Support Vector Machines on General Confidence Functions

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Abstract

We present a generalized view of support vector machines that does not rely on a Euclidean geometric interpretation nor even positive semidefinite kernels. We base our development instead on the *confidence matrix*—the matrix normally determined by the direct (Hadamard) product of the kernel matrix with the label outer-product matrix. It turns out that alternative forms of confidence matrices are possible, and indeed useful. By focusing on the confidence matrix instead of the underlying kernel, we can derive an intuitive principle for optimizing example weights to yield robust classifiers. Our principle initially recovers the standard quadratic SVM training criterion, which is only convex for kernel-derived confidence measures. However, given our generalized view, we are then able to derive a principled relaxation of the SVM criterion that yields a convex upper bound. This relaxation is *always* convex and can be solved with a linear program. Our new training procedure obtains similar generalization performance to standard SVMs on kernel-derived confidence functions, but achieves even better results with indefinite confidence functions.

1 Introduction

Support vector machines were originally derived from purely geometric principles [10, 1]: given a labeled training set, one attempts to solve for a consistent linear discriminant that maximizes the minimum Euclidean distance between any data point and the decision hyperplane. Specifically, given $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_t, y_t)$, $y \in \{-1, +1\}$, the goal is to determine a $(\mathbf{w}, b)$ such that $\min_i y_i(\mathbf{w}^\top \mathbf{x}_i + b)/\|\mathbf{w}\|$ is maximized. Vapnik [10] famously proposed this principle and formulated a convex quadratic program for efficiently solving it. With the addition of slack variables the dual form of this quadratic program can be written

$$\min_{\boldsymbol{\alpha}} \quad \frac{1}{2}\sum_{ij} \alpha_i \alpha_j y_i y_j \mathbf{x}_i^\top \mathbf{x}_j - \sum_i \alpha_i \quad \text{subject to} \quad 0 \leq \boldsymbol{\alpha} \leq \beta,\ \boldsymbol{\alpha}^\top \mathbf{y} = 0 \quad (1)$$

where the dual variables $\boldsymbol{\alpha}$ behave as weights on the training examples.

One of the key insights behind the support vector machine approach is that the training vectors appear only as inner products in both training and classification, and therefore can be abstracted away by a general kernel function. In this case, the kernel function, $k(\mathbf{x}_i, \mathbf{x}_j)$,

simply reports inner products $\langle \boldsymbol{\phi}(\mathbf{x}_i), \boldsymbol{\phi}(\mathbf{x}_j) \rangle$ in some arbitrary feature (Hilbert) space. Combining the kernel abstraction with the ν-SVM formulation of [9, 2] one can re-express (1) in the more general form

$$\min_{\boldsymbol{\alpha}} \quad \boldsymbol{\alpha}^\top (K \circ \mathbf{y}\mathbf{y}^\top)\boldsymbol{\alpha} \quad \text{subject to} \quad 0 \leq \boldsymbol{\alpha} \leq \beta,\ \boldsymbol{\alpha}^\top \mathbf{y} = 0,\ \boldsymbol{\alpha}^\top \mathbf{e} = 1 \qquad (2)$$

where K is the *kernel matrix*, $K_{ij} = \langle \boldsymbol{\phi}(\mathbf{x}_i), \boldsymbol{\phi}(\mathbf{x}_j) \rangle$, the matrix $\mathbf{y}\mathbf{y}^\top$ is the *label matrix*, the vector $\mathbf{e}$ consists of all 1's, and $\circ$ denotes componentwise matrix multiplication (Hadamard product).

Although (2) appears to be a very general formulation of the weight training problem for $\boldsymbol{\alpha}$, it is in fact quite restrictive: for (2) to be convex, the combined matrix $K \circ \mathbf{y}\mathbf{y}^\top$ must be positive semidefinite, implying that K itself must be conditionally positive semidefinite.[1] Thus, it is commonly assumed that support vector machines should be applied on conditionally positive semidefinite kernels K.

