

Assignment #4 Due Friday April 4<sup>th</sup> at 4:00 PM

1. In Figure 1 on the last page there are three cameras where the distance between the cameras is  $B$ , and all three cameras have the same focal length  $f$ . The disparity  $d_L = x_0 - x_L$ , while the disparity  $d_R = x_R - x_0$ . Show that  $|d_L| = |d_R|$ . You should prove this relationship holds mathematically by using the appropriate equations. **2 mark**
2. Consider two points  $A$  and  $B$  in a simple stereo system. Point  $A$  projects to  $A_l$  on the left image, and  $A_r$  on the right image. Similarly there is a point  $B$  which projects to  $B_l$  and  $B_r$ . Consider the order of these two points in each image on their epipolar lines. There are two possibilities; either they ordered on the epipolar lines in the same order; for example they appear as  $A_l, B_l$  and  $A_r, B_r$ , or they are in opposite order, such as  $B_l, A_l$  and  $A_r, B_r$ . Place the two 3d points  $A$  and  $B$  in two different locations in a simple stereo diagram which demonstrates these two possibilities. (Draw a different picture for each situation). **2 mark**
3. There is a simple stereo system with one camera placed above the other camera in the  $y$  direction (not the  $x$  direction is as usual) by a distance of  $b$ . In such a case there is no rotation between the cameras, only a translation by a vector  $T = [0, b, 0]$ . First compute the essential matrix  $E$  in this case. You are given a point  $p_1$  in camera co-ordinates in the first image as  $(x_1, y_1, f)$ , and a matching point  $p_2$  in the second image where  $p_2$  is  $(x_2, y_2, f)$ . Write the equation of the epipolar line that contains the matching point  $p_2$  in camera co-ordinates in the second image. In this case you are given  $p_1$  and you have computed  $E$ , and you need to write the equation of the line that contains  $p_2$  (the free variables are  $x_2, y_2$ ) using  $p_1$  and the elements of  $E$  as the fixed

variables. Now repeat the entire process again for the case where  $T = [b,b,0]$  (a translation of 45 degrees to the right in the  $x,y$  plane), and finally where  $T = [0,0,b]$  (a translation straight ahead in the  $Z$  direction). For the particular case where  $p_1 = (0, 1, f)$  what is the equation of the epipolar line for all three situations? And where  $p_1 = (1, 1, f)$  what is the equation of the epipolar line in these three situations? Draw the epipolar lines for all three cases, you just need to show the basic shape of the epipolar lines.

**4 marks**

4. In simple stereo  $Z = f B/d$ . If the baseline  $B$  is 0.5 meter,  $Z$  is 2.0 meters, and  $f$  is 50 millimeters what is the value of  $d$  in mm? Repeat this process for  $Z = 1$  meter, and 0.5 meter to get two more values for  $d$  in mm (same  $f$  and  $B$ ). **1/3 mark** If we are measuring depth at a given value of  $Z$  then we have a certain accuracy in the measurement which depends on how much error there is in computing the disparity. In turn, the error in disparity computation depends on how accurately we can locate a feature, such as a line or corner in the image. In practice, the error in computing the location of a feature is fixed; it does not change regardless of the value of  $Z$  for that feature. Assume that this error in disparity  $d$  for locating a feature is  $\pm 1$ mm. In other words, for a computed value of disparity  $d$ , the true value is  $d$  plus or minus 1mm. So for each of the three given values of  $d$  computed above, compute two new values for  $z$ ;  $z$  high when  $d = d - 1$  mm, and  $z$  low when  $d = d + 1$ mm. **1/3 mark** Now in each of the three cases above compute  $Z_{diff} = Z_{high} - Z_{low}$ . You will now have three values for  $Z_{diff}$ . We call these three values  $Z_{diff}(2 \text{ meters})$ ,  $Z_{diff}(1 \text{ meter})$  and  $Z_{diff}(0.5 \text{ meter})$ . **1/3 mark** In these three cases we have cut the value of the value of  $Z$  by one half, from 2 meter, to 1 meter to 0.5 meters. Hypothesize a relationship that seems to hold (approximately) on the ratio of the  $Z_{diff}$ s when we cut  $Z$  in half. Look at the ratio of  $Z_{diff}(2 \text{ meters})/Z_{diff}(1 \text{ meter})$  and

$Z_{\text{diff}}(1 \text{ meter})/Z_{\text{diff}}(0.5 \text{ meter})$ . If the number these ratios are converging to is not obvious, then repeat the experiment again but with  $Z_{\text{diff}}(0.25 \text{ meter})$  and consider the ratio which is  $Z_{\text{diff}}(0.5 \text{ meter})/Z_{\text{diff}}(0.25 \text{ meter})$ . Guess the obvious number! Another way to say this is to consider the following statements, where  $X$  is that same number; If we are doing stereo measurements at a given distance  $Z$  we expect a certain accuracy in the measurements ( $\pm \Delta Z$ ). If we now measure at one half the distance of  $Z$ , which is  $Z/2$  we expect our accuracy to improve by a factor of  $X$ . So when we cut our depth in half, then depth resolution improves by a factor of  $X$ .

**$\frac{1}{2}$  mark – 2 marks in total**

**Figure 1**

