# Hough Transform

COMP 4102A

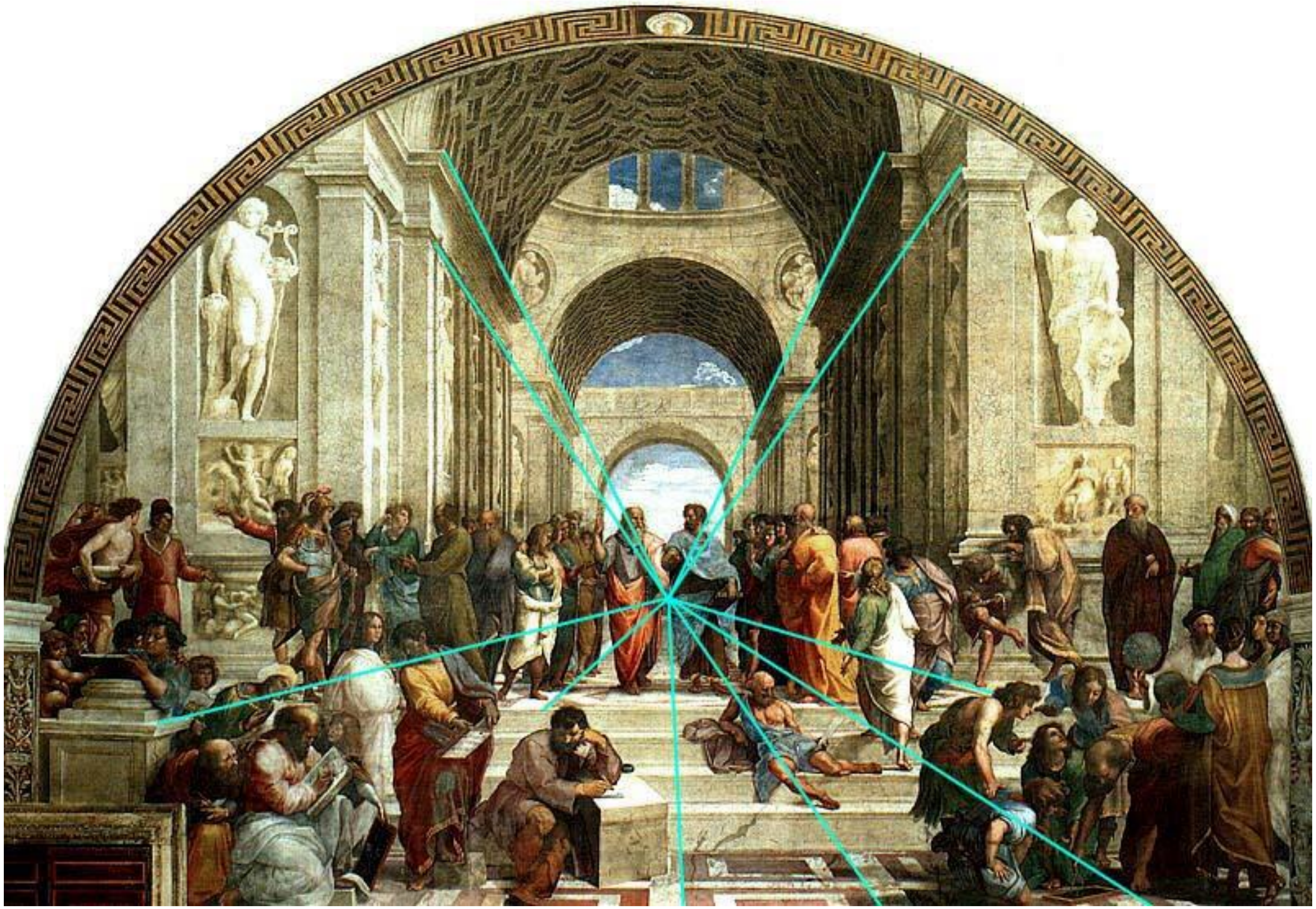
Gerhard Roth

Winter 2014

Version 1

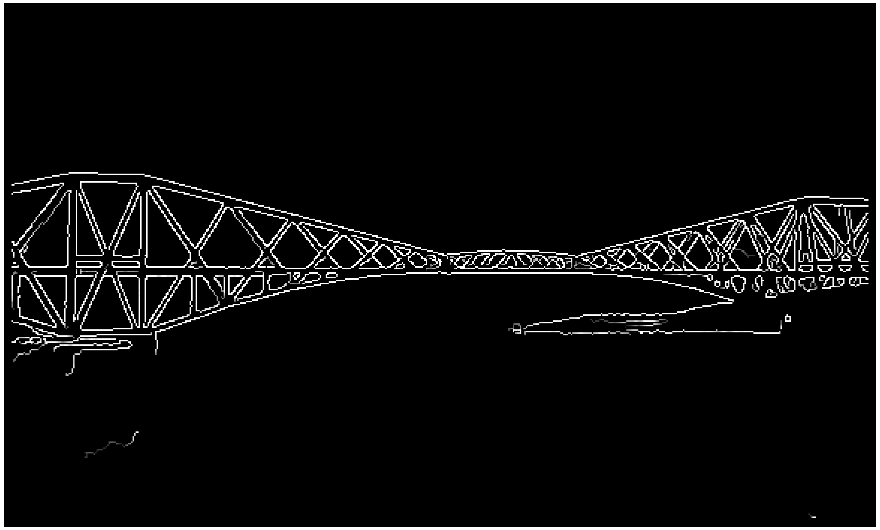
# Lines

# Lines



Rafael, The School of Athens (1518)

# Line Detection



The problem:

- How many lines?
- Find the lines.

# Equations for Lines

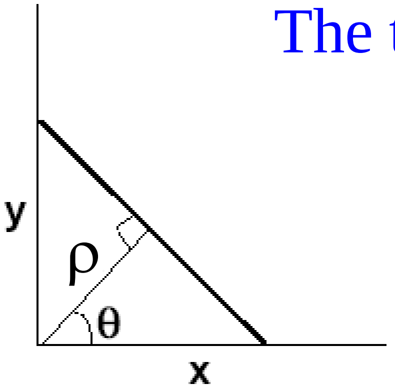
The slope-intercept equation of line

$$y = mx + b$$

What happens when the line is vertical? The slope *a* goes to infinity.

A better representation – the polar representation

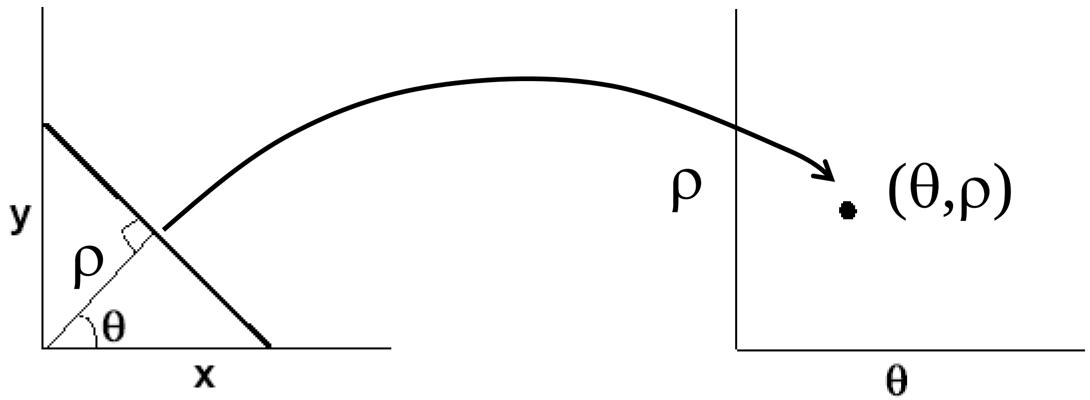
The two parameters $\rho, \theta$ defining line are bounded



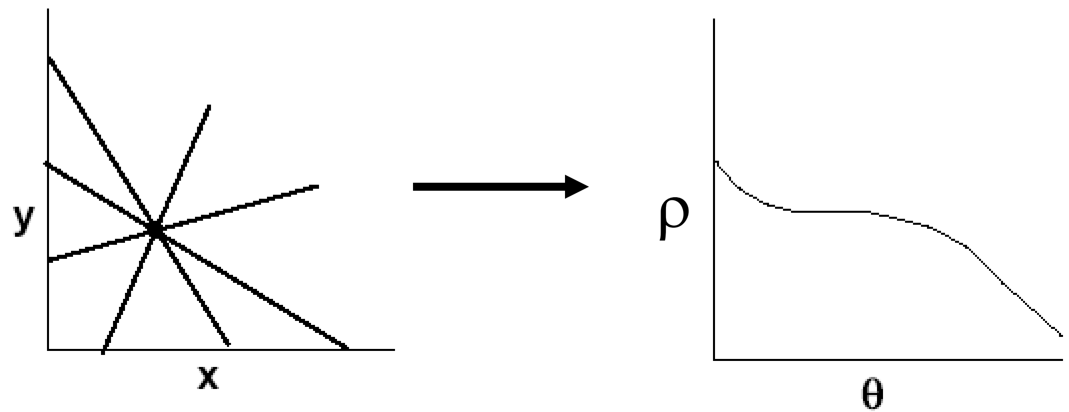

$$\rho = x\cos\theta + y\sin\theta$$

# Hough Transform: line-parameter mapping

A line in the plane maps to a point in the θ-ρ space.