Although the restriction to conditional positive semidefiniteness might not appear onerous, it is actually problematic in many natural situations. First, as [8] notes, verifying that a putative kernel function $k(\cdot, \cdot)$ is conditionally positive semidefinite can be a significant challenge. Second, as many authors note [8, 7, 5] using indefinite kernels and only approximately optimizing (2) can often yield similar or even better results than using conventional positive semidefinite kernels. (A frequently used example is the hyperbolic tangent kernel $\tanh(a\langle \mathbf{x}_i, \mathbf{x}_j \rangle + b)$.) Third, adding conditional positive semidefiniteness as a constraint causes tremendous difficulty when attempting to *learn* a kernel (similarity measure) directly from data.

In fact, it is this third difficulty that is the main motivation for this research. We are interested in *learning* similarity measures from data that we can then use to train accurate classifiers. One can easily devise natural ways of doing this (we elaborate on one approach below), but unfortunately in these cases ensuring positive semidefiniteness ranges from hard to impossible. To date, most successful attempts at learning conditionally positive semidefinite kernels have been reduced to taking convex combinations of known conditionally positive semidefinite kernels [6, 3]. But we would like to consider a wider range of techniques for learning similarities, and therefore seek to generalize (2). Our goal is to develop an efficient weight optimization procedure for $\boldsymbol{\alpha}$ that does not require a positive semidefinite matrix $K \circ \mathbf{y}\mathbf{y}^\top$, while still preserving the desirable generalization and sparseness properties achieved by standard SVM training.

Below in Section 2 we show how the standard kernel classifier can be generalized to consider more abstract *confidence functions* $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j)$ that play the same role as the usual kernel-label combination $y_i y_j k(\mathbf{x}_i, \mathbf{x}_j)$. We then briefly outline some natural approaches for learning confidence functions from data in Section 3. The approach we propose there is very simple, but effective. Nevertheless, it suffers from the drawback of not being able to ensure a positive semidefinite matrix for optimization. Section 4 then outlines our main development. Given the general confidence function viewpoint, we derive an $\boldsymbol{\alpha}$-weight optimization procedure from intuitive, strictly *non*-geometric first principles. The first procedure we derive simply recovers the classical quadratic objective, but from a new perspective. With this re-derivation in hand, we are then able to formulate a novel relaxation of the standard SVM objective that is both principled while also being guaranteed to be convex. Finally, in Section 5 we present experimental results with this new training principle, showing similar performance to standard SVM training with standard kernel functions, but obtaining stronger performance using indefinite confidence functions learned from data.

[1] A symmetric matrix K is conditionally positive semidefi nite if $\mathbf{z}^\top K \mathbf{z} \geq 0$ for all $\mathbf{z}$ such that $\mathbf{z}^\top \mathbf{e} = 0$. K need only be conditionally positive semidefi nite to ensure $K \circ \mathbf{y}\mathbf{y}^\top$ is positive semidefinite because of the assumption $\boldsymbol{\alpha}^\top \mathbf{y} = 0$. That is, if $(\boldsymbol{\alpha} \circ \mathbf{y})^\top \mathbf{e} = \boldsymbol{\alpha}^\top \mathbf{y} = 0$, then we immediately obtain $\boldsymbol{\alpha}^\top (K \circ \mathbf{y}\mathbf{y}^\top)\boldsymbol{\alpha} = (\boldsymbol{\alpha} \circ \mathbf{y})^\top K (\boldsymbol{\alpha} \circ \mathbf{y}) \geq 0$.

2 Confidence function classification

Our goal is to develop a learning and classification scheme that can be expressed more abstractly than the usual formulation in terms of $y_i y_j k(\mathbf{x}_i \mathbf{x}_j)$. We do this via the notion of a *confidence function*, $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j)$, which expresses a numerical confidence that the label pair $y_i y_j$ is in fact correct for the input pair $\mathbf{x}_i$ and $\mathbf{x}_j$. A large confidence value expresses certainty that the label pair is correct, while a small value correspondingly expresses a lack of confidence that the label pair is correct (or certainty that the label pair is wrong). We make no other assumptions about the confidence function, although it is usually presumed to be symmetric: $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j) = c(y_j y_i | \mathbf{x}_j \mathbf{x}_i)$.

