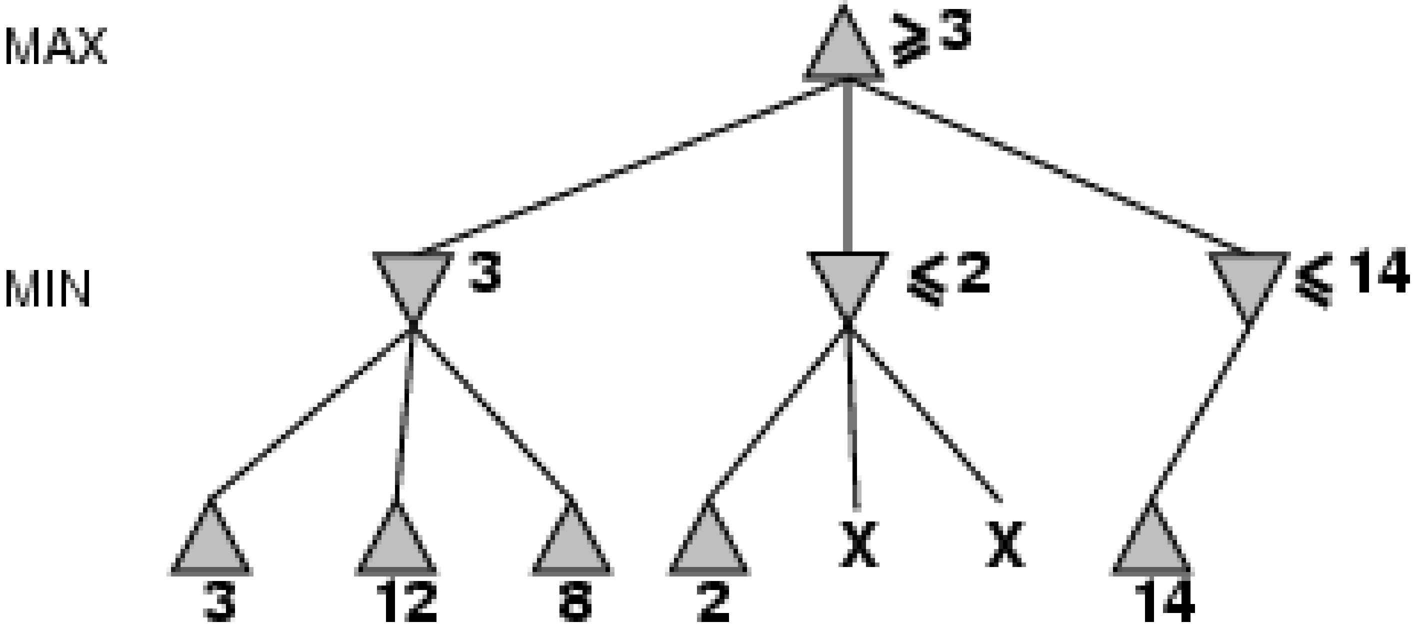
Example: α-β Pruning



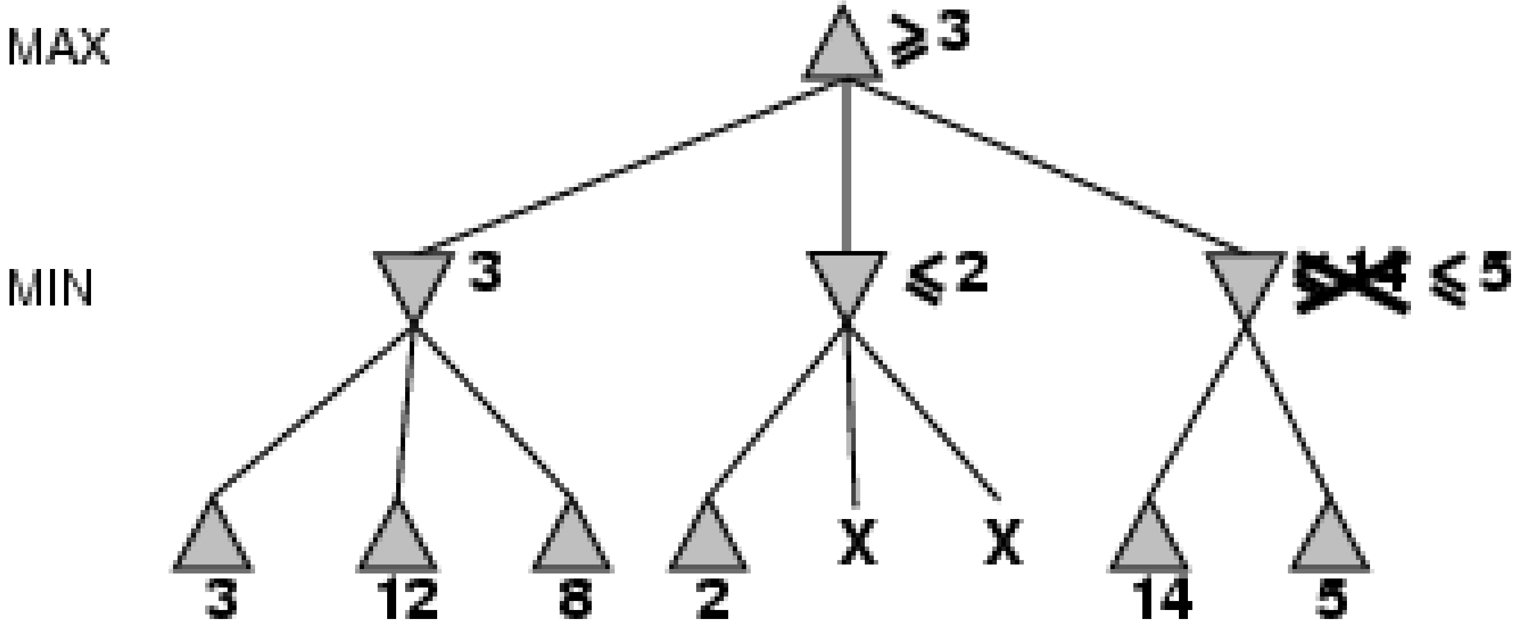
Example: α-β Pruning



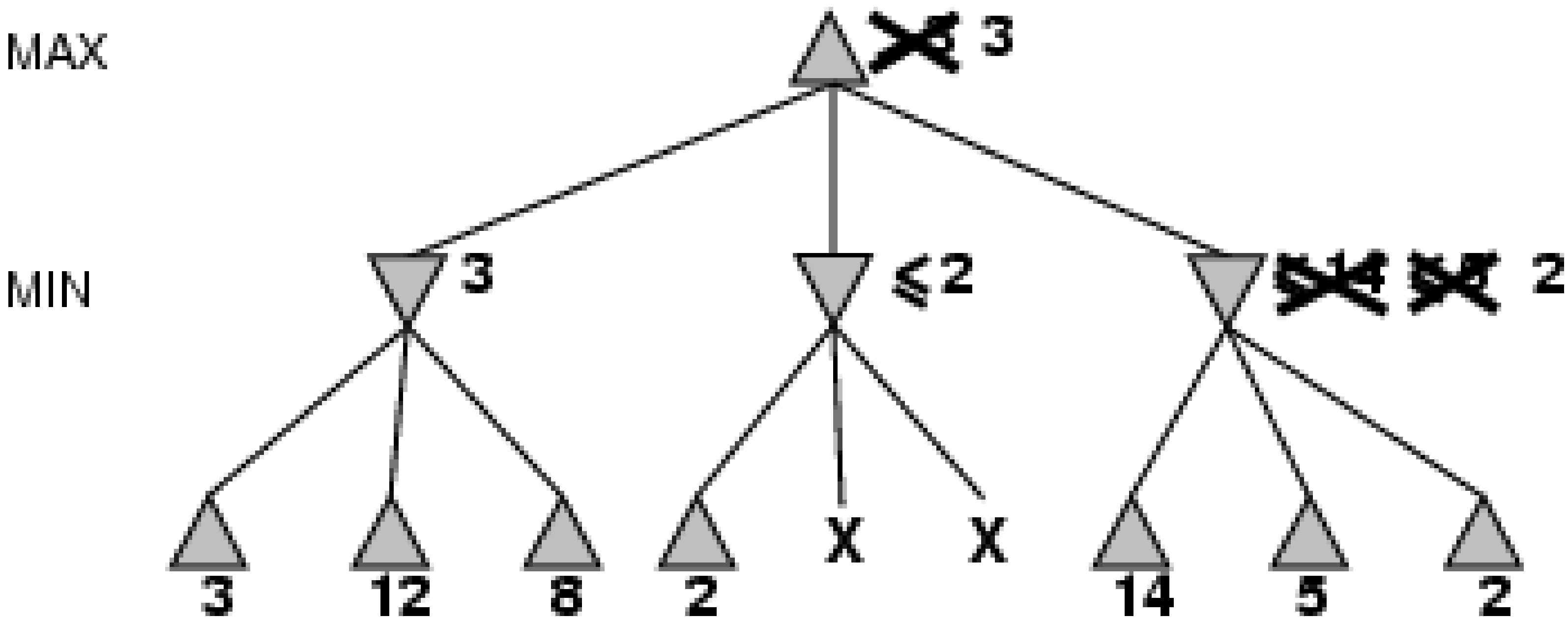
Example: α-β Pruning



Example: α-β Pruning



Example: α-β Pruning



The α-β Algorithm

- From Russell and Norvig
- Only Change from Mini-Max: The lines in Green

α = best score for MAX so far
β = best score for MIN so far

game = game description
state = current state in game

function MAX-VALUE(*state, game, α, β*) **returns** *a utility value*

if CUTOFF-TEST(*state,*) **then return** EVAL(*state)*
for each *s* **in** SUCCESSORS(*state*) **do**
α := MAX(*α,* MIN-VALUE(*s, game, α, β*))
if α ≥ β **then return** α /*Only line that is different*/
end
return α

function MIN-VALUE(*state, game, α, β*) **returns** *a utility value*

if CUTOFF-TEST(*state*) **then return** EVAL(*state)*
for each *s* **in** SUCCESSORS(*state*) **do**
β := MIN(*β,* MAX-VALUE(*s, game, α, β*))
if β ≤ α **then return** β /*Only line that is different*/
end
return β

The α-β Algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

The α-β Algorithm

```
function MIN-VALUE(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for  MAX along the path to state