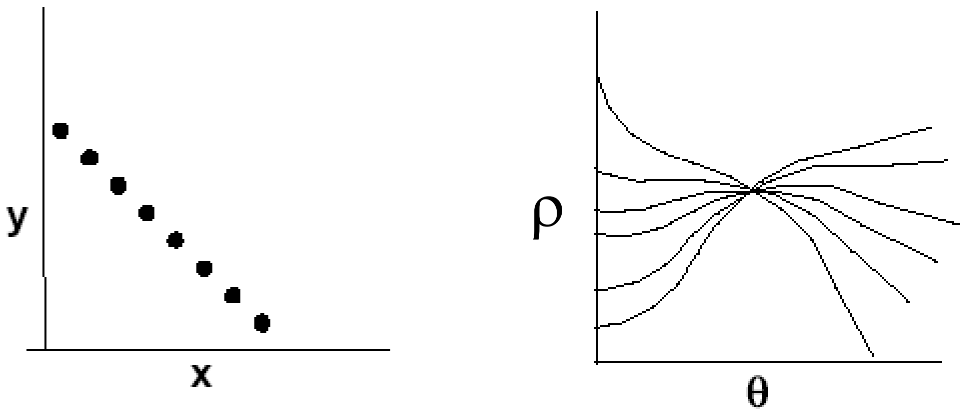



All lines passing through a point map to a sinusoidal curve in the θ-ρ (parameter) space.




$$\rho = x\cos\theta + y\sin\theta$$

# Mapping of points on a line




Points on the same line define curves in the parameter space that pass through a single point.

Main idea: transform edge points in *x*-*y* plane to curves in the parameter space. Then find the points in the parameter space that has many curves passing through it.

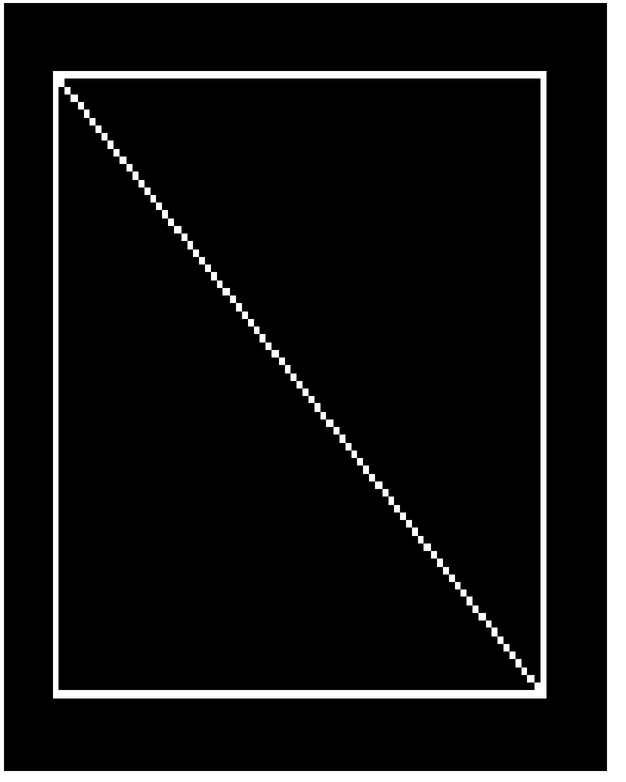
# Hough Idea

- Each straight line in this image can be described by an equation

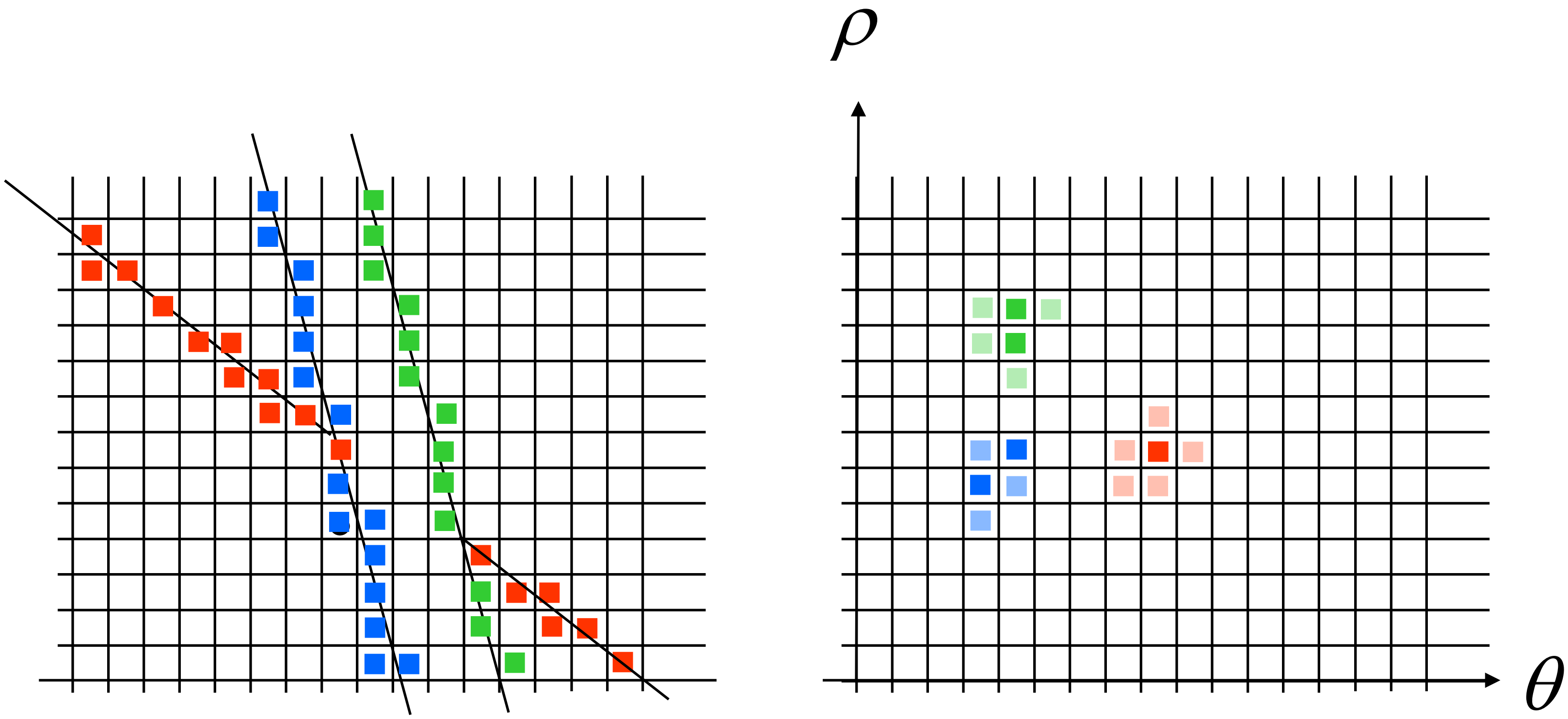
- Each white point if considered in isolation could lie on an infinite number of straight lines

