Filtering (II)

Dr. Gerhard Roth

COMP 4900C
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Image Filtering

Modifying the pixels in an image based on some functions of a local neighbourhood of the pixels

N(p)

p

10	30	10
20	11	20
11	9	1

$f(p)$

	5.7	

Linear Filtering – convolution

The output is the linear combination of the neighbourhood pixels

$$I_A(i,j) = I * A = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} A(h,k) I(i-h, j-k)$$

The coefficients come from a constant matrix A, called kernel.
This process, denoted by '*', is called (discrete) convolution.

1	3	0
2	10	2
4	1	1

Image

*

1	0	-1
1	0.1	-1
1	0	-1

Kernel

=

	5	

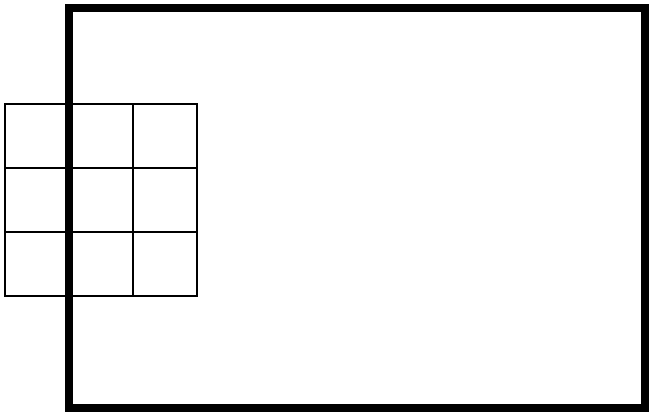
Filter Output

Handle Border Pixels

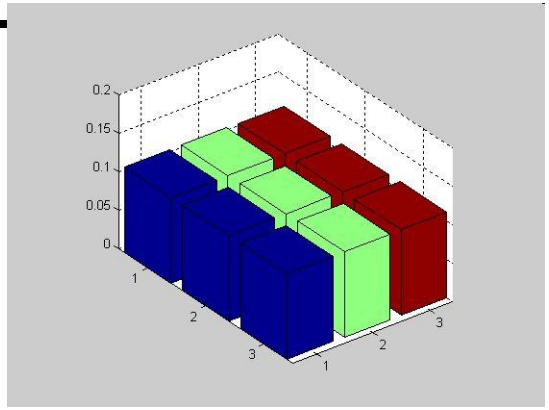
Near the borders of the image, some pixels do not have enough neighbours. Two possible solutions are:

- Set the value of all non-included pixels to zero.

- Set all non-included pixels to the value of the corresponding pixel in the input image.



Smoothing by Averaging

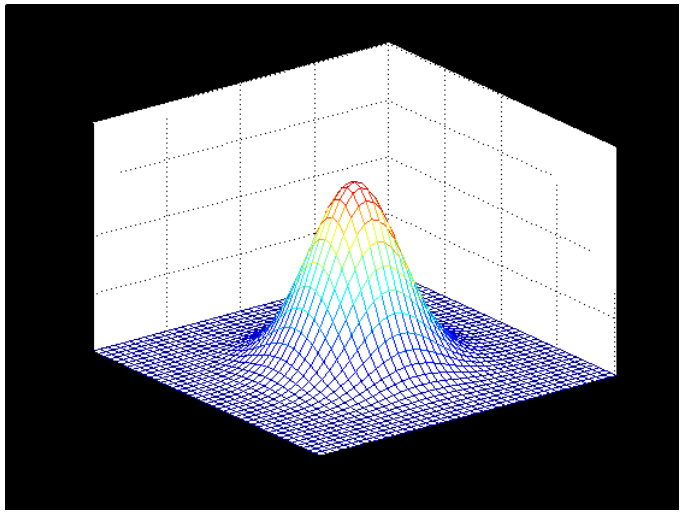


$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Convolution can be understood as weighted averaging.

