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# Camera Calibration

COMP4102A

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# Finding camera parameters (intrinsic)

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- Can use the EXIF tag for any digital image
  - Has focal length  $f$  in millimeters but not the pixel size
  - But you can get the pixel size from the camera manual
  - There are only a finite number of different pixels sizes because number of sensing element sizes is limited
  - If there is not a lot of image distortion due to optics then this approach is sufficient (only linear calibration)
- Can perform explicit camera calibration
  - Put a calibration pattern in front of the camera
  - Take a number of different pictures of this pattern
  - Now run the calibration algorithm (different types)
  - Result is intrinsic camera parameters (linear and non-linear) and the extrinsic camera parameters of all the images

# Explicit camera calibration

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- Use a calibration pattern with known geometry
  - In Opencv use a checkerboard
  - Other systems use special targets with known 3d geometry
- Write equations linking co-ordinates of the projected points, and the camera parameters
- From images of the calibration target
  - Intrinsic camera parameters
    - (depend only on camera characteristics)
  - Extrinsic camera parameters
    - (depend only on position camera)
  - In OpenCV the calibration process finds  $f_x$ ,  $f_y$ ,  $o_x$ ,  $o_y$ , along with the distortion parameters
  - We study a method that does not find the distortion parameters

# Calibration using known 3d geometry

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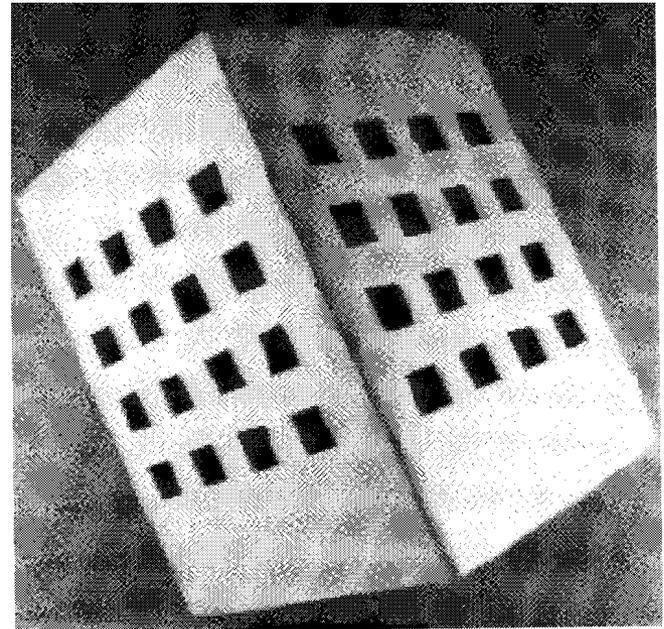
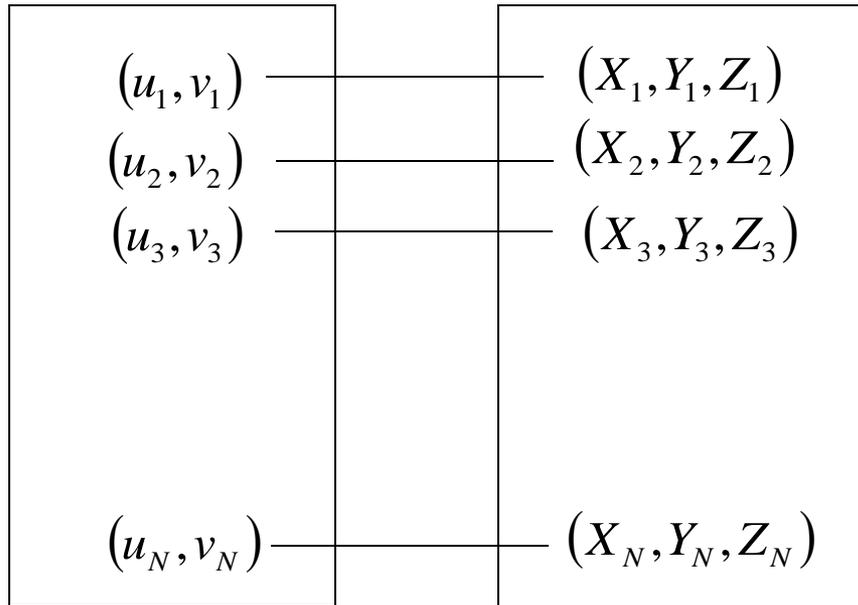
- Use a calibration pattern with known 3d geometry (often a box, not planar)
- Write projection equations linking known coordinates of a set of 3-D points and their projections and solve for camera parameters
- Given a set of one or more images of the calibration pattern estimate
  - Intrinsic camera parameters
    - (depend only on camera characteristics)
  - Extrinsic camera parameters
    - (depend only on position camera)
- We do not estimate distortion parameters