- **In the Hough transform each point votes for every line it could be on**

- The lines with the most votes win

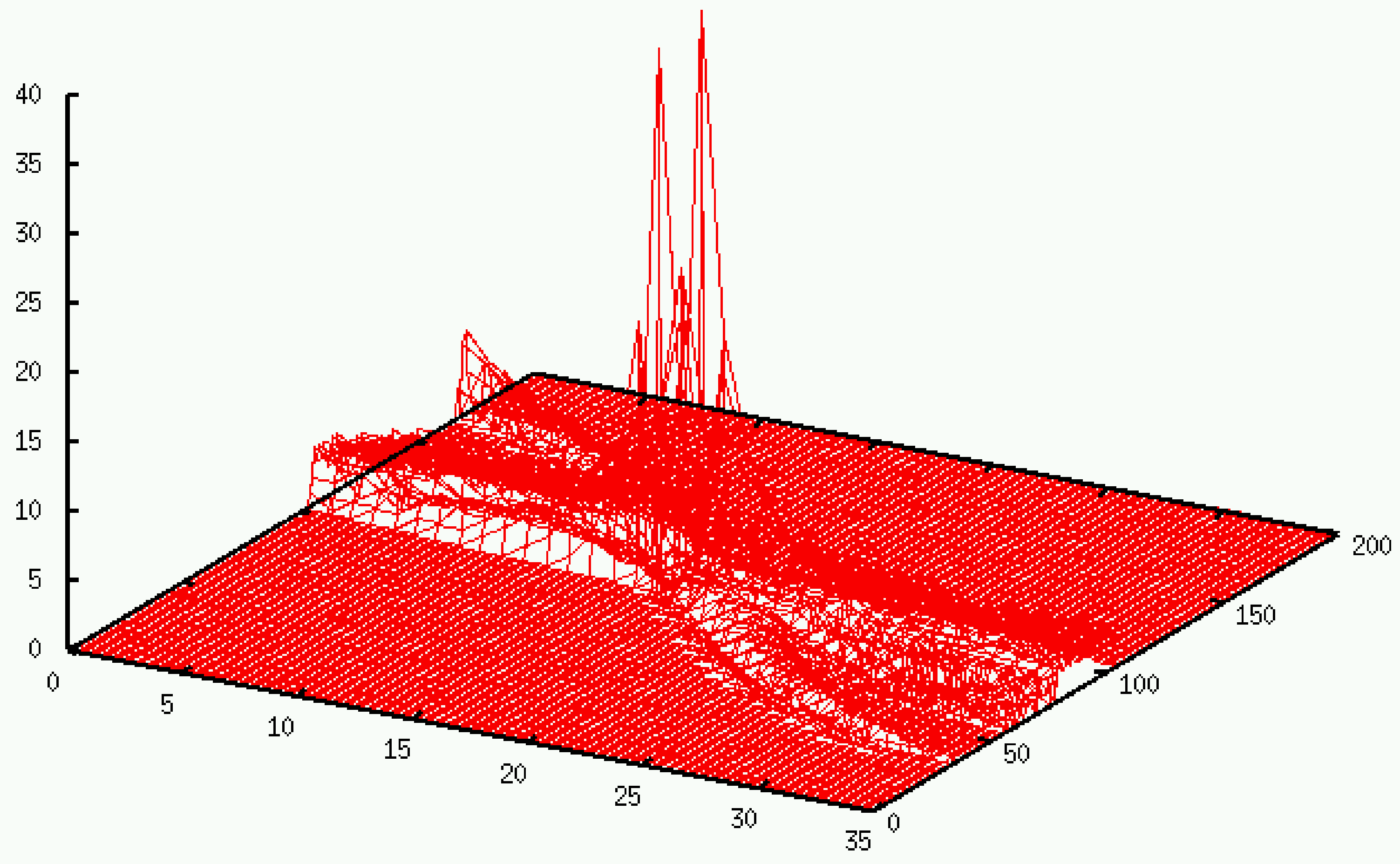


# Quantize Parameter Space

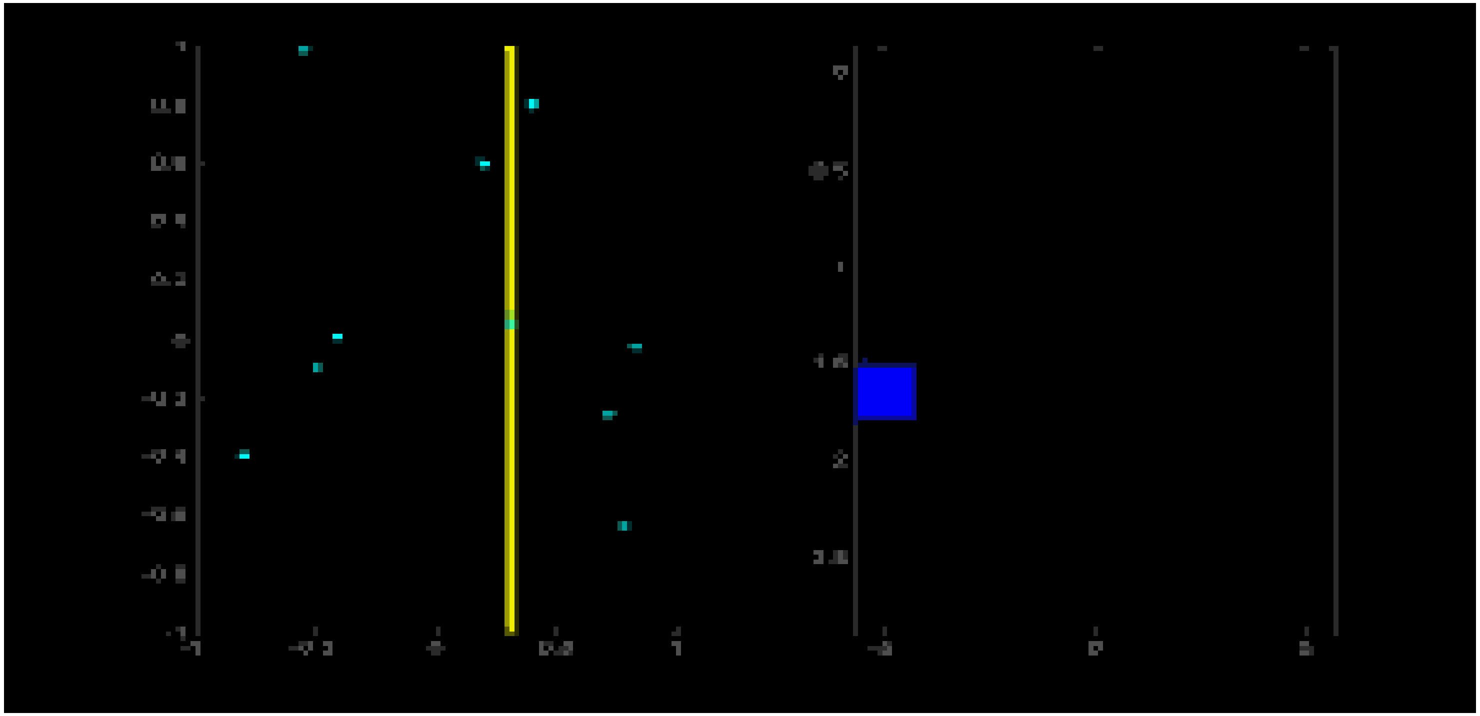



Detecting Lines by finding maxima / clustering in parameter space.