Although the notion of a pairwise confidence function might seem peculiar, it is in fact exactly what the $y_i y_j k(\mathbf{x}_i, \mathbf{x}_j)$ values provide to the SVM. In particular, if we make the analogy $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j) = y_i y_j k(\mathbf{x}_i, \mathbf{x}_j)$ and assume $y \in \{-1, +1\}$, one can see that $y_i y_j k(\mathbf{x}_i, \mathbf{x}_j)$ behaves as a simple form of confidence function: the value is relatively large if either $y_i = y_j$ and $\mathbf{x}_i$ and $\mathbf{x}_j$ are similar under the kernel k, or if $y_i \neq y_j$ and $\mathbf{x}_i$ and $\mathbf{x}_j$ are *dissimilar* under the kernel. We therefore refer to the matrix $C = K \circ \mathbf{y}\mathbf{y}^\top$ as the *confidence matrix*.

Proposition 1 *If the entries of the confidence matrix $K \circ \mathbf{y}\mathbf{y}^\top$ are strictly positive, then the training data is linearly separable in the feature space defined by K.*

This proposition clearly shows that high confidence values translate into an accurate classifier on the training data. In fact, it is confidences, not similarities, that lie at the heart of support vector machines: The SVM methodology can be recast strictly in terms of confidence functions, abstracting away the notion of a kernel entirely, without giving up anything (except the Euclidean geometric interpretation). To illustrate, consider the standard SVM classifier: Assuming a vector of training example weights, $\boldsymbol{\alpha}$, has already been obtained from the quadratic minimization (2), the standard classification rule can be rewritten strictly in terms of confidence values

$$\begin{aligned} \hat{y} &= \text{sign}\Big(\Big(\sum_j \alpha_j y_j k(\mathbf{x}, \mathbf{x}_j)\Big) + b\Big) &&= \arg\max_y \Big(\sum_j \alpha_j y y_j k(\mathbf{x}, \mathbf{x}_j)\Big) + by \\ &&&= \arg\max_y \Big(\sum_j \alpha_j c(y y_j | \mathbf{x}\mathbf{x}_j)\Big) + by \quad (3) \end{aligned}$$

Thus a test example $\mathbf{x}$ is classified by choosing the label y that exhibits the largest weighted confidence when paired against the training data.

Quite obviously, the SVM training algorithm itself can also be expressed strictly in terms of a confidence matrix over training data.

$$\min_{\boldsymbol{\alpha}} \quad \boldsymbol{\alpha}^\top C \boldsymbol{\alpha} \quad \text{subject to} \quad 0 \leq \boldsymbol{\alpha} \leq \beta,\ \boldsymbol{\alpha}^\top \mathbf{y} = 0,\ \boldsymbol{\alpha}^\top \mathbf{e} = 1 \qquad (4)$$

This is just a rewriting of (2) with the substitution $C = K \circ \mathbf{y}\mathbf{y}^\top$, which does not change the fact that the problem is convex if and only if C is positive semidefinite. However, the formulation (4) is still instructive. Apparently the SVM criterion is attempting to reweight the training data to *minimize* expected confidence. Why? Below we argue that this is in fact an incorrect view of (4), and suggest that, alternatively, (4) can be interpreted as attempting to *maximize* the robustness of the classifier against changes in the training labels. With this different view, we can then formulate an alternative training criterion—a relaxation of (4)—that still attempts to maximize robustness, but is convex for *any* confidence matrix C. This allows us to advance our goal of *learning* confidence functions from data, while still being able to use SVM training of the example weights without having to ensure positive semidefiniteness.

Before turning to the interpretation and relaxation of (4), we first briefly consider a natural approach to learning confidence functions, which further motivates this research.

3 Learning confidence functions

There are many natural ways to consider learning a confidence function $c(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$ from training data. In [4] we investigated a particularly simple technique that achieves reasonable results. Given training labels, one can just straightforwardly learn to predict label pairs $y_i y_j$ given their corresponding input vectors $\mathbf{x}_i$ and $\mathbf{x}_j$. Concretely, given examples $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_t, y_t)$, it is easy to form the set of training pairs $\{(\mathbf{x}_i\mathbf{x}_j, y_i y_j)\}$ from the original data, which doubles the number of input features and squares the number of training labels and classes. (Subsampling can always be used to reduce the size of this training set.) Given such pairwise training data, standard probabilistic models can be learned to predict the probability of a label pair given the input vectors.