           β, the value of the best alternative for  MIN along the path to state

   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← +∞
   for a, s in SUCCESSORS(state) do
      v ← MIN(v, MAX-VALUE(s, α, β))
      if v ≤ α then return v
      β ← MIN(β, v)
   return v
```

Improving Game Playing

- Increase Depth of Search
- Have better heuristic for game state evaluation

Changing Levels of Difficulty

- Increase Depth of Search

Resource Limits

- Suppose we have 100 secs, explore 10^4 nodes/sec
  - 10^6 nodes per move

- Standard approach:
  - Cutoff test: Depth limit (perhaps add quiescence search)
- Evaluation function:
  - Estimated desirability of position

Quiescence search

- Quiescence search: Study moves that are noisy
- They appear good, but moves around them - bad
- Investigate them with a localized leaf search
- Attempt to identify delaying tactics and change the seemingly-good value of the node
- A very natural extension of Mini-Max
- Simply run search again at a leaf node until that leaf node becomes quiet
- As with iterative deepening, running time of the algorithm won’t increase by more than a constant

Evaluation Functions

- Chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- Example: $w_1 = 9$ with
  $f_1(s)$ = (number of white queens) – (number of black queens)
  etc.

Cutting-Off Search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?* (Have I reached a *Cutoff* Point)
2. *Utility* is replaced by *Eval*

Does it work in practice?

$b^m = 10^6$, b=35 → m=4

4-ply lookahead is a hopeless Chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov

Real Deterministic Games

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
  - Used a precomputed endgame database
  - Defining perfect play for all positions involving 8 or fewer pieces on the board - a total of 444 billion positions.
- Chess: Deep Blue defeated human world champion Kasparov in a six-game match in 1997.
  - Deep Blue searches 200 million positions per second
  - Uses very sophisticated evaluation
  - Undisclosed methods for extending some lines of search up to 40 ply.

Real Deterministic Games

- Othello: Human champions refuse to compete against computers, who are too good.

Things to Remember: Games

- Games are fun to work on!
- They illustrate several important points about AI
- Perfection is unattainable
- Must approximate paths and solutions
- Good idea to think about what to think about