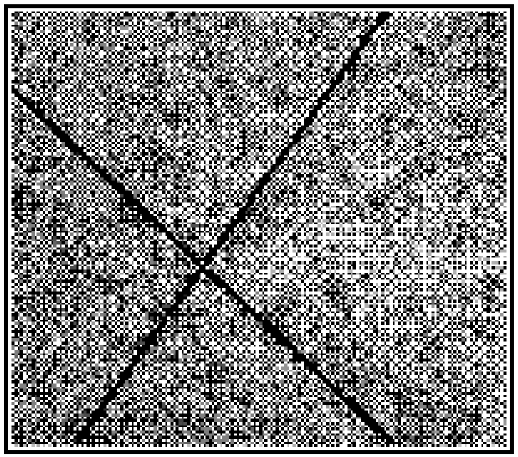
# Parameter space – 3D view



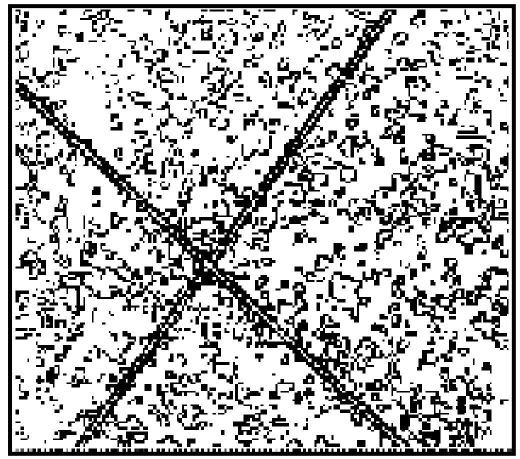
# A Voting Scheme



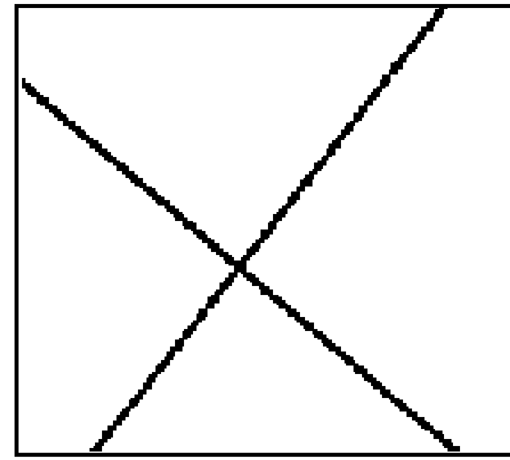
# Hough Processing



**Image**



**Edge detection**

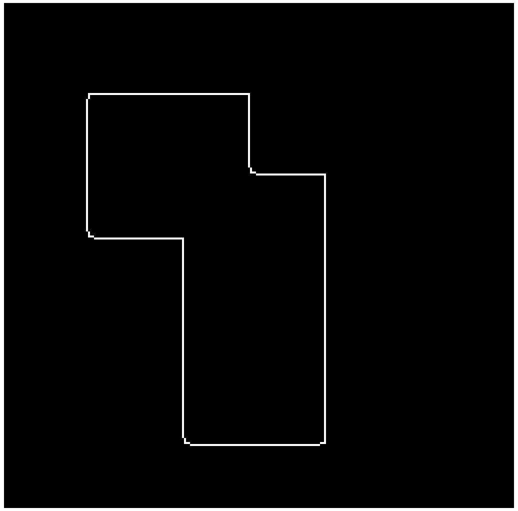


**Hough Transform**

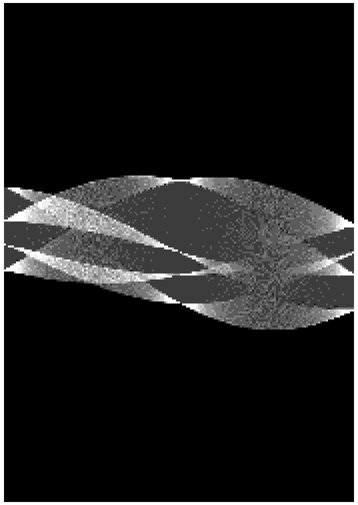
- Find the edges in the image (Canny operator common)
- Use each edge point to vote in the accumulator space
  - Accumulator space also called the Hough Space
- Find the peak(s) in the accumulator space

# Examples

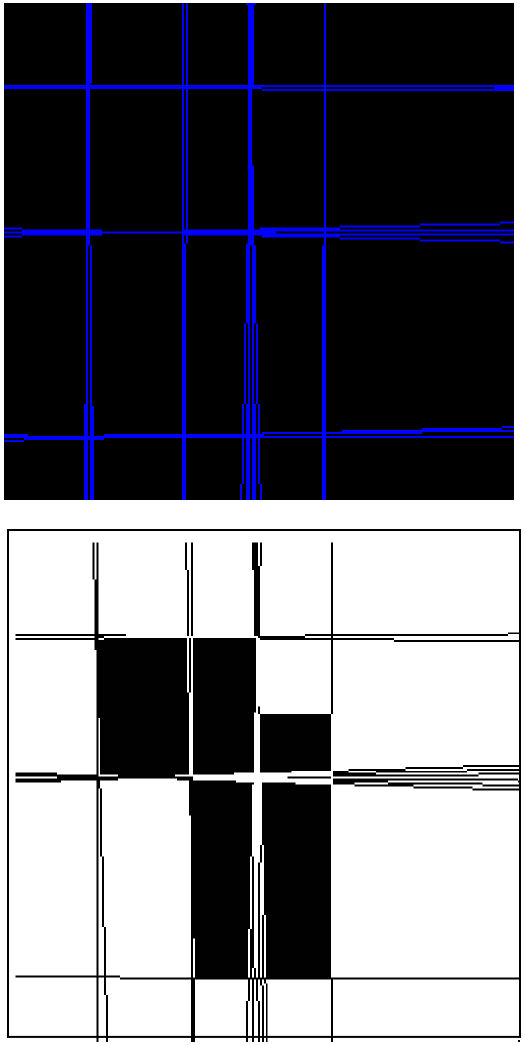
input image



Hough space

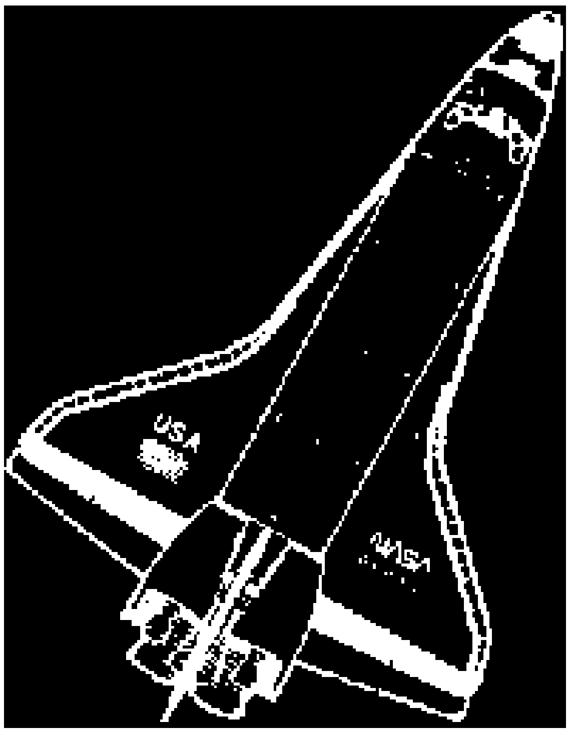


lines detected

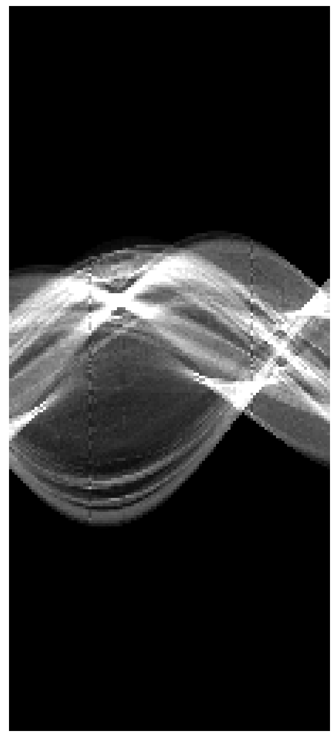


# Examples

input image




Hough space



lines detected

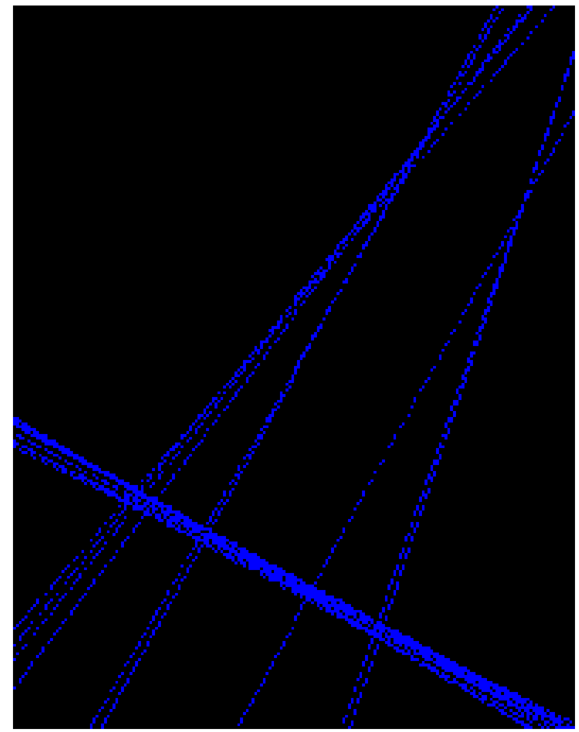
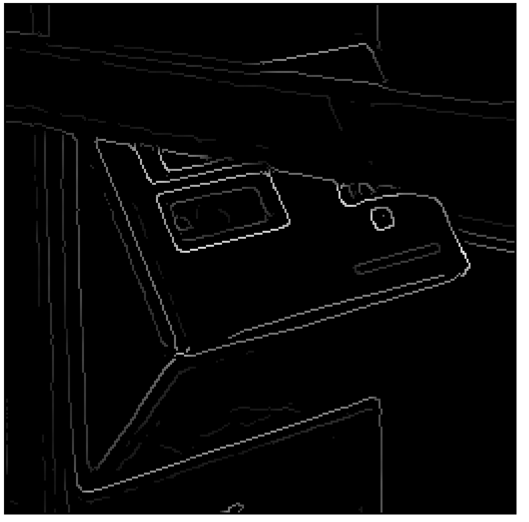