In [4] we considered logistic regression and naive Bayes classifiers for learning pairwise predictors. For example, the logistic regression classifiers were trained to maximize the conditional likelihood $P(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$ of the observed label pairs given the conjoined vector of inputs $\mathbf{x}_i\mathbf{x}_j$. Once learned, the pairwise model was used to classify test inputs $\mathbf{x}$ by maximizing the product $\hat{y} = \arg\max_y \prod_j P(yy_j|\mathbf{x}\mathbf{x}_j)$.[2] Clearly, this is equivalent to using a confidence function $c(y_i y_j|\mathbf{x}_i\mathbf{x}_j) = \log P(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$ and classifying with respect to uniform example weights $\boldsymbol{\alpha}$. Surprisingly, [4] obtained good classification results with this simple approach. In this paper, we are interested in the connection to support vector machines and attempt to improve the basic method by optimizing the $\boldsymbol{\alpha}$-weights.

Note that, as a confidence measure, using a log probability model, $\log P(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$, is a very natural choice. It can be trained easily from the available data, and performs quite well in practice. However, using a log probability model for a confidence function raises a significant challenge: $\log P(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$ is always non-positive and therefore any confidence matrix C it produces, since it is strictly non-positive, cannot be positive semidefinite. Ironically, the very confidence functions that appear to be most natural in this context, are precisely the ones ruled out by the positive semidefinite constraint. This problem motivates us to reformulate the quadratic optimization criterion (4), so that convexity can be achieved more generally while preserving the effective generalization properties.

4 Optimizing training example weights: An alternative view

Given a confidence classifier (3) it is natural to consider adjusting the training example weights $\boldsymbol{\alpha}$ to improve accuracy. At first glance, the quadratic minimization criterion (4) used by SVMs appears to be adjusting the example weights to *minimize* the confidence of the training labels y_i. However, we can argue that this interpretation is misleading. In fact, standard kernel-based confidence functions have a special property that masks a key issue: how confidences change when a training label is flipped. For the classifier (3), it is not the absolute confidence that counts, but rather the *relative* confidences between the correct label and the incorrect label. That is, we would like the confidence of a correct label to be larger than the confidence of a wrong label. For kernel-based confidence functions it turns out that the relationship between the relative confidences is greatly restricted.

Observation 1 *Let $\overline{y}$ denote a label flip, $-y$. If $c(y_i y_j|\mathbf{x}_i\mathbf{x}_j) = y_i y_j k(\mathbf{x}_i\mathbf{x}_j)$ then*

$$\sum_j \alpha_j c(yy_j|\mathbf{x}\mathbf{x}_j) + by \quad = \quad -\sum_j \alpha_j c(\overline{y}y_j|\mathbf{x}\mathbf{x}_j) - b\overline{y} \tag{5}$$

[2] [4] also considered other techniques for classification, including correlating the predictions on the test data in a transductive manner, but we do not pursue these extensions here.

However, the relationship (5) is obviously not true in general. For example, it is violated by any probabilistic confidence function defined by $c(y_i y_j|\mathbf{x}_i\mathbf{x}_j) = \log P(y_i y_j|\mathbf{x}_i\mathbf{x}_j)$.

Thus for kernel-based confidence functions, the confidence in the opposite label is always just the negation of the confidence in the current label.

We now show how the concept of minimizing sensitivity to label flips on the training data recovers the classical SVM training criterion. Consider a training example $(\mathbf{x}_i, y_i)$ and an arbitrary current set of weights $\boldsymbol{\alpha}$. The current confidence in the training label y_i is

$$\Big(\sum_j \alpha_j c(y_i y_j|\mathbf{x}_i\mathbf{x}_j)\Big) + by_i$$

Now consider the change in the confidence in y_i that would result if a single training label y_k was actually mistaken. That is, if the current value of y_k is incorrect and should have been given the opposite sign, then the mistake we are making in y_i's confidence is

$$\begin{aligned} \Delta_k c(y_i) &= \alpha_k c(y_k y_i|\mathbf{x}_k\mathbf{x}_i) - \alpha_k c(\overline{y_k} y_i|\mathbf{x}_k\mathbf{x}_i) && (6)\\ &= 2\alpha_k c(y_k y_i|\mathbf{x}_k\mathbf{x}_i) && (7)\end{aligned}$$

