Estimate the First and Second Order Marginals

1. The joint probability distribution of the random vector $\mathbf{V} = [V_1, V_2, \ldots, V_d]^T$ in terms of conditional probabilities is given as:

$$P(\mathbf{V}) = P(V_1)P(V_2|V_1)P(V_3|V_1, V_2) \ldots P(V_d|V_1, V_2, \ldots, V_{d-1}),$$

2. Each V_i is conditioned on an increasing number of other variables.

3. Approximation: Use only the lower-order marginals:

$$P_a(\mathbf{V}) = \prod_{i=1}^{d} Pr(V_i|V_{j(i)})$$

$P_a(\mathbf{V})$ is the approximated form of $P(\mathbf{V})$. V_i conditioned on $V_{j(i)}$.

Create a Fully Connected Undirected Graph

The dependence of the variables can be represented as

$$\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{W})$$

- **V** is a finite set of vertices;
  - V_j represents the random variables.
- **E** is a finite set of edges;
  - $\langle V_i, V_j \rangle$ represents an edge between the vertices V_i and V_j.
- **W** is a finite set of weights;
  - $w_{i,j}$ is the weight assigned to the edge $\langle V_i, V_j \rangle$.

Create a Fully Connected Undirected Graph

The values of weights can be calculated based on either:

1. Expected Mutual Information Measure:

$$I^*(V_i, V_j) = \sum_{v_i, v_j} Pr(v_i, v_j) \log \frac{Pr(v_i, v_j)}{Pr(v_i)Pr(v_j)} \ .$$

2. Chi-Square Metric:

$$I_\chi(V_i, V_j) = \sum_{v_i, v_j} \frac{(Pr(v_i, v_j) - P(v_i)P(v_j))^2}{P(v_i)P(v_j)} \ .$$

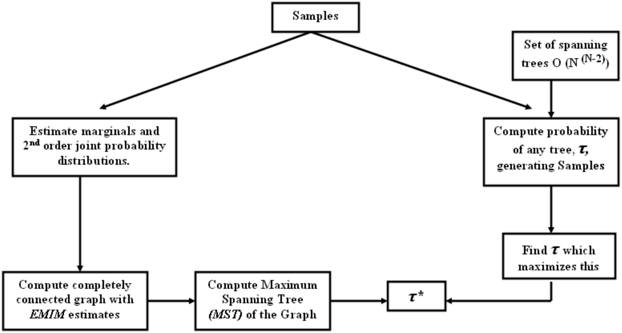
3. Assuming Normality (Correlation Matrix):

$$I^*(V_i, V_j) = -\frac{1}{2} \log(1 - \rho_{ij}^2) \ .$$

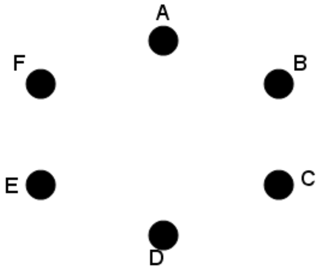
Create a Fully Connected Undirected Graph

1. The connected graph consists of a large number of trees ($\bigcirc(d^{(d-2)})$).
2. Each tree represents a unique approximated form of $P(\mathbf{V})$.
3. The best “dependence tree” $\Rightarrow$ the Maximum Spanning Tree.
4. Maximum Likelihood method - Estimates the unknown probabilities.
5. The *MLE* of best dependence tree - Compute the *MST* of the graph.

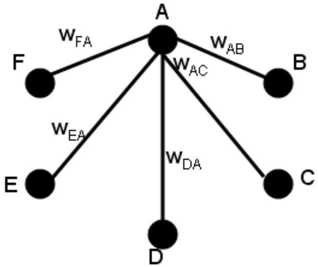
ML Estimate of the Best Dependence Tree



Example



Example



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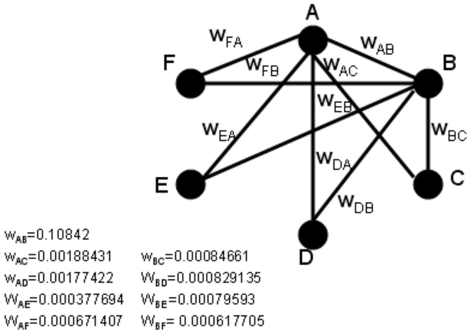
w_{AC}=0.00188431
w_{AD}=0.00177422
W_{AE}=0.000377694
W_{AF}=0.000671407

Example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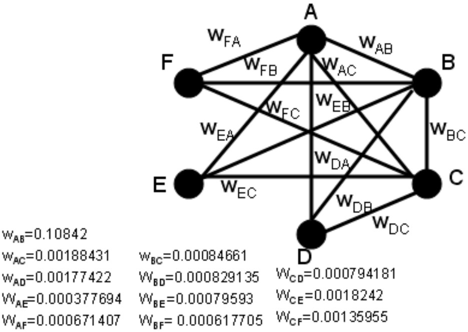
Example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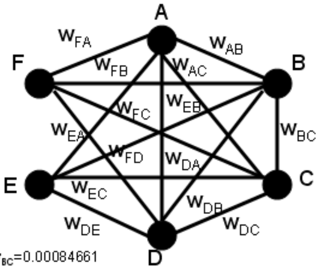
Example



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W_{CE}=0.0018242
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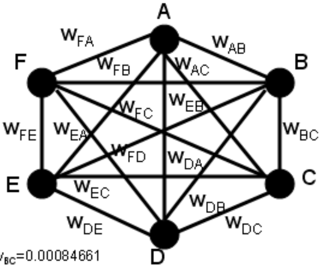
W_{DF}=0.000441118

