

COMP 4900D: Assignment 1
Due: Thursday, Feb. 2, 2012

- (1 mark) Scaling a vector $\begin{bmatrix} x & y \end{bmatrix}^T$ in the plane can be achieved by
$$x' = sx \text{ and } y' = sy$$
where s is a scalar.
 - Write out the matrix form of this transformation.
 - Write out the transformation matrix using homogeneous coordinates.
 - If the transformation also includes a translation
$$x' = sx + t_x \text{ and } y' = sy + t_y$$
Write out the new transformation matrix using homogeneous coordinates.
- (1 mark) A 3d rotation matrix has 9 (3 by 3) entries, and a 2d rotation matrix has 4 (2 by 2) entries. How many actual degrees of freedom are there in a 3d or 2d rotation? In other words, what is the minimum number of parameters necessary to uniquely specify a 3d rotation and a 2d rotation (answer with 2 numbers)? What does this imply about the relationship between the entries of a rotation matrix?
- (2 mark) In the thin lens equation (as shown in Figure 2.4 of Ch 2, p. 21, in the book *Introductory Techniques for 3d Computer Vision* which is on the CD) first prove that $Zz = f^2$ (Hint use this diagram along with the characteristics of similar triangles). Then use this equation along with the fact that $Z^\wedge = Z + f$, $z^\wedge = z + f$ to prove the thin lens equation, which is equation 2.2 in the book.
- (2 marks) A pinhole camera has focal length $f = 500$, pixel sizes $s_x = s_y = 1$, and its principal point is at $(o_x, o_y) = (320, 240)$. The world coordinate frame and the camera coordinate frame can be related by $X_c = RX_w + T$, where
$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$
 - Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.
 - What are the pixel coordinates of the world point $X_w = \begin{bmatrix} 150 & 200 & 400 \end{bmatrix}^T$
- (2 marks) Using the same R , T , f , s_x , s_y , and X_w as given in Question 3 write an OpenCV program that projects this single 3d point using the given camera parameters. This program should call the routine `cvProjectPoints2` but you need to convert the R matrix to the appropriate format. There is a slight problem with this routine. For reasons too complex to explain you should define the 3d object point as `object_points = cvCreateMat(3, 3, CV_32F);` and the 2d image points as `image_points = cvCreateMat(3, 2, CV_32F);`. There is only one point to project (not three) to project but this routine has some problems