Note that (7) holds only under the kernel-based restriction (5), but in this special case the confidence penalty is just twice the original confidence. If (5) does not hold, then (6) can be used. The sum of the local confidence changes measures the overall sensitivity of the classification of training label y_i to possible mislabelings of other data points

$$\Delta c(y_i) = \sum_k \Delta_k c(y_i) \quad (8)$$

The smaller this value, the less likely y_i is to be misclassified due to a mislabeling of some other data point. That is, the sensitivity to label flips should be minimized if the classifier is to be made more robust. Nevertheless, there might be a tradeoff between the sensitivities of different training examples. Therefore, as a final step, we consider minimizing the overall *weighted* sensitivity of the training labels. This yields the minimization objective

$$\begin{aligned} \sum_i \alpha_i \Delta c(y_i) &= \sum_i \alpha_i \sum_k \alpha_k \Big(c(y_k y_i|\mathbf{x}_k\mathbf{x}_i) - c(\overline{y_k} y_i|\mathbf{x}_k\mathbf{x}_i)\Big) && (9)\\ &= 2\sum_{ik} \alpha_i\alpha_k c(y_k y_i|\mathbf{x}_k\mathbf{x}_i) && (10)\end{aligned}$$

Again, (10) only holds under the kernel-based restriction (5), but if this is violated, the more general form (9) can be used.

Therefore, if we are using a kernel-based confidence function, we recover exactly the same training criterion as the standard SVM (4). What is interesting about this derivation is that it does not require any reasoning about Euclidean geometry or even feature (Hilbert) spaces. The argument is only about adjusting the example weights to reduce the sensitivity of the classifier (3) to any potential mistakes in the training labels. That is, from the perspective of $\boldsymbol{\alpha}$-weight optimization, it is only the reflection property (5) and the desire to minimize sensitivity to mislabeled training examples that yields the same minimization objective as standard SVMs. The remaining constraints in (4) are also easily justified in this context: It is natural to assume that the example weights form a convex combination, and therefore $0 \leq \boldsymbol{\alpha}$, $\boldsymbol{\alpha}^\top \mathbf{e} = 1$. It is also natural to preserve class balance in the reweighting, hence $\boldsymbol{\alpha}^\top \mathbf{y} = 0$. Finally, as a regularization principle, it makes sense to limit the magnitude of the largest weights so that too few examples do not dominate the classifier, hence $\boldsymbol{\alpha} \leq \beta$.

Of course, rederiving an old criterion from alternative principles is not a significant contribution. However, what is important about this perspective is that it immediately suggests

principled alternatives to the SVM criterion that can still reduce sensitivity to potential training label changes. Our goal is to reformulate the objective to avoid a quadratic form, since this prevents effective optimization on indefinite confidence functions, which are the confidence functions we are most interested in (Section 3). It turns out that just a minor adjustment to (10) yields just such a procedure.

Given the goal of minimizing the sensitivity to training label changes, previously we sought to minimize the *weighted* sensitivity, using the same weights being optimized, which leads to the quadratic form (10). However, instead of minimizing weighted sensitivity, one could instead be more conservative and attempt to minimize the *maximum* sensitivity of any label in the training set. That is, we would like to adjust the example weights so that the worst sensitivity of any training label y_i to potential mislabellings of other examples is minimized. This suggestion immediately yields our proposal for a new training criterion

$$\min_{\boldsymbol{\alpha}} \quad \max_{i} \quad \sum_{k} \alpha_k \Big(c(y_k y_i | \mathbf{x}_k \mathbf{x}_i) - c(\overline{y_k} y_i | \mathbf{x}_k \mathbf{x}_i) \Big) \tag{11}$$
$$\text{subject to} \quad 0 \leq \boldsymbol{\alpha} \leq \beta,\ \boldsymbol{\alpha}^\top \mathbf{y} = 0,\ \boldsymbol{\alpha}^\top \mathbf{e} = 1$$

Once $\boldsymbol{\alpha}$ has been optimized, the offset b can be chosen to minimize training error.

