# Corner (Interest Point) Detection 

COMP 4900C
Winter 2011
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## Motivation: Corners for Recognition



Image search: find the book in an image.

## Motivation: Corners for Recognition



## Corners for Augmented Reality



## Motivation: Corners for Robotics



## Motivation: 2D map built using corners



## Motivation: Build a panorama



Input images


Output vanorama 1


## Motivation: Build a Panorama

M. Brown and D. G. Lowe. Recognising Panoramas. ICCV 2003

## How do we build panorama?

## We need to match (align) images



## Matching with Corners

-Detect corner points in both images


## Matching with Corners

-Detect corner points in both images
-Find corresponding corner pairs by comparing the corner descriptors


## Matching with Corners

-Detect feature points in both images
-Find corresponding corner pairs by comparing the corner descriptors
-Use these pairs to align images


## More motivation...

Corner points are used also for:

- Image alignment (homography, fundamental matrix)
- 3D reconstruction, Object recognition, Indexing and database retrieval, Robot navigation, ... other
Corners define repeatable points for matching
Not just intersection of two lines (pure corner) but pixels which have a "corner like" structure
Corners sometimes called interest points because pixels that are "corner like" are interesting
Observe that in the region around a corner the gradient has two or more distinct values


## Corner Feature

Corners are image locations that have large intensity changes in more than one direction

For a pixel which is a corner shifting a window centered on that pixel in any direction should give a large change in the average intensity in that window.


## Harris Detector: Basic Idea


"flat" region:
no change in all directions

"edge":
no change along the edge direction

"corner":
significant change in all directions
C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

## Examples of Corner Features



## Corners in Calibration pattern



## Corners in a Basement room



## Change of Intensity

The intensity change at a given pixel in the direction (u,v) can be quantified by sum-of-squared-difference (SSD) of all pixels in a nbhd of that window, and the associated pixel shifted by ( $\mathrm{u}, \mathrm{v}$ )

$$
D(u, v)=\sum_{i, j}(I(i+u, j+v)-I(i, j))^{2}
$$



Here $\mathrm{i}, \mathrm{j}$ range over all the pixels in the nbhd and the difference between the original pixel and shifted pixel is summed.
Different $\mathrm{D}(\mathrm{u}, \mathrm{v})$ function exists for every pixel in the image.

## Change Approximation

If $u$ and $v$ are small, by Taylor's theorem:

$$
I(i+u, j+v) \approx I(i, j)+I_{x} u+I_{y} v
$$

where $\quad I_{x}=\frac{\partial I}{\partial x} \quad$ and $\quad I_{y}=\frac{\partial I}{\partial y}$
therefore

$$
\begin{aligned}
(I(i+u, j+v)-I(i, j))^{2} & =\left(I(i, j)+I_{x} u+I_{y} v-I(i, j)\right)^{2} \\
& =\left(I_{x} u+I_{y} v\right)^{2} \\
& =I_{x}^{2} u^{2}+2 I_{x} I_{y} u v+I_{y}^{2} v^{2} \\
& =\left[\begin{array}{ll}
u & v]\left[\begin{array}{cc}
I_{x}^{2} & I_{x} I_{y} \\
I_{x} I_{y} & I_{y}^{2}
\end{array}\right]\left[\begin{array}{c}
u \\
v
\end{array}\right]
\end{array},=\right.\text {. }
\end{aligned}
$$

## Gradient Variation Matrix

$$
D(u, v)=\left[\begin{array}{ll}
u & v
\end{array}\right]\left[\begin{array}{cc}
\sum I_{x}^{2} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y}^{2}
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right]
$$

This function is a rotated ellipse. Ellipse $D(u, v)=$ const

$$
C=\left[\begin{array}{cc}
\sum I_{x}^{2} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y}^{2}
\end{array}\right]
$$

Matrix $C$ characterizes how intensity changes in a certain direction. Each entry is computed by summing the appropriate values over every pixel in the neighbourhood around the given pixel

## Eigenvalue Analysis - simple case

First, consider case where:

$$
C=\left[\begin{array}{ll}
\sum I_{x}^{2} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y}^{2}
\end{array}\right]=\left[\begin{array}{cc}
\lambda_{1} & 0 \\
0 & \lambda_{2}
\end{array}\right]
$$

This means dominant gradient directions align with x or y axis
If either $\lambda$ is close to 0 , then this is not a corner, so look for locations where both are large.

The bigger the smallest $\lambda$ the more "corner like" is that pixel in the image

## Eigenvalue Analysis - simple case

$$
\begin{gathered}
D(u, v)=\left[\begin{array}{ll}
u & v
\end{array}\right]\left[\begin{array}{cc}
\lambda_{1} & 0 \\
0 & \lambda_{2}
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right] \\
D(u, v)=\lambda_{1} u^{2}+\lambda_{2} v^{2}
\end{gathered}
$$

Here $\lambda_{1}$ is the rate of change in direction of $u$
$\lambda_{2}$ is the rate of change in direction of $v$
If both $\lambda$ are small, we have a constant region,
If only one $\lambda$ is large have an edge,
If both $\lambda$ large is a corner (smallest $\lambda$ is large)

## General Case

It can be shown that since C is symmetric it can be diagonalized, which means finding $Q$ to rotate and rewrite C as:

$$
C=Q^{T}\left[\begin{array}{ll}
\lambda_{1} & 0 \\
0 & \lambda_{2}
\end{array}\right] Q
$$