# Estimating camera parameters

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- Projection matrix

Calibration pattern



# Camera parameters

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- Intrinsic parameters (K matrix)
  - There are 5 intrinsic parameters
  - Focal length  $f$
  - Pixel size in x and y directions,  $s_x$  and  $s_y$
  - Principal point  $o_x, o_y$
- But they are not independent
  - Focal length  $f_x = f / s_x$  and  $f_y = f / s_y$
  - Principal point  $o_x, o_y$
  - This makes four intrinsic parameters
- Extrinsic parameters [R| T]
  - Rotation matrix and translation vector of camera
  - Relations camera position to a known frame
  - [R|T] are the extrinsic parameters
- Projection matrix
  - 3 by 4 matrix  $P = K [R | T]$  is called projection matrix

# Projection Equations

## Projective Space

- Add fourth coordinate
  - $P_w = (X_w, Y_w, Z_w, 1)^T$
- Define  $(u, v, w)^T$  such that
  - $u/w = x_{im}, v/w = y_{im}$

$$\begin{pmatrix} x_{im} \\ y_{im} \end{pmatrix} = \begin{pmatrix} u/w \\ v/w \end{pmatrix}$$



$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{M}_{int} \mathbf{M}_{ext} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

## 3x4 Matrix $\mathbf{E}_{ext}$

- Only extrinsic parameters
- World to camera

$$\mathbf{M}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^T & T_x \\ \mathbf{R}_2^T & T_y \\ \mathbf{R}_3^T & T_z \end{bmatrix}$$

## 3x3 Matrix $\mathbf{E}_{int}$

- Only intrinsic parameters
- Camera to frame

$$\mathbf{M}_{int} = \begin{bmatrix} -f_x & 0 & o_x \\ 0 & -f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

## Simple Matrix Product! Projective Matrix

$$\mathbf{M} = \mathbf{M}_{int} \mathbf{M}_{ext}$$

- $(X_w, Y_w, Z_w)^T \rightarrow (x_{im}, y_{im})^T$
- Linear Transform from projective space to projective plane
- $\mathbf{M}$  defined up to a scale factor – 11 independent entries

# Two different calibration methods

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- Both use a set of 3d points and 2d projections
- Direct approach (called Tsai method)
  - Write projection equations in terms of all the parameters
    - That is all the unknown intrinsic and extrinsic parameters
  - Solve for these parameters using non-linear equations
- Projection matrix approach
  - Compute the projection matrix (the 3x4 matrix M)

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix}$$

- Compute camera parameters as closed-form functions of M

# •Two different calibration methods

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- Both approaches work with same data
  - Projection matrix approach is simpler to explain than the direct approach
- Direct approach requires an extra step
  - There are also other calibration methods
- But all calibration methods
  - Use patterns with know geometry or shape
  - Take multiple views of theses patterns
  - Match the information across the different views
- Perform some mathematics to calculate the intrinsic and extrinsic camera parameters
- We look at simplified case of only one view!