Proposition 2 *The objective (11) is convex for any confidence function c, and moreover is an upper bound on (4).*

The proof of this proposition is obvious. The minimization objective is a maximum of linear functions of $\boldsymbol{\alpha}$, and hence is convex. Given the constraints $0 \leq \boldsymbol{\alpha}$, $\boldsymbol{\alpha}^\top \mathbf{e} = 1$ we immediately have $\max_i f(i) \geq \sum_i \alpha_i f(i)$.

As a practical matter, (11) can be solved by a simple linear program

$$\min_{\boldsymbol{\alpha}, \delta} \ \delta \quad \text{subject to} \quad \delta \geq \sum_{k} \alpha_k \Big(c(y_k y_i | \mathbf{x}_k \mathbf{x}_i) - c(\overline{y_k} y_i | \mathbf{x}_k \mathbf{x}_i) \Big) \quad \forall i$$
$$0 \leq \boldsymbol{\alpha} \leq \beta,\ \boldsymbol{\alpha}^\top \mathbf{y} = 0,\ \boldsymbol{\alpha}^\top \mathbf{e} = 1 \tag{12}$$

This formulation provides a convex relaxation of the SVM criterion for *any* confidence function $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j)$, including the probabilistic confidence functions discussed above.

5 Experimental results

We implemented the new weight optimization scheme based on linear programming (12) and, where possible, compared it to standard SVM quadratic minimization (4) as well as using uniform weights. (Although using uniform weights appears to be naive, for high quality confidence measures such as those learned by training reasonable probability models, uniform weighting can still achieve highly competitive generalization performance [4].) We compared the different weight optimization schemes on a variety of confidence functions, including those defined by standard positive semidefinite kernels (linear dot product and RBF), as well as the sigmoid kernel $\tanh$, and two probabilistic confidence models trained using naive Bayes and logistic regression respectively. We compared the test accuracy of these various algorithms on the set of two-class UCI data sets. All of our experimental results are averages over 5 times repeats with training size =100 or 4/5 of the data size.

In the first study we compared how the different weight optimization schemes performed using the positive semidefinite functions determined by the linear and RBF kernels respectively. Table 1 shows that the new weight optimization scheme (12) achieves comparable generalization performance to standard quadratic training. The uniform weighting strategy is clearly inferior in the linear kernel case. Though it is better in the RBF kernel case, the

Table 1: *Comparison of the test accuracy of different $\boldsymbol{\alpha}$-weight optimizers ($\boldsymbol{\alpha}$0: uniform weighting; $\boldsymbol{\alpha}$1: linear optimization; $\boldsymbol{\alpha}$2: quadratic optimization) on UCI data sets, using the positive semidefinite confidence functions defined by linear (L) and RBF ($k(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2)$) kernels.*

	L-$\boldsymbol{\alpha}$0	**L-$\boldsymbol{\alpha}$1**	**L-$\boldsymbol{\alpha}$2**	**RBF-$\boldsymbol{\alpha}$0**	**RBF-$\boldsymbol{\alpha}$1**	**RBF-$\boldsymbol{\alpha}$2**
australian	0.6132	0.8149	0.8434	0.7288	0.7837	0.7132
breast	0.9280	0.9691	0.9698	0.9465	0.9712	0.9623
cleve	0.6173	0.7939	0.8020	0.8173	0.8194	0.7918
corral	0.5714	0.9000	0.9286	0.9071	0.9357	1.0000
crx	0.5461	0.8495	0.8373	0.6492	0.7696	0.6767
diabetes	0.6512	0.7542	0.7051	0.7410	0.7048	0.7299
flare	0.8292	0.8292	0.7602	0.7998	0.8151	0.8292
german	0.7000	0.7053	0.6660	0.4984	0.6280	0.7002
glass2	0.7556	0.7810	0.8190	0.8921	0.8667	0.8635
heart	0.7800	0.8235	0.8353	0.7635	0.7941	0.7741
hepatitis	0.8125	0.8625	0.8625	0.7250	0.8250	0.8250
mofn-3-7	0.7794	0.7814	0.8047	0.7038	0.7760	0.8565
pima	0.6512	0.7629	0.7359	0.7722	0.7009	0.6943
vote	0.6149	0.9307	0.9349	0.8806	0.9128	0.8752
average	0.7036	0.8256	0.8218	0.7732	0.8074	0.8066

results are still not comparable to the linear and quadratic weighting scheme. The problem is that the confidence functions are only weakly informative here, and simply averaging them still yields a sensitive classifier. It is encouraging to note that our convex relaxation retains most of the benefit of the original quadratic objective in this case.