So every C is simply a rotated version of the simple case:

Other eigenvector is in the direction of the slowest change

One eigenvector is in the direction


## Harris Detector



## Harris Detector

Find points where smallest eigenvalue is >threshold


## Harris Detector

Take only the points of local maxima of the smallest eigenvalue


## Harris Detector



## Gradient Orientation



## Corner Detection Summary

- if this is a region of constant intensity, both eigenvalues will be very small.
- if it contains an edge, there will be one large and one small eigenvalue (the eigenvector associated with the large eigenvalue will be parallel to the image gradient).
- if it contains edges at two or more orientations (i.e., a corner), there will be two large eigenvalues (the eigenvectors will be parallel to the image gradients).
- Eigenvectors encode edge directions, eigenvalues edge strength



## Corner Detection Algorithm

Algorithm
Input: image $f$, threshold $t$ for $\lambda_{2}$, size of $Q$
(1) Compute the gradient over the entire image $f$
(2) For each image point $p$ :
(2.1) form the matrix $C$ over the neighborhood $Q$ of $p$
(2.2) compute $\lambda_{2}$, the smaller eigenvalue of $C$
(2.3) if $\lambda_{2}>t$, save the coordinates of $p$ in a list $L$
(3) Sort the list in decreasing order of $\lambda_{2}$
(4) Scanning the sorted list top to bottom: delete all the points that appear in the list that are in the same neighborhood $Q$ with $p$

Step (3) and (4) is a type of non-maxima suppression (can be done other ways)

## Invariance of Corner Detector

- Compute the corners for a given picture
- Take new picture different from the original
- Still look at the same object or region in the new picture
- Change something (camera orientation, scale, lighting, etc.)
- Compute the corners for the new picture
- Corner detector is invariant if corners are the same in the corresponding parts of the image between old and new
- Invariance is desirable but not easy to get
- Often have some type of invariance (but not every type)
- Harris corners are invariant to rotations and translations in the camera plane, but not to other transformation (i.e. scale)
- Scale invariance is very difficult to achieve
- Has been achieved with SIFT and SURF descriptors


## Blur and Lighting Change



## Orientation and Zoom/Rotation



## Harris Detector Rotation Invariance

- Invariant to rotation in the image plane
- These are different than rotations out of the camera plane!


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

## Harris Detector Rotation Invariance

- Repeatability with image plane rotation




## Comparing corners - Corner descriptor

- To match corners extract a description of the corner (this is also called a corner descriptor)
- Need this to compare two corners in different images
- Use the set of pixels in region (nbhd) around the corner and compute a high dimensional vector from these features
- Up to you what you compute, but it should be invariant!
- Can simply use the pixels in a neighbourhood around each corner as the descriptor
- To compare two descriptors take the sum of squares difference of pixels in a small window around each corner
- This is a very easy but is not invariant
- There are better corner descriptors than this one
- Want invariance for the corner detection process and for the descriptor associated with each corner


## Scale Invariance - Find natural scale!

Bad Scale choice means wrong descriptor


Good scale choice Means correct
Descriptor


## SIFT/SURF Features - in OpenCV

- Bot the feature detection and the feature descriptor are invariant to scale
- Means the pixels used to compute the feature descriptor change with the feature scale

- Box around the feature changes scale appropriately


## Harris relative to SIFT/SURF features

- For each feature Harris detector returns
- Pixel location, corner strength and corner orientation
- Size of nbhd used to for the feature descriptor not specified
- SIFT/SURF are scale invariant so they also
- Include a scale (size of the nbhd window around the feature)
- Compute complex descriptor from pixels in this nbhd
- In other words, the nbhd changes to cover the same pixels as we change the distance of the camera to the feature
- So SURF/SIFT descriptor is invariant to scale
- As we change camera distance still have the same corner descriptor because pixels in the nbhd change appropriately
- This is scale invariance, not true for Harris corners


## Small motions/large motions

- Consider two images
- Extract corners (SIFT/SURF or Harris) and then match using some feature descriptor
- Harris features work only for some motions (rotation in camera plane, translation)
- Descriptors usually pixels in a small nbhd around the corner
- SIFT/SURF features work for larger motions, and for different types of motions
- Can handle blur, lighting, compression, all motion in the camera plane, and some motions out of the camera plane
- But they are slower to compute and the matching of the descriptors associated with each feature is also slower


## Successful SIFT/SURF matching



## Viewpoint Change - SURF



## Increasing Blur - SURF



## Scale/Rotate in Camera Plane - SURF



## Decreasing Brightness - SURF



## JPEG Compression - SURF



## Harris Corners

- Are the simplest corners to find which
- Can be extracted in real-time and are they are invariant to translation and rotation in image plane
- They are not invariant to scale, only good for small motion
- Finding corners is only half the work
- To match you need a corner descriptor which is a high dimensional vector derived from the pixels in a window centered at that corner pixel (what size is the window?)
- Both the corner location and the corner descriptor should be as invariant as possible
- Invariant to scale, orientation, lighting, etc.
- Should find same corners and very similar descriptor
- Scale invariant corners now exist and are common
- David Lowe (UBC) created SIFT corners, SURF is faster SIFT

