COMP 4102A: Assignment 1
Due: Monday, Feb. 4 at 9:00 AM, 2013

1. (1/2 mark) Why are homogeneous co-ordinates used in computer vision? I want to know what the use of homogenous co-ordinates makes possible in terms of camera models.

2. (1 mark) Consider the case where a point P = (X,Y,Z) projects to the image plane. Using the matrix below which defines the projection of this point for a camera with focal length f to prove that every other point on the line from the origin through the point P projects to the same image point.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \qquad \begin{aligned} x &= u/w \\ y &= v/w \end{aligned}$$

3. (1 mark) A 3d rotation matrix has 9 (3 by 3) entries, and a 2d rotation matrix has 4 (2 by 2) entries. How many actual degrees of freedom are there in a 3d or 2d rotation? In other words, what is the minimum number of parameters necessary to uniquely specify a 3d rotation and a 2d rotation (answer with 2 numbers)? What does this imply about the relationship between the entries of a rotation matrix?

4. (2 mark) In the thin lens equation (as shown in Figure 2.4 of Ch 2, p. 21, in the book Introductory Techniques for 3d Computer Vision which is on the CD) first prove that Zz = f*f (Hint use this diagram along with the characteristics of similar triangles). Then use this equation along with the fact that Z^ = Z + f, z^ = z + f to prove the thin lens equation, which is equation 2.2 in the book. What are the two values of z^ for the cases where Z^ is plus infinity, and where Z^ = f.

5. (1 1/2 marks) A pinhole camera has focal length $f = 500$, pixel sizes $s_x = s_y = 1$, and its principal point is at $(o_x, o_y) = (320,240)$. The world coordinate frame and the camera coordinate frame can be related by $X_c = RX_w + T$, where

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$

   (a) Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.
   (b) What are the pixel coordinates of the world point $X_w = \begin{bmatrix} 150 & 200 & 400 \end{bmatrix}^T$

6. (2 marks) Using the same R, T, f, sx, sy, and Xw as given in Question 3 write an OpenCV program that projects this single 3d point using the given camera parameters. This program should call the routine `cvProjectPoints2` but you need to convert the R matrix to the appropriate format. There is a slight problem with this routine. For reasons to complex to explain you should define the 3d object point as `object_points = cvCreateMat(3, 3, CV_32F);` and the 2d image points as `image_points = cvCreateMat(3, 2, CV_32F);`. There is only one point to project (not three) to project but this routine has some problems when it projects only one point. So just ignore the other nine points, and only set up the one point to be projected. The program should print or display the value of Xw, and then the projected x and y pixel positions after calling the appropriate routines. The final pixel values should be approximately equal to what you calculated by hand in Question 3. Include the source code of your program and some proof that the program prints the correct results (copy the printed output or print the results to a file). We do not need the program executable. An old tutorial on OpenCV is in http://www.cs.iit.edu/~agam/cs512/lect-notes/opencv-intro/opencv-intro.html

7. (1 mark) What is the time complexity of a convolution with an n by n sized kernel when using a direct convolution with a 2d mask, and when using a separable kernel. Each of the answers is in the form of O(?) where ? is an expression in n.

8. (1 mark) A square in the 2d plane has the x and y co-ordinates of (0, 0), (1, 0), (0, 1) and (1, 1). Apply a scaling of 0.5 units in the x direction, followed by a translation of three units in the x direction and three units in the y direction. First write down this transformation in homogeneous co-ordinates, and then apply it to the four corners of the square. Then write the new x and y co-ordinates of the four corners.