A more interesting test of the method is on indefinite confidence functions, such as those determined by $\tanh$ and $\log P(y_i y_j | \mathbf{x}_i \mathbf{x}_j)$. In these cases, the quadratic objective is non-convex and cannot be solved by standard quadratic optimizers. However, as mentioned, our relaxation remains convex in this case. [8] suggests more sophisticated approach to training in these cases, but their methods are substantially more technical than the simple technique proposed here. Table 2 shows the results of our linear weight optimization procedure and the the uniform weighting on the indefinite confidence functions. Clearly the probabilistic (trained) confidence functions yield effective classifiers, and they perform better than the standard SVMs with both linear and RBF kernels. Even the results for the sigmoid kernel with linear optimization weighting are comparable to the results of the standard SVM with RBF kernel. Moreover, even uniform weighting already achieves good results for these confidence functions.

The main benefit of the new approach is the ability to reliably optimize example weights for a wider range of confidence functions. We believe this is a useful advantage over SVM training because most natural confidence functions, in particular *learned* confidence functions, are not usually positive semidefinite. Learned confidence functions have a wider potential for generalization improvement over using fixed kernels, as our results suggest, since the accuracies obtained by using the probabilistic confidence functions tend to exceed those of other techniques in our experiments.

6 Conclusion

We have introduced a simple generalization of support vector machines based on the notion of a *confidence function* $c(y_i y_j | \mathbf{x}_i \mathbf{x}_j)$. This view allows us to think of SVM training as at-

Table 2: *Comparison of the test accuracy of different $\boldsymbol{\alpha}$-weight optimizers ($\boldsymbol{\alpha}$0: uniform weighting; $\boldsymbol{\alpha}$1: linear optimization) on UCI data sets, using the indefinite confidence functions* $\tanh$ *($k(\mathbf{x}_i, \mathbf{x}_j) = tanh(0.001 \cdot \mathbf{x}_i \mathbf{x}_j - 1)$), naive Bayes (NB), and logistic regression (LR).*

	LR-$\boldsymbol{\alpha}$0	**LR-$\boldsymbol{\alpha}$1**	**NB-$\boldsymbol{\alpha}$0**	**NB-$\boldsymbol{\alpha}$1**	$\tanh$**-$\boldsymbol{\alpha}$0**	$\tanh$**-$\boldsymbol{\alpha}$1**
australian	0.8502	0.8502	0.8461	0.8471	0.6488	0.8193
breast	0.9585	0.9328	0.9715	0.9691	0.9605	0.9684
cleve	0.8020	0.8041	0.8398	0.8459	0.7582	0.7622
corral	0.8714	0.8929	0.9000	0.9000	0.8786	0.7071
crx	0.8452	0.8452	0.8416	0.8347	0.6058	0.8365
diabetes	0.7440	0.7413	0.7605	0.7545	0.6512	0.7027
flare	0.8197	0.8199	0.8306	0.8207	0.8292	0.8273
german	0.7189	0.7182	0.7144	0.7133	0.7084	0.6969
glass2	0.8063	0.8127	0.8730	0.8952	0.7524	0.7683
heart	0.8129	0.8118	0.8224	0.8271	0.8200	0.8165
hepatitis	0.9000	0.9000	0.9250	0.9375	0.7750	0.8250
mofn-3-7	0.9111	0.9026	0.8873	0.9239	0.7794	0.7802
pima	0.7425	0.7425	0.7575	0.7587	0.6512	0.7156
vote	0.9176	0.9188	0.9194	0.9122	0.8669	0.9200
average	0.8357	0.8352	0.8492	0.8529	0.7633	0.7961

tempting to minimize the sensitivity of the classifier to perturbations of the training labels. From this perspective, we can not only rederive the standard SVM objective without appealing to Euclidean geometry, we can also devise a new training objective that is convex for arbitrary, not just positive semidefinite, confidence functions. Of course, other optimization objectives are possible, and perhaps superior ones could still be developed. An important research direction is to develop a generalization theory for our relaxed training procedure that is analogous to the theory that has already been developed for SVMs.

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