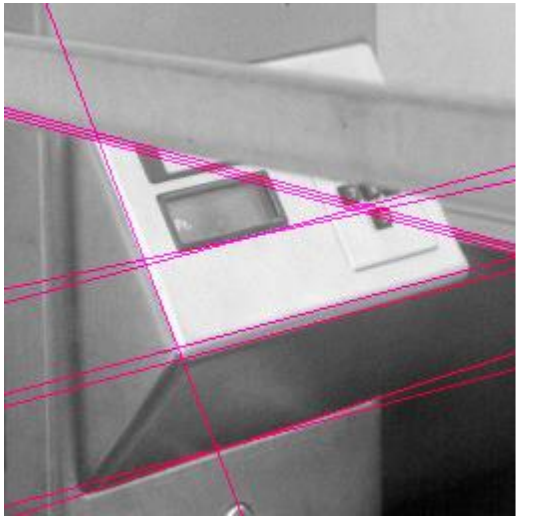
Image credit: NASA Dryden Research Aircraft Photo Archive

# Examples

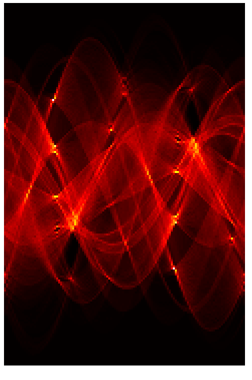
Original



Edge Detection



Found Lines



Parameter Space

# Algorithm

1. Quantize the parameter space

   int P[0, $\rho_{max}$][0, $\theta_{max}$]; // accumulators

2. For each edge point ($x$, $y$) {

   For ($\theta = 0$; $\theta <= \theta_{max}$; $\theta = \theta + \Delta\theta$) {

   $\rho = x\cos\theta + y\sin\theta$ // round off to integer

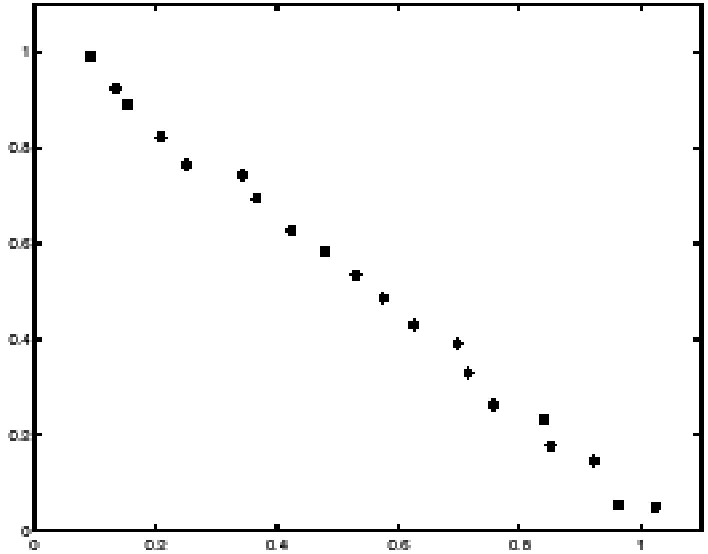
   (P[$\rho$][$\theta$])++;

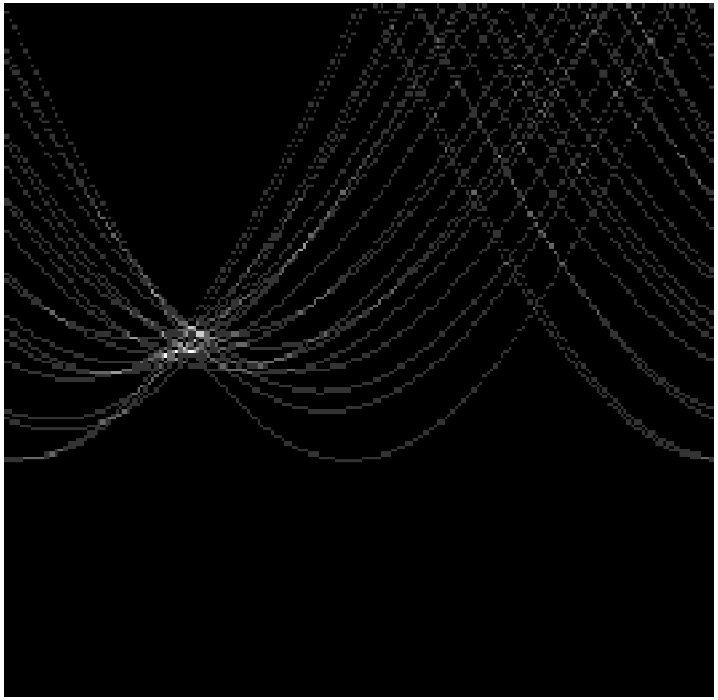
   }

   }

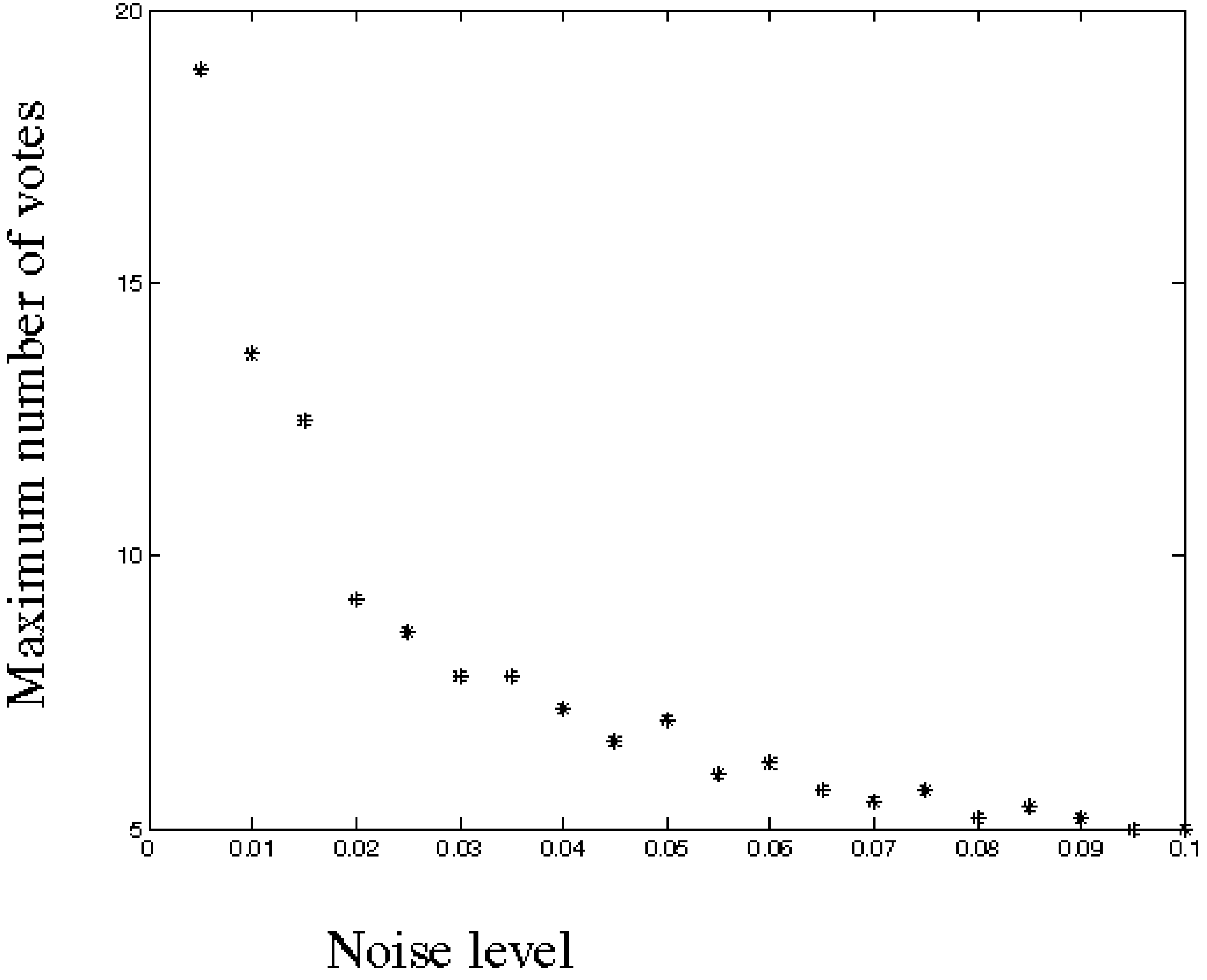
3. Find the peaks in P[$\rho$][$\theta$].