Example



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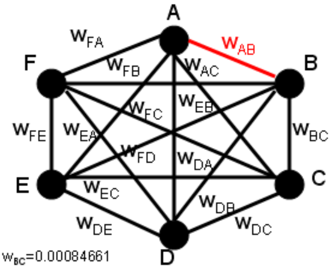
Example



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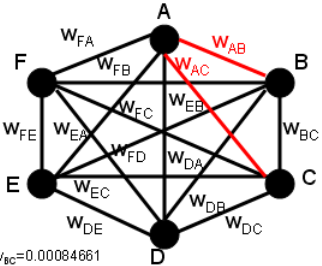
Example



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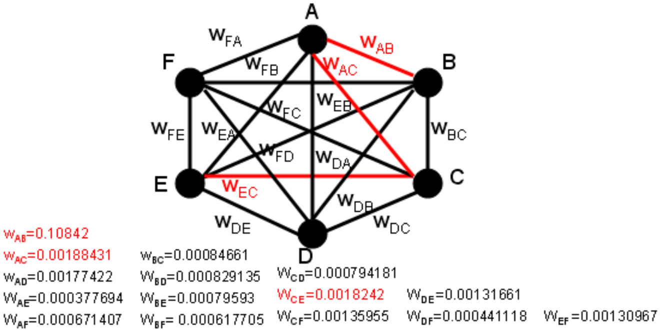
Example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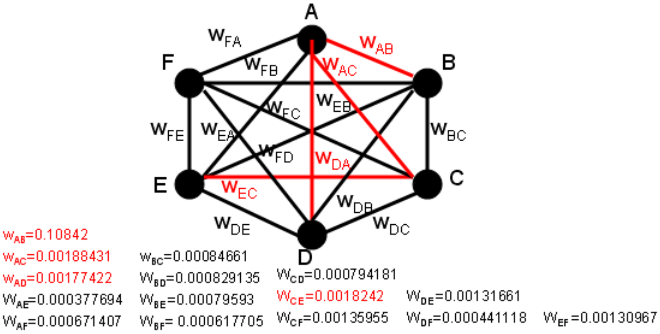
Example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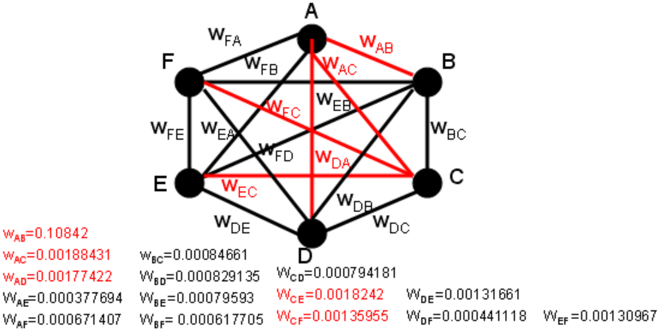
Example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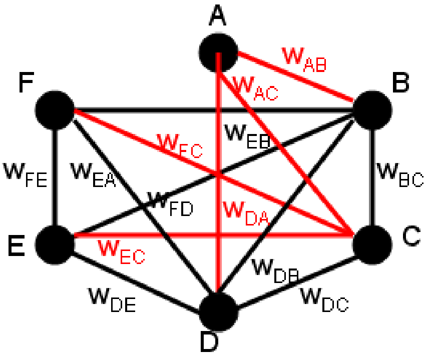
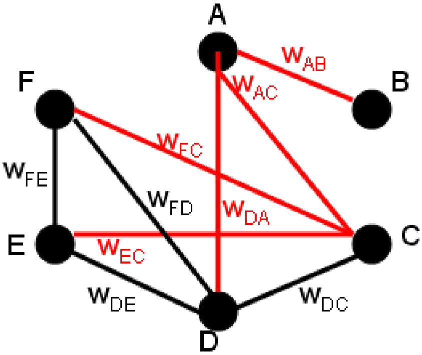
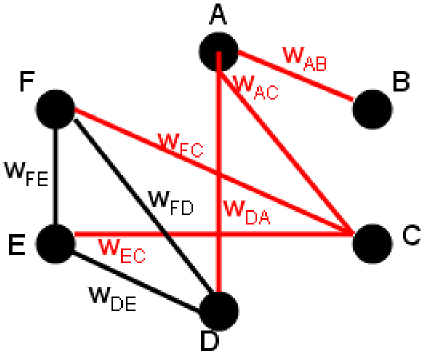


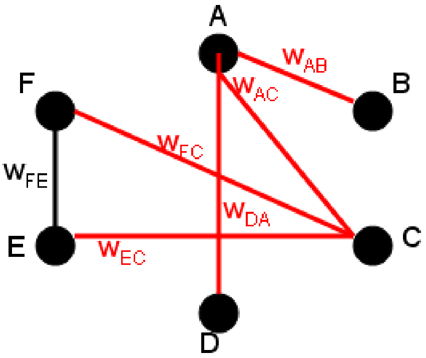
Example



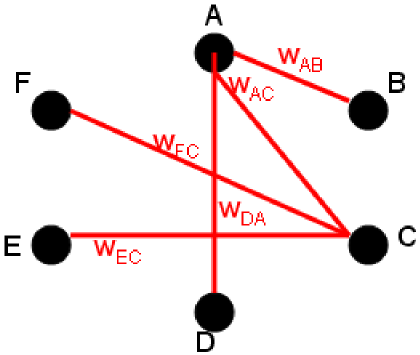
Example



Example



Example



Example