Gaussian Filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{\left(x^2+y^2\right)}{2\sigma^2}\right)$$

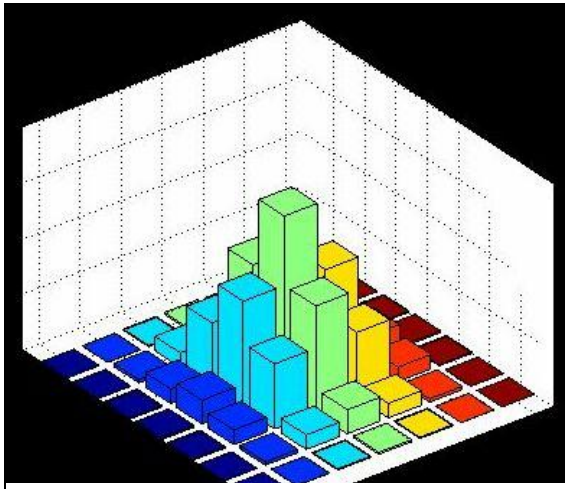


Discrete Gaussian kernel:

$$G(h,k) = \frac{1}{2\pi\sigma^2}e^{-\frac{h^2+k^2}{2\sigma^2}}$$

where $G(h,k)$ is an element of an $m \times m$ array

Gaussian Filter



$$* \quad \frac{1}{273} \begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix} =$$

$$\sigma = 1$$

Gaussian Kernel is Separable

$$I_G = I * G =$$

$$= \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} G(h,k) I(i-h,\, j-k) =$$

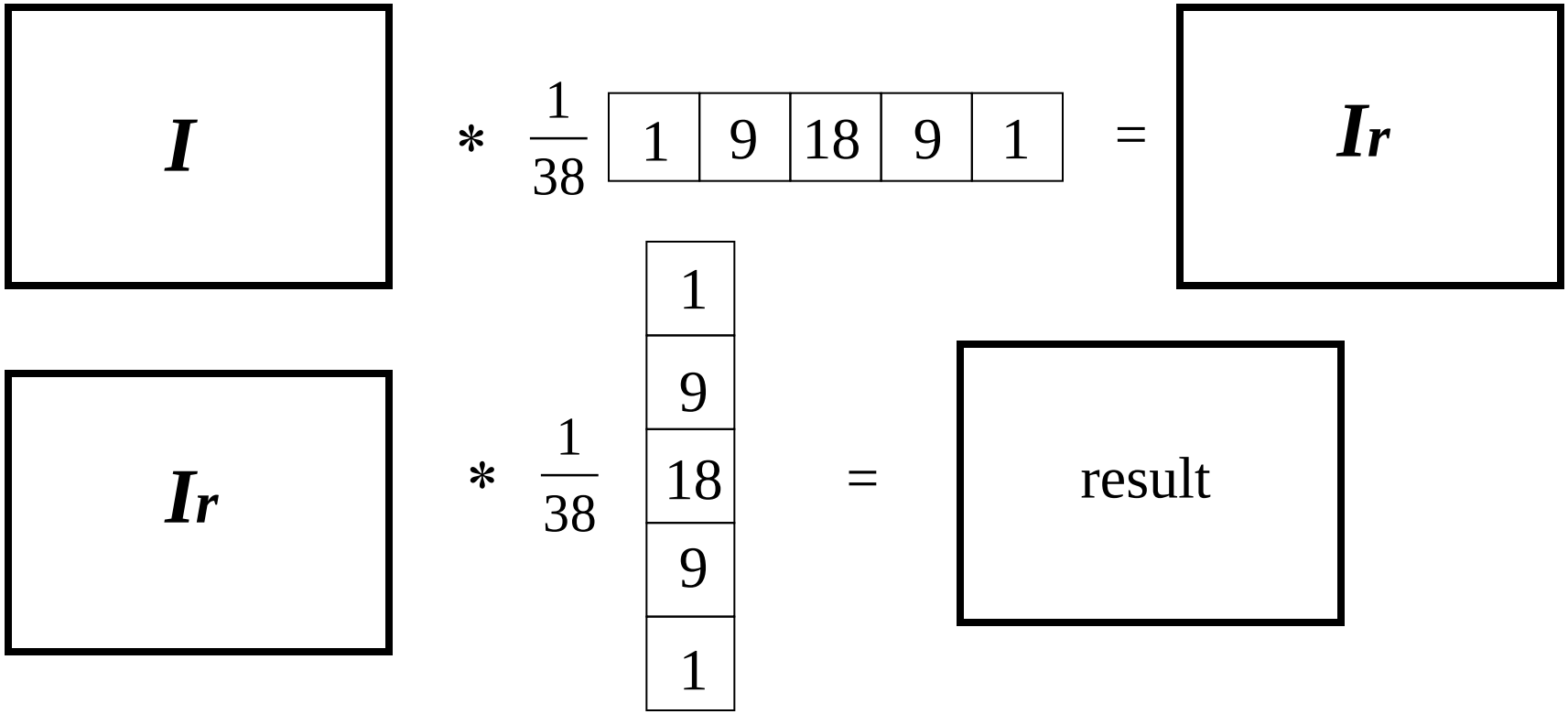
$$= \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} e^{-\frac{h^2+k^2}{2\sigma^2}} I(i-h,\, j-k) =$$