# Cell Size

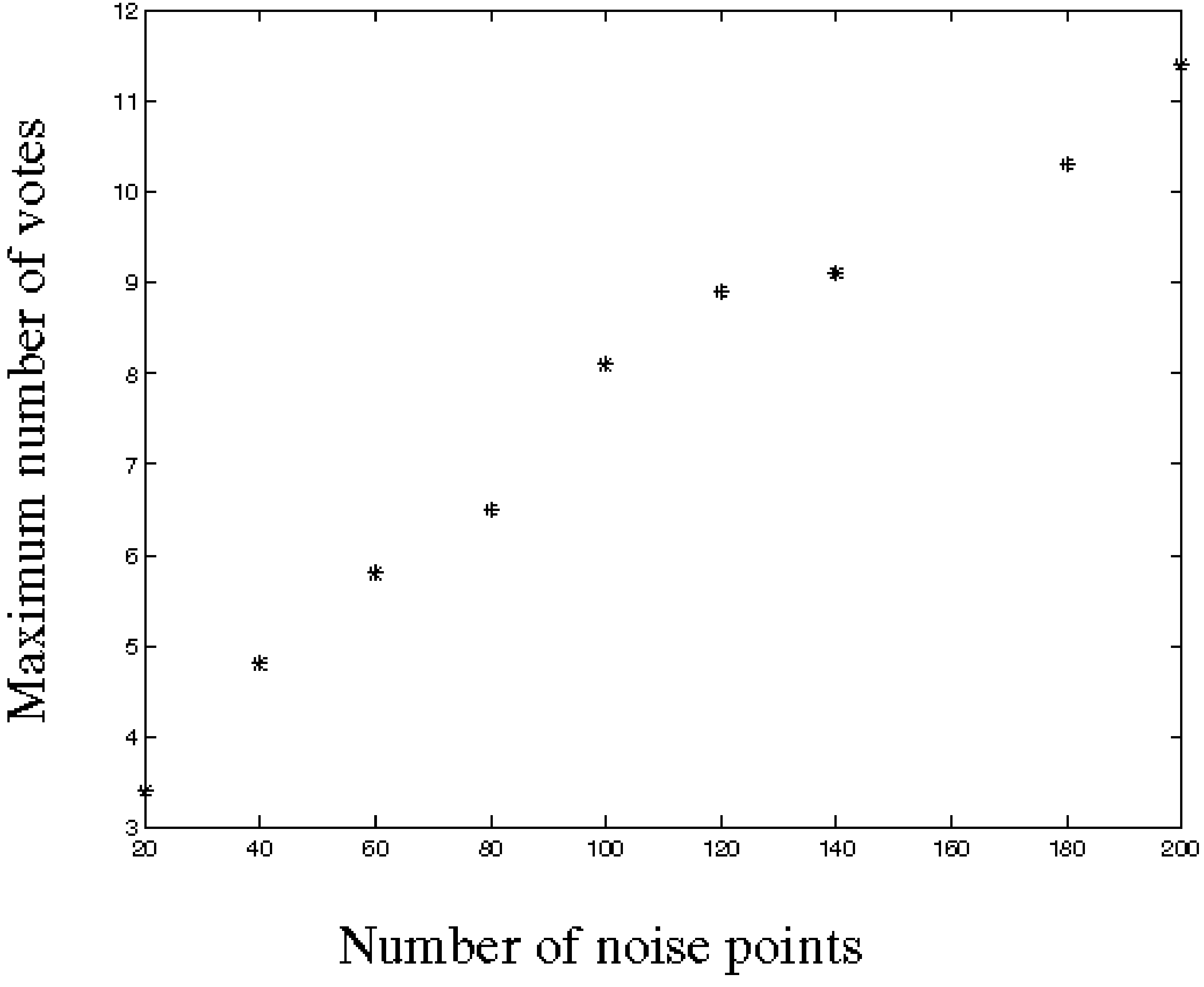


Choose the parameter cell size such that the algorithm is robust to noise.




Fewer votes land in a single bin when noise increases.




Adding more clutter increases number of bins with false peaks.

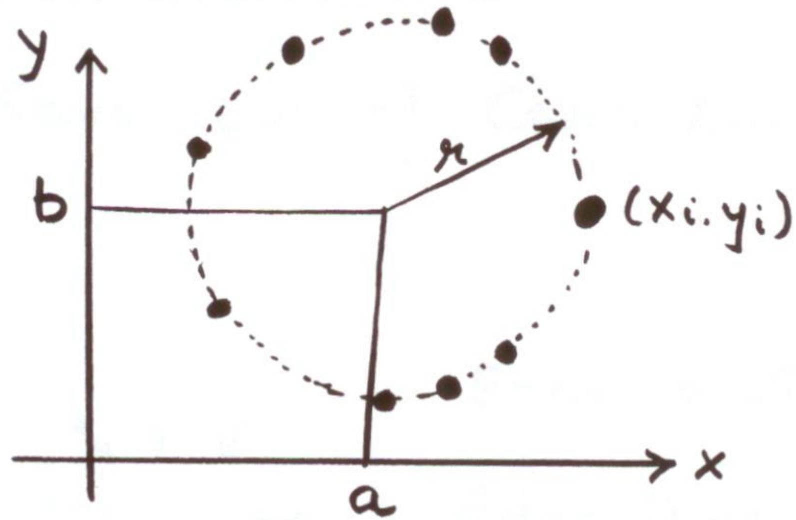
# Finding Circles by Hough Transform

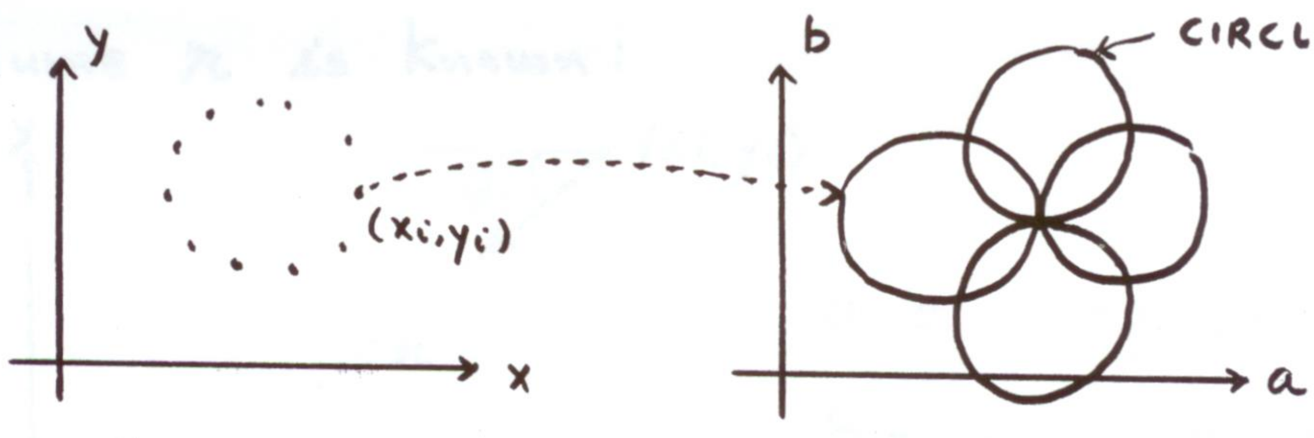
Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)

Accumulator Array $A(a,b)$

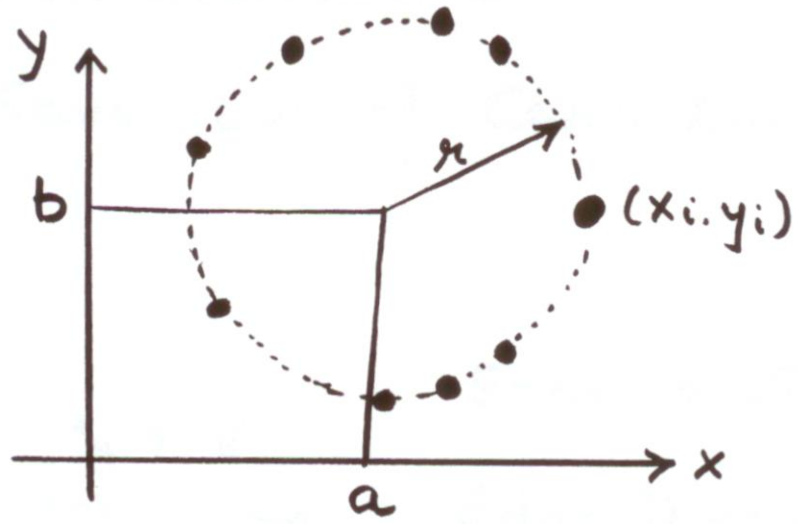


# Finding Circles by Hough Transform

Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

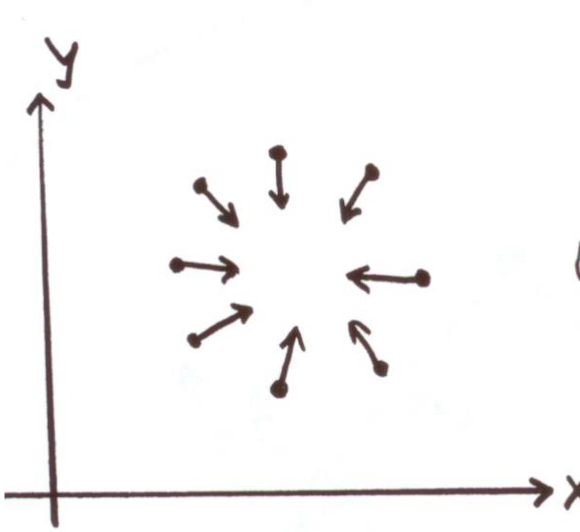



If radius is not known: 3D Hough Space!

