Consider a two-class problem in which the class conditional distributions are both normally distributed in 3-dimensions with means $M_1$ and $M_2$, where:

$$M_1 = [3 \quad 1 \quad 4], \text{ and, } \quad M_2 = [-3 \quad 1 \quad -4].$$

The covariance matrices $\Sigma_1$ and $\Sigma_2$ are:

$$\Sigma_1 = \begin{bmatrix}
    a^2 & \beta ab & \alpha ac \\
    \beta ab & b^2 & \beta bc \\
    \alpha ac & \beta bc & c^2
\end{bmatrix}$$

and

$$\Sigma_2 = \begin{bmatrix}
    c^2 & \alpha bc & \beta ac \\
    \alpha bc & b^2 & \alpha ab \\
    \beta ac & \alpha ab & a^2
\end{bmatrix}$$

(a) Write a program to generate Gaussian random vectors assuming that you only have access to a function which generates Uniform random variables.

(b) Using the strategy taught in class, write a program to simultaneously diagonalize both the distributions. Print out the diagonalizing matrices for a few cases, and in particular, for the case of $a=2$, $b=3$, $c=4$ and $\alpha=0.1$, $\beta=0.2$. Show the intermediate covariance matrices in the process.

(c) Generate 200 points of each distribution for the case of $a=2$, $b=3$, $c=4$ and $\alpha=0.1$, $\beta=0.2$. Before diagonalization and plot them in the $(x_1-x_2)$ and $(x_1-x_3)$ domains. These points are 200 3-D vectors, but the projected points in the $(x_1-x_2)$ and $(x_1-x_3)$ domains must be plotted graphically.

(d) Consider the same 200 generated in (b) above for the case of $a=2$, $b=3$, $c=4$ and $\alpha=0.1$, $\beta=0.2$ after diagonalization and plot them in the $(x_1-x_2)$ and $(x_1-x_3)$ domains. Again remember that these points are 200 3-D vectors, but the points in the $(x_1-x_2)$ and $(x_1-x_3)$ domains must be plotted graphically.