$$= \sum_{h=-m/2}^{m/2} e^{-\frac{h^2}{2\sigma^2}} \sum_{k=-m/2}^{m/2} e^{-\frac{k^2}{2\sigma^2}} I(i-h,\, j-k)$$

since $$e^{-\frac{h^2+k^2}{2\sigma^2}} = e^{-\frac{h^2}{2\sigma^2}} e^{-\frac{k^2}{2\sigma^2}}$$

Gaussian Kernel is Separable

Convolving rows and then columns with a 1-D Gaussian kernel.



The complexity increases linearly with m instead of with m^2.

Gaussian vs. Average

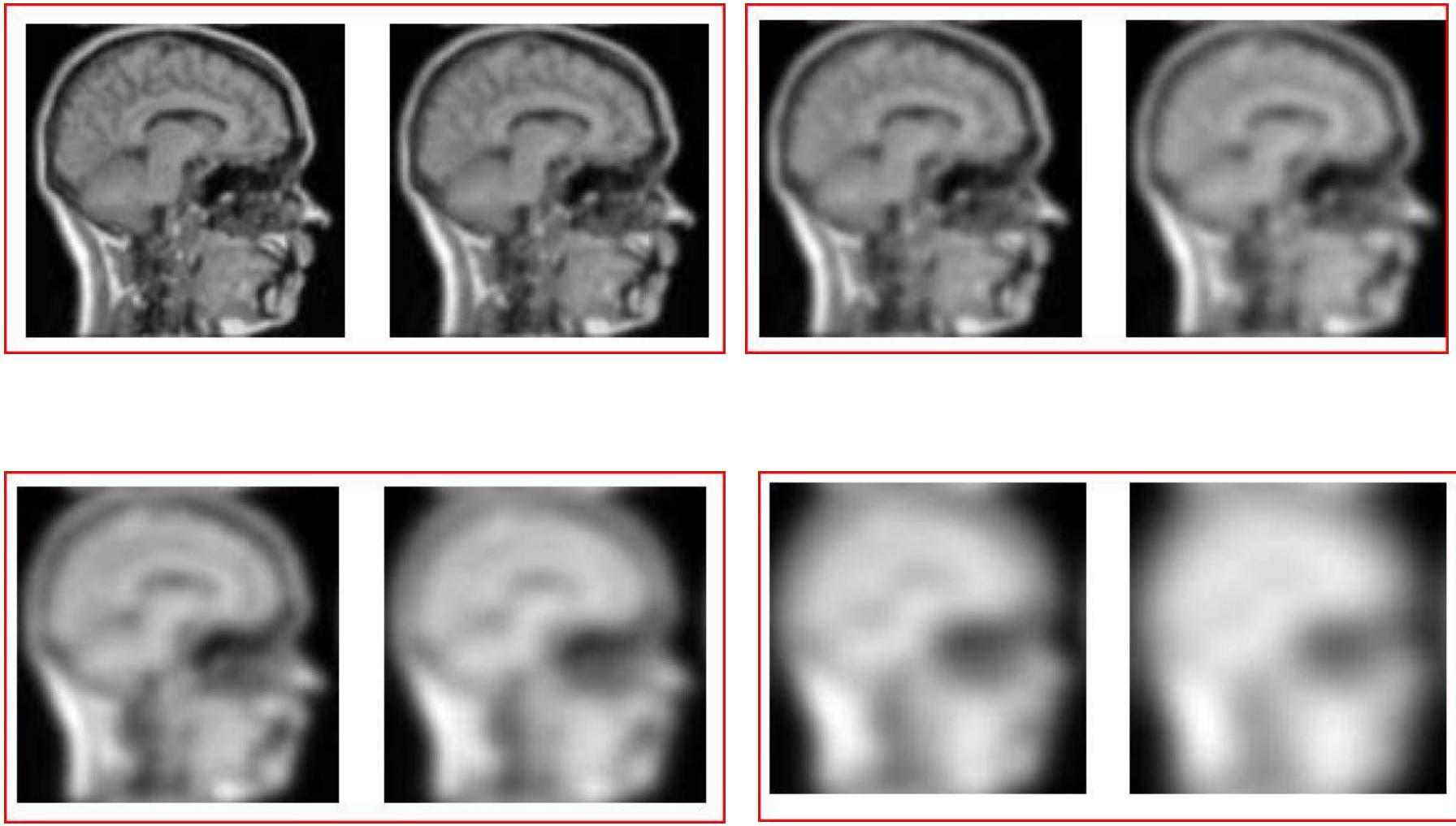
•Gaussian filter is optimal for removing Gaussian noise