Use Accumulator array $A(a, b, r)$

# Using Gradient Information

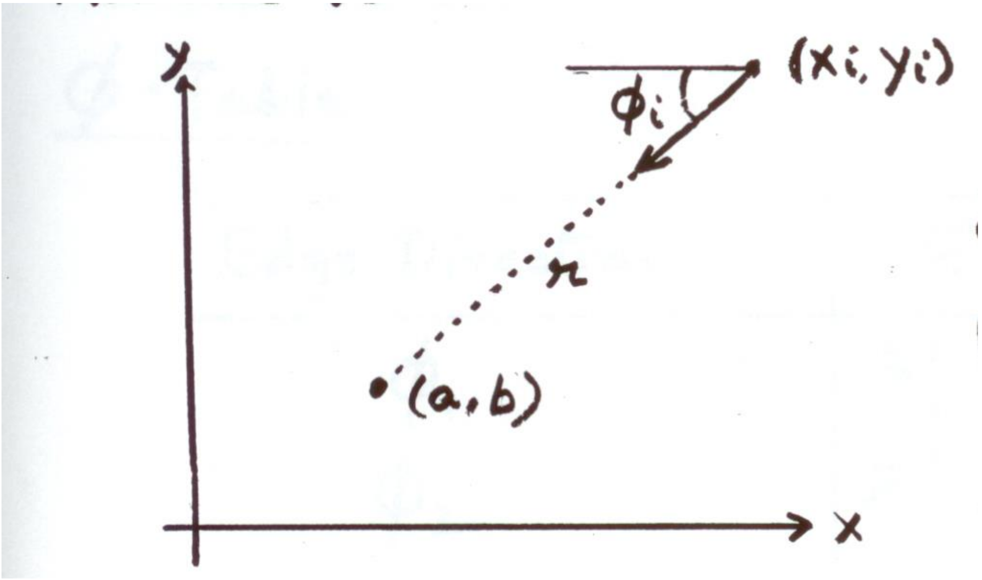
- Gradient information can save lot of computation:




Edge Location $(x_i, y_i)$

Edge Direction $\phi_i$

Assume radius is known:



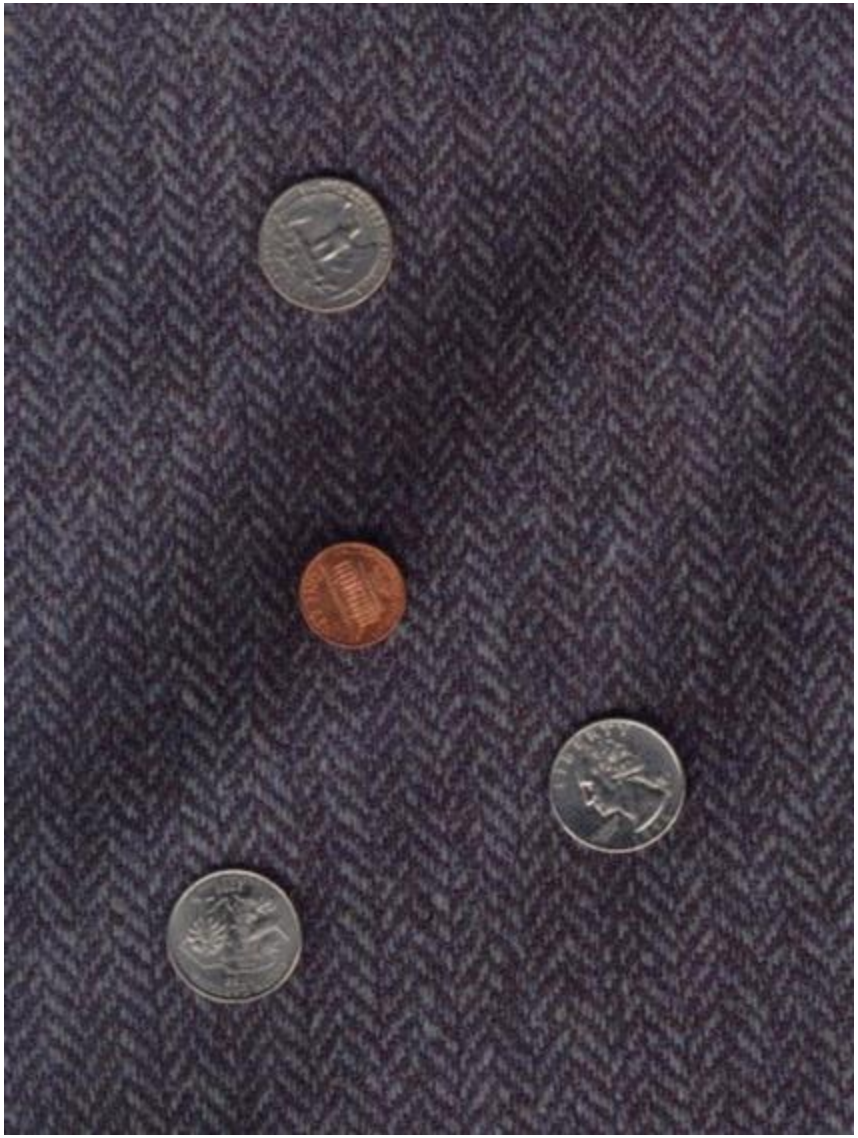

$$a = x - r\cos\phi$$

$$b = y - r\sin\phi$$

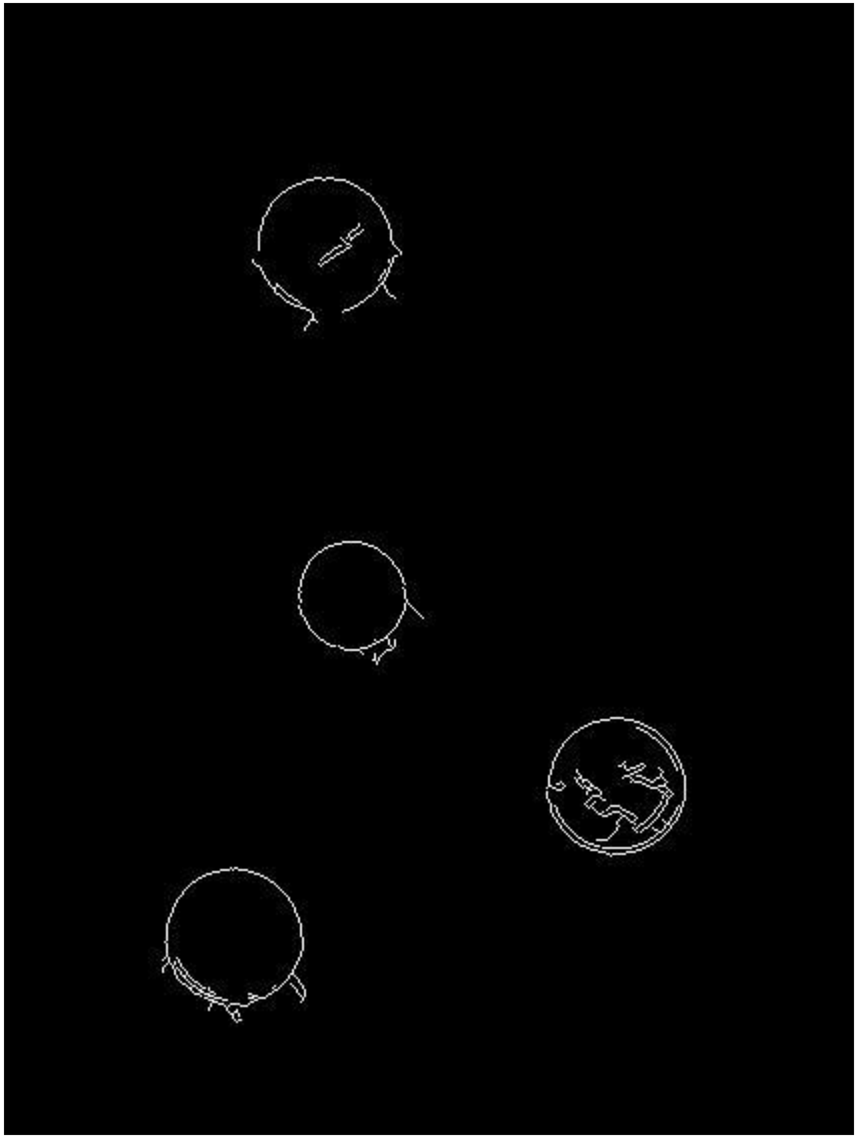
Need to increment only one point in Accumulator!!
If radius is not known, accumulator is 2d using gradients