# Estimating the projection matrix

## World – Frame Transform

- Drop “im” and “w”
- N pairs  $(x_i, y_i) \leftrightarrow (X_i, Y_i, Z_i)$

$$x_i = \frac{u_i}{w_i} = \frac{m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

$$y_i = \frac{u_i}{w_i} = \frac{m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

## Linear equations of m

- 2N equations, 11 independent variables
- $N \geq 6$ , SVD  $\Rightarrow$  m up to a unknown scale

$$\mathbf{A}\mathbf{m} = \mathbf{0}$$

$$\mathbf{A} = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1Z_1 & -y_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & & & & \end{bmatrix}$$

$$\mathbf{m} = [m_{11} \quad m_{12} \quad m_{13} \quad m_{14} \quad m_{21} \quad m_{22} \quad m_{23} \quad m_{24} \quad m_{31} \quad m_{32} \quad m_{33} \quad m_{34}]^T$$

# Homogeneous System

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- M linear equations of form  $A\mathbf{x} = 0$
- If we have a given solution  $\mathbf{x}_1$ , s.t.  $A\mathbf{x}_1 = 0$  then  $c * \mathbf{x}_1$  is also a solution  $A(c * \mathbf{x}_1) = 0$
- Need to add a constraint on  $\mathbf{x}$ ,
  - Basically make  $\mathbf{x}$  a unit vector  $\mathbf{x}^T \mathbf{x} = 1$
- Can prove that the solution is the eigenvector corresponding to the single zero eigenvalue of that matrix  $A^T A$ 
  - This can be computed using eigenvector of SVD routine
  - Then finding the zero eigenvalue (actually smallest)
  - Returning the associated eigenvector

# Decompose projection matrix

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- 3x4 Projection Matrix  $\mathbf{M}$  computed previously
  - Both intrinsic (4) and extrinsic (6) – 10 parameters

$$\mathbf{M} = \begin{bmatrix} -f_x r_{11} + o_x r_{31} & -f_x r_{12} + o_x r_{32} & -f_x r_{13} + o_x r_{33} & -f_x T_x + o_x T_z \\ -f_y r_{21} + o_y r_{31} & -f_y r_{22} + o_y r_{32} & -f_y r_{23} + o_y r_{33} & -f_y T_y + o_y T_z \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

## From $\mathbf{M}$ to parameters (p134-135)

- Find scale  $|\gamma|$  by using unit vector  $\mathbf{R}_3^T$
- Determine  $T_z$  and sign of  $\gamma$  from  $m_{34}$  (i.e.  $q_{43}$ )
- Obtain  $\mathbf{R}_3^T$
- Find  $(O_x, O_y)$  by dot products of Rows  $q_1, q_3, q_2, q_3$ , using the orthogonal constraints of  $\mathbf{R}$
- Determine  $f_x$  and  $f_y$  from  $q_1$  and  $q_2$  All the rests:  $\mathbf{R}_1^T, \mathbf{R}_2^T, T_x, T_y$

# Calibration Summary

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- **Comparison of methods**
  - Direct approach requires extra step to find  $O_x$ ,  $O_y$
  - Projection approach finds  $O_x$ , and  $O_y$  at same time
    - Is simpler mathematically than the direct approach
  - Both methods require a refit to find a “valid”  $R$  matrix
- **There are other calibration methods**
  - Zhang approach uses flat plane (implemented in OpenCV)
  - Plane must be flat, but do not need 3D co-ordinates
- **But all calibration methods**
  - Have some known targets with known 3D geometry or shape
  - Take a number of images of these targets
  - From these measurements calculate the camera parameters
  - Are essential for further processing like reconstruction