Gaussian Smoothing

Smoothing by Averaging

Gaussian Scale Space (increasing σ)

Gaussian Scale Space (increasing σ)



Noise Filtering

After Averaging

Gaussian Noise

After Gaussian Smoothing

Noise Filtering

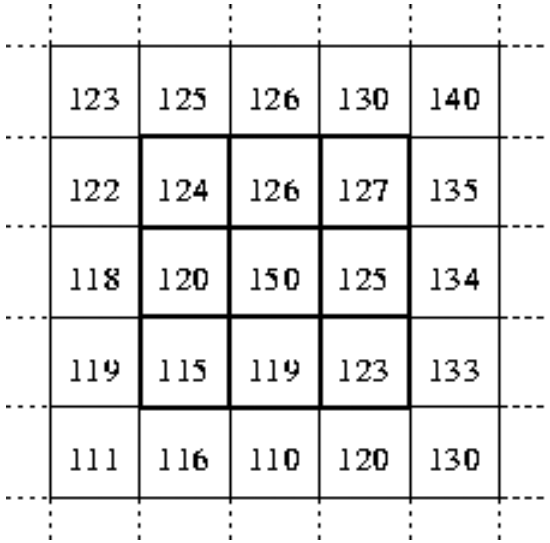
After averaging

Salt-and-pepper noise

After Gaussian smoothing

Nonlinear Filtering – median filter

Replace each pixel value $I(i, j)$ with the median of the values found in a local neighbourhood of (i, j).

