

COMP 4102A: Assignment 1  
Due: Friday, Feb. 7 at 4:00 PM, 2014

1. (1/2 mark) Why are homogeneous co-ordinates used in computer vision? I want to know what the use of homogenous co-ordinates makes possible in terms of camera models.
2. (1/2 mark) Does a pin hole camera (no lens) have an infinite depth of field or not? Explain the answer.
3. (1/2 mark) For a camera with a lens is the depth of field infinite or not? Explain the answer.
4. (1/2 mark) Are three dimensional rotations expressed as  $R_x$ ,  $R_y$ , and  $R_z$  (rotations around the  $x$ ,  $y$  and  $z$  axis) commutative? That is, does the order in which they are applied matter. Explain the answer.
5. (1/4 mark) How does the field of view change if the focal length is varied (but the lens and the size of the sensing element stays the same)?
6. (1/4 mark) Under what conditions is weak perspective projection a good approximation to the true perspective projection?
7. (1 mark) Consider the case where a point  $P = (X, Y, Z)$  projects to the image plane. Using the matrix below which defines the projection of this point for a camera with focal length  $f$  to prove that every other point on the line from the origin through the point  $P$  projects to the same image point. Hint: Write down the parametric equation of the line from the origin through the 3d point  $X, Y, Z$ .

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \begin{array}{l} x = u / w \\ y = v / w \end{array}$$

8. (1/2 mark) A 3d rotation matrix has 9 (3 by 3) entries, and a 2d rotation matrix has 4 (2 by 2) entries. How many actual degrees of freedom are there in a 3d or 2d rotation? In other words, what is the minimum number of parameters necessary to uniquely specify a 3d rotation and a 2d rotation (answer with 2 numbers)? What does this imply about the relationship between the entries of a rotation matrix?
9. (2 mark) In the thin lens equation (as shown in Figure 2.4 of Ch 2, p. 21, in the book *Introductory Techniques for 3d Computer Vision* which is on the CD) first prove that  $Zz = f^2$  (Hint use this diagram along with the characteristics of similar triangles). Then use this equation along with the fact that  $Z^\wedge = Z + f$ ,  $z^\wedge = z + f$  to prove the thin lens equation, which is equation 2.2 in the book. What are the two values of  $z^\wedge$  for the cases where  $Z^\wedge$  is plus infinity, and where  $Z^\wedge = f$ .

10. (1 mark) A pinhole camera has focal length  $f = 500$ , pixel sizes  $s_x = s_y = 1$ , and its principal point is at  $(o_x, o_y) = (320, 240)$ . The world coordinate frame and the camera coordinate frame can be related by  $X_c = RX_w + T$ , where

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$

- (a) Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.
- (b) What are the pixel coordinates of the world point  $X_w = [150 \ 200 \ 400]^T$
11. (2 marks) Using the same R, T, f,  $s_x$ ,  $s_y$ , and  $X_w$  as given in Question 10 write an OpenCV program that projects this single 3d point using the given camera parameters. This program should call the routine `cvProjectPoints2` but according to the documentation you need to convert the R matrix to the appropriate format but I have found this routine works even when R is a 3 by 3 matrix. There is also a slight problem with this routine. For reasons too complex to explain you should define the 3d object point as `object_points = cvCreateMat(3, 3, CV_32F);` and the 2d image points as `image_points = cvCreateMat(3, 2, CV_32F);`. There is only one point to project (not three) to project but this routine has some problems when it projects only one point. So just ignore the other points, and only set up the one point to be projected. The program should print or display the value of  $X_w$ , and then the projected x and y pixel positions after calling the appropriate routines. The final pixel values should be approximately equal to what you calculated by hand in Question 3. Include the source code of your program and some proof that the program prints the correct results (copy the printed output or print the results to a file). We do not need the program executable. An old tutorial on OpenCV is in <http://www.cs.iit.edu/~agam/cs512/lect-notes/opencv-intro/opencv-intro.html>
12. (1/2 mark) What is the time complexity of a convolution with an N by N sized kernel when using a direct convolution with a square 2d mask, and when using a separable kernel. Each of the answers is in the form of  $O(?)$  where ? is an expression in N.
13. (1/2 mark) Mark each of the following as true or false. A) When we smooth an image with a Gaussian filter we are performing a low-pass filter which is accentuating the low frequencies in the image. B) All possible image filters are also linear image filters. C) A median filter is a linear image filter. D) A Gaussian filter is a linear image filter.