# Finding Coins

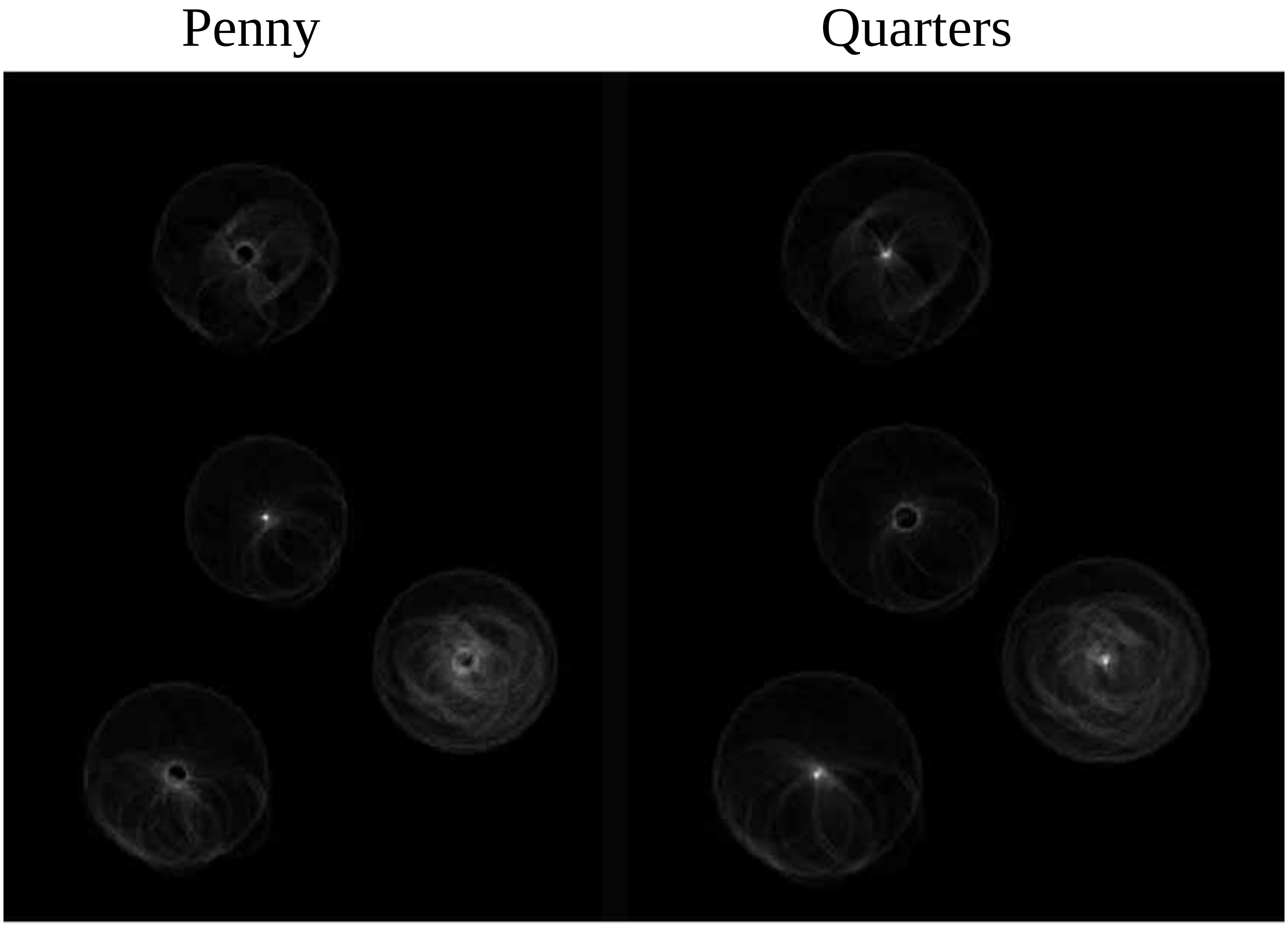
Original



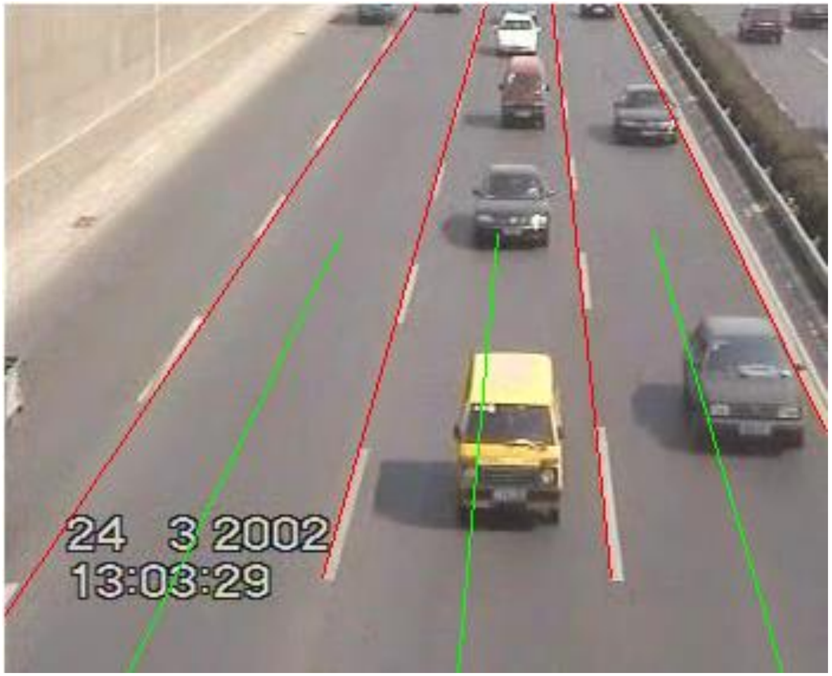
Edges (note noise)



# Finding Coins (Continued)



# Application: Lane Detection



# Hough Characteristics

- Detects all the curves in an image at once
- Running time proportional to the number of edge points that are in the image
- Can deal with disconnected edge points
  - Does not assume (require) any connectivity for edges
- Accumulator dimension (space) proportional to number of parameters that define the curve
  - Works well for lines (only 2d accumulator array necessary)
- Not easy to extend to more complex curves because of the space requirements
  - Can use image gradient to decrease space requirements
- Using gradients works well for circles

# Probabilistic Hough Transform

- Given a set of p edge points in an image
  - Goal is to find a particular curve (line, or circle)
  - Idea is that given n edge points (n is 2 for line or 3 for circle) we can create a unique curve through just these points
- Do while we have enough edge points
  - For K times (a parameter)
    - Choose n random edge points (2 for line and 3 for circle)
    - Create a unique line or circle through these points
    - Count the number of edge points that are within d pixels (another parameter) of that unique line or circle
    - Save the best curve (has most points within distance d)
  - Endfor
- remove the edge points found for best curve
- Enddo

# Probabilistic Hough Transform

- Two parameters distance d, and #samples K
  - Distance d is typically set in range 1 to 5 pixels
  - #samples K depends on how many curves you expect there to be in the image
- Given expectation of at most n curves in the image you can compute a value for K
  - K is an exponential function of n, the degrees of freedom (dof) of the curve, which is 2 for a line and 3 for a circle
- Running time O(n K p) where K is number of samples, and p is the number of edge points
- Space requirements are low so you could use this for complex curves (like ellipse, 5 dof)

# Probabilistic HT relative to ordinary HT

- Ordinary HT space requirements where q is grid size are $q^2$ for line and $q^3$ for a circle
  - Running time is O(q p) with p edge points
- Probabilistic HT space requirements are simply O(p), the number of edge points
  - Running time is O(n K p), n curves p edge points, K samples
- Which is faster for lines and circles?
  - Depends on how many lines and circles exist (n)
  - Remember for Prob. HT value of K is an exponential function of the number of expected lines or circles
  - With a small number of curves K is small, and Prob. HT is faster, large number of curves K is large and HT is faster
- For curves like ellipse Prob. HT is only choice