

COMP 4900C: Assignment 1  
Due: Tuesday, Feb. 3, 2011

1. (1 mark) The rotation matrix in the x-y plane is

$$R(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}$$

- (a) Verify  $R(\phi_1)R(\phi_2) = R(\phi_1 + \phi_2)$  from the identities for  $\cos(\phi_1 + \phi_2)$  and  $\sin(\phi_1 + \phi_2)$ .  
(b) What is  $R(\phi)R(-\phi)$ ?  
(c) What is  $R(\phi)R(\phi)^T$ ?

2. (1 mark) Scaling a vector  $\begin{bmatrix} x & y \end{bmatrix}^T$  in the plane can be achieved by

$$x' = sx \text{ and } y' = sy$$

where  $s$  is a scalar.

- (a) Write out the matrix form of this transformation.  
(b) Write out the transformation matrix for homogeneous coordinates.  
(c) If the transformation also includes a translation

$$x' = sx + t_x \text{ and } y' = sy + t_y$$

Write out the transformation matrix for homogeneous coordinates.

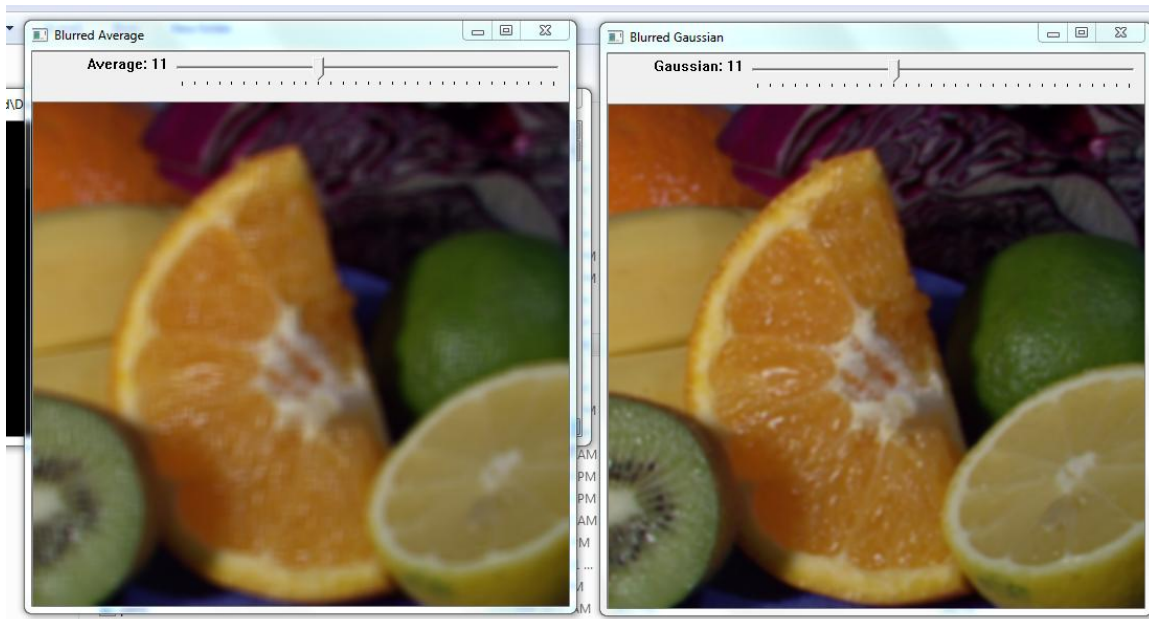
3. (2 marks) A pinhole camera has focal length  $f = 500$ , pixel sizes  $s_x = s_y = 1$ , and its principal point is at  $(o_x, o_y) = (320, 240)$ . The world coordinate frame and the camera coordinate frame can be related by  $X_c = RX_w + T$ , where

$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$

- (a) Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.  
(b) What are the pixel coordinates of the world point  $X_w = \begin{bmatrix} 150 & 200 & 400 \end{bmatrix}^T$
4. (2 marks) Using the same  $R$ ,  $T$ ,  $f$ ,  $s_x$ ,  $s_y$ , and  $X_w$  as given in Question 3 write an OpenCV program that projects this single 3d point using the given camera parameters. This program should call the routine `cvProjectPoints2`. There is a slight problem with this routine. For reasons too complex to explain you should define the 3d object point as `object_points = cvCreateMat(10, 3, CV_32F)`; and the 2d image points as `image_points = cvCreateMat(10, 2, CV_32F)`; . There is only one point to project (not ten) but this routine has some problems when it projects only one point. So just ignore the other nine points, and only set up the one point to be projected. The program should print the value of  $X_w$ , and then the projected x and y pixel positions after calling the appropriate

routines. The final pixel values should be approximately equal to what you calculated by hand in Question 3.

- (2 marks) Write a program that takes the input image fruits.jpg (as found in the OpenCV C examples directory) and blurs it with a Gaussian filter, and also with an ordinary averaging filter. The program should display two windows where the slider above each window adjusts the size of the kernel or mask. Use the routine `cvSmooth`, and note that in this routine the mask size is called aperture width. Remember that even though the minimum slider value is zero, the aperture minimum width is 3, and it must be an odd number. The maximum value for the slider should be 31.



- (2 marks) The projection equation is a 3 by 4 matrix defined as  $P$ , and when applied it transforms the points from 3D space to the image plane as follows  $(x, y, w)^T = P(X, Y, Z, 1)^T$ . Consider the figure below, where the point being imaged is on a plane in the world. Prove that in this special case  $(x, y, w)^T = M(X, Y, 1)^T$  where  $M$  is now a 3 by 3 matrix. Write out the definition of  $M$  in terms of the entries of the matrix  $P$ . Hint: You can put the world co-ordinate frame wherever you want, so choose the world co-ordinate frame appropriately. When I say that  $P$  is a projection matrix, just use the matrix with the entries as below:

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