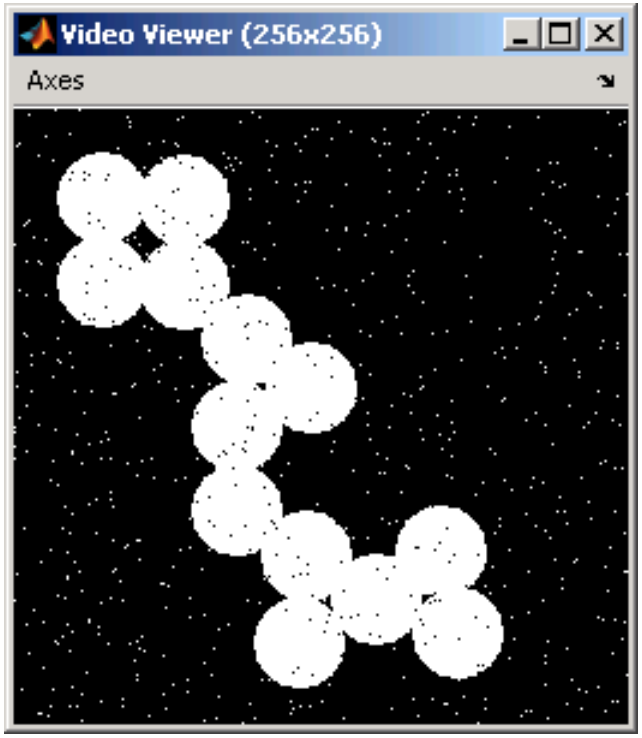


Neighbourhood values:

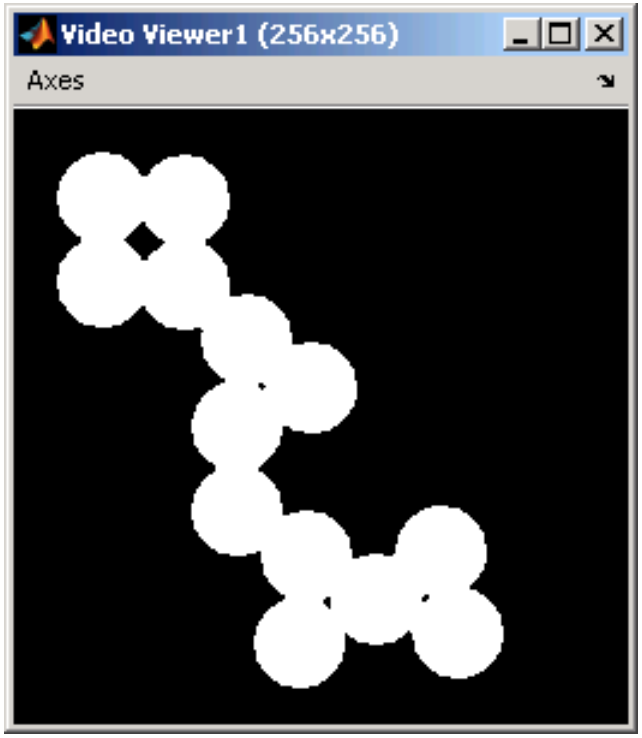
115, 119, 120, 123, 124,
125, 126, 127, 150

Median value: 124

Median Filter

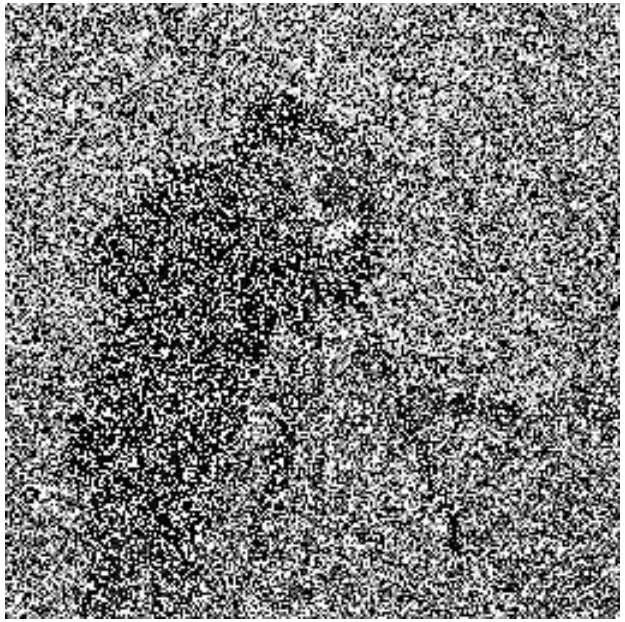


Salt-and-pepper noise



After median filtering

Remove noise and preserve edges!



Salt-and-Pepper Noise Removal by Median-type Noise Detectors and Edge-preserving Regularization

Raymond H. Chan, Chung-Wa Ho, and Mila Nikolova

IEEE Transactions on Image Processing, 14 (2005), 1479-1485.