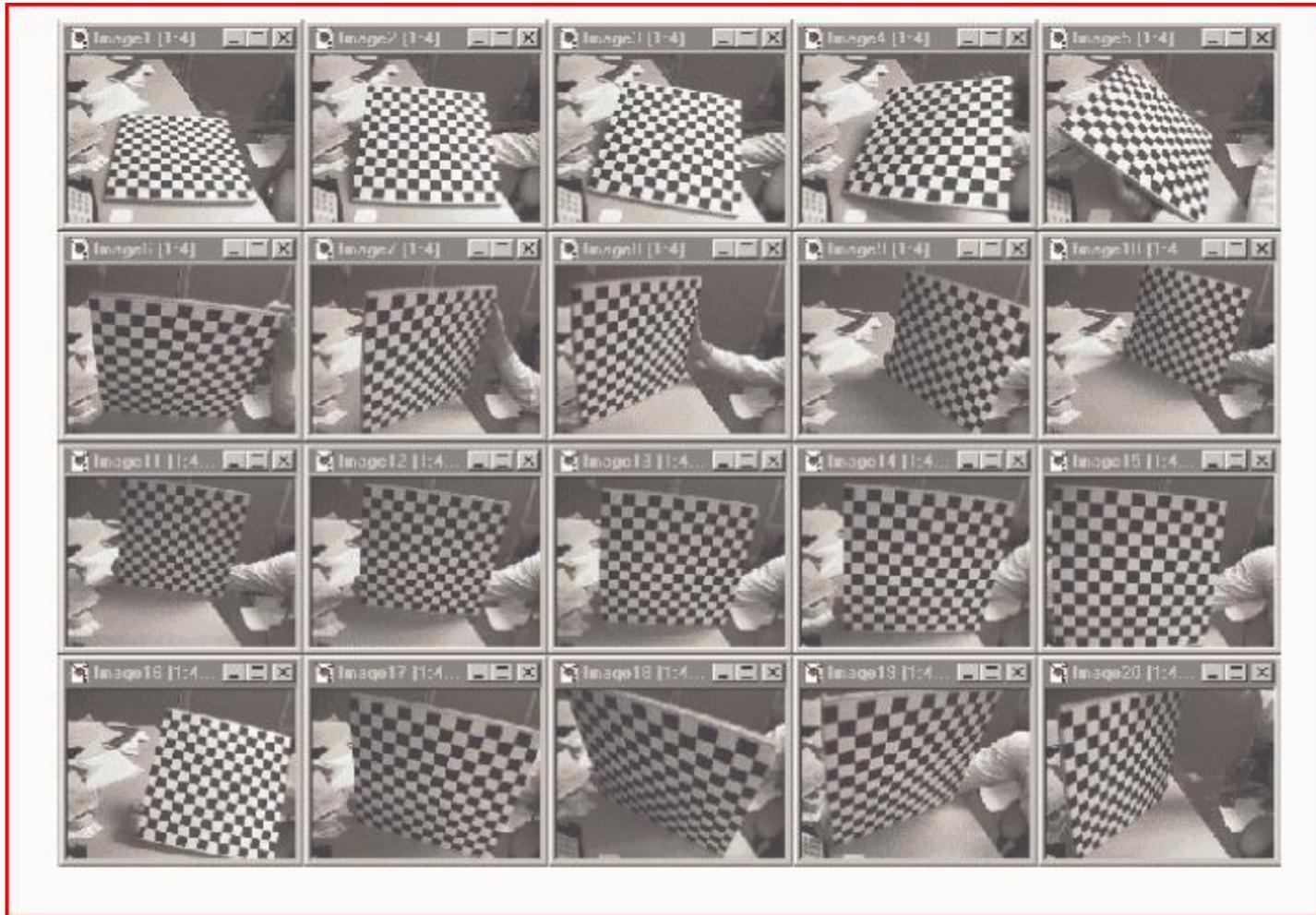
# Multiple View/Camera Calibration

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- Previous math describes the calibration process for a single image
  - We usually take multiple images of the same calibration target (from a variety of different views)
  - Simultaneously find all extrinsic parameters and all the intrinsic parameters of the single camera
- Also calibrate radial distortion using fact that there are straight lines in the pattern
- OpenCV code can do this using a checkerboard pattern
- Zhang's algorithm is used most in practice

# Input set of 2d Calibration Patterns

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# Final Camera positions and the pattern

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