Assignment #4 Due on date of Second Midterm, Tuesday April 5th

1. In Figure 1 on the last page there are three cameras where the distance between the cameras is B, and all three cameras have the same focal length f. The disparity $dL = x0 - xL$, while the disparity $dR = xR - x0$. Show that $|dL| = |dR|$. You should prove this relationship holds mathematically by using the appropriate equations. **1 mark**

2. Consider two points A and B in a simple stereo system. Point A projects to Al on the left image, and Ar on the right image. Similarly there is a point B which projects to Bl and Br. Consider the order of these two points in each image on their epipolar lines. There are two possibilities; either they ordered on the epipolar lines in the same order; for example they appear as Al, Bl and Ar Br, or they are in opposite order, such as Bl, Al and Ar,Br. Place the two 3d points A and B in two different locations in a simple stereo diagram which demonstrates these two possibilities. (Draw a different picture for each situation). **1 mark**

3. The equation of a simple stereo system is $z = f\,T / d$. In this question assume that $f\,T = 1$ which means that $z = 1 / d$. Assume that the only source of error in a simple stereo system is the error in estimating the disparity, and that this error is exactly one pixel, and it does not change with the actual disparity value. So if the stereo system says the disparity is 5 pixels it is really between 4 and 6 pixels. Similarly, if the stereo system says the disparity is 10 pixels then it is really between 9 and 11 pixels. The error in estimating Z at a given disparity d due to this one pixel error in estimating the disparity is called ErrorZ. For a given value of disparity d, this error is estimated by the formula ErrorZ-1(d pixels) = $\| z(d - 1) - z(d + 1) \|$. Compute ErrorZ-1(5 pixels), ErrorZ-1(10 pixels), and the ratio of the two, which is

ErrorZ-1(5 pixels) /ErrorZ-1(10 pixels). Repeat this entire process but now assume that the error in calculating the disparity is ½ pixel, so that ErrorZ-1/2(d pixels) = $\| z(d - 1/2) - z(d + 1/2)\|$. Again compute ErrorZ-1/2(5 pixels), ErrorZ-1/2(10 pixels). Now compute the ratio of the two, which is ErrorZ-1/2(5 pixels) /ErrorZ-1/2(10 pixels). Looking at this ratio hypothesize a relationship between error in Z and error in disparity which holds as the error in disparity approaches zero. In other words, given a small fixed error in computing the disparity, how does the resulting error in computing Z change if the disparity d doubles. Verify that your hypothesis is true by computing the derivative of Z with respect to disparity d in the case where $z = 1/d$, which represents the change in depth over the change in disparity (in the limit as the change in disparity goes to zero). The theory should agree with the practice.
2 marks

4. There is a simple stereo system with one camera placed above the other camera in the y direction (not the x direction is as usual) by a distance of b. In such a case there is no rotation between the cameras, only a translation by a vector T = [0,b,0]. First compute the essential matrix E in this case. You are given a point p_1 in camera co-ordinates in the first image as (x_1,y_1,f), and a matching point p_2 in the second image where p_2 is (x_2,y_2,f). Write the equation of the epipolar line that contains the matching point p_2 in camera co-ordinates in the second image. In this case you are given p_1 and you have computed E, and you need to write the equation of the line that contains p_2 (the free variables are x_2,y_2) using p_1 and E as the fixed variables. Now repeat the entire process again for the case where T = [b,b,0] (a translation of 45 degrees to the right in the x,y plane), and finally where T = [0,0,b] (a translation straight ahead in the Z direction). For the particular case where $p_1 = (0, 1, f)$ what is the equation of the epipolar line for all three situations? And where $p_1 = (1, 1, f)$ what is the equation of the epipolar line in these

three situations? Draw the epipolar lines for all three cases, you just need to show the basic shape of the epipolar lines.
4 marks

5. If F is the fundamental matrix of the camera-pair (P, Q) then what is the fundamental matrix of the camera pair (Q, P).
½ mark

6. There are two stereo images; in the first there is a point p which matches to a point q in the second image. If for a point p in the first image the associated epipolar line in the second image is Fp then what is the associated epipolar line in the second image for the point q. **½ mark**

7. In question 4 above when T = [0,0,b] (a translation straight ahead in the Z direction) you computed the essential matrix E. For this case use the essential matrix to prove that the epipole (a point p such that E p = **0**) is (0, 0, f). **1 mark**

Figure 1