Choices

- In many applications, it becomes useful to choose or pick domain values, possibly depending on other values
- For that, we can introduce a new "choice" construct or operator
- So as with PCs, it will be possible to eliminate it (by defining it) if wanted (coming)

  But it may be useful to have it as given
- A program Π may have a "choice rule" with a "choice operator": (*)

  $$P(x, y, z) \longleftarrow Q(\ldots, x, \ldots), S(\ldots, y, \ldots), R(\ldots, z, \ldots), \mathit{Choice}((x, y), z)$$

  For each combination of values (x, y), non-deterministically choose a unique value for z and put the trio (x, y, z) into P

  (while satisfying the other conditions in the body)

- Example: Consider the relational table that does not satisfy the functional dependency $R:\ AB \rightarrow C$

R	A	B	C
	a	b	c
	a	c	e
	a	b	d

- Repairing R: (by tuple deletion)

  $R'(x,y,z) \leftarrow\ R(x,y,z), choice((x,y),z)$.

  $R(a,b,c)$. Etc.

- Two stable models (two repairs):

  $\{R'(a,b,c), R'(a,c,e), R(a,b,c), \ldots\}$
  $\{R'(a,b,d), R'(a,c,e), R(a,b,c), \ldots\}$

- Different choices for z appear in different stable models
- Use of choice rules increases number of SMs
- No negation in this example?

  Or in the rule (*) above?

- Negation, actually unstratified, is implicit in choice rules
  It will reappear when we define the “choice operator” by means of regular rules
- So, no essential need for the *choice* operator (but nice, intuitive and useful having it!)
- Replace choice rule (*) by:

  $$P(x,y,z) \longleftarrow Q(\ldots,x,\ldots), S(\ldots,y,\ldots), R(\ldots,z,\ldots), \mathit{Chosen}(x,y,z)$$

- Next, define the *Chosen* predicate with two extra rules:

  $$\begin{aligned} \mathit{Chosen}(x,y,z) \longleftarrow\ & Q(\ldots,x,\ldots), S(\ldots,y,\ldots), R(\ldots,z,\ldots), \\ & \mathit{not}\ \mathit{DiffChoice}(x,y,z) \\ \mathit{DiffChoice}(x,y,z) \longleftarrow\ & \mathit{Chosen}(x,y,z'),\ z' \neq z \end{aligned}$$

- Unstratified negation!
  choice operator is an extra source of unstratified negation

- The last two rules ensure that, for every pair of values (x, y) that satisfies the body, the predicate $Chosen(x, y, z)$ satisfies the functional dependency: $xy \rightarrow z$

- In (*) the choice operator can (or must) be replaced by the new predicate *Chosen* that forces the expected functional dependency

- Some systems do not support the choice operator, so *Chosen* (defined as above) has to be used instead

  Exercise: In the example above, replace the *choice* operator by its corresponding and concrete *Chosen* predicate (adding its definition, of course); and compute the repairs of the DB with DLV

  Exercise: Give a general program to solve the "Hamiltonian Cycle" problem

The Answer Set Programming Paradigm

- Normal programs with stable model semantics can be used to solve hard (and easy) combinatorial problems
- We use the more general notion of "Logic Programs with Answer-Set Semantics" (more below)
- Determining if there are stable models, and computing one (in the positive case) is good enough (brave semantics can be used)
- Answer-Set Programming is a new (logic) programming paradigm: Specify the problem's conditions

  The underlying "solver" will find a solution (if any), or report when none exists

  Cf. Brewka,G., Eiter, T. and Truszczynski, M. Answer Set Programming at a Glance. Comm. of the ACM, 2011, 54(12), pp. 93-103.
- A form of declarative programming (as opposed to imperative)

- NPs (and more generally ASPs) are commonly used to implicitly specify by means of a general program $\Pi^{\mathcal{G}}$ all the solutions of a general (usually combinatorial) problem $\mathcal{P}$

- A specific instance I (input) for problem $\mathcal{P}$ is usually represented by means of a specific EDB E for $\Pi^{\mathcal{G}}$

- The SMs of the combined specification $\Pi^{\mathcal{G}} \cup E$ become the solutions for $\mathcal{P}$ with instance I

  E.g. $\mathcal{P}$ could be 3-GC, and I a concrete graph (c.f. page 23)

- Solutions are computed using a general underlying *solver*

  In many applications finding one model is good enough

- Commonly, the *solver* internally transforms the problem (so as SM computation) into SAT, the problem of determining the truth assignments that make a propositional formula true

  The latter a crucial problem in many areas of computer science

- SAT is a sufficiently and necessarily difficult problem; complete for the complexity class *NP*
- The computational data complexity of normal programs matches that of *NP*-complete problems (more below)

  Then, normal programs can be used to solve any problem in the class *NP*
- The solutions can be computed using general implementations of the SM semantics, e.g. CLINGO or DLV
- ASPs are successfully used in different areas and problems
  1. Logic-based knowledge representation in general

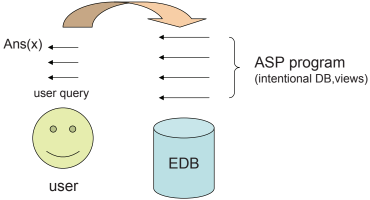
     In particular, representation of commonsense knowledge and commonsense reasoning
  2. Solution of hard combinatorial problems
  3. Many applications in data management

     ASPs extend Datalog

- In data management, ASP can be used to define complex views and express complex queries

  E.g. R' on page 29

- The ASP goes on top of a RDB



The stable models of EDB + ASP program are extended with query atoms $Ans(a)$ and their specification by means of extra rules

- Sometimes we need (or welcome) an extension of NPs

  To specify finitely many, but usually mutually exclusive, alternatives

- The SM semantics can be extended to disjunctive programs

Disjunctive Stable Model Semantics

- Now we admit rules of the form

$$B_1 \vee \cdots \vee B_k \longleftarrow A_1, \cdots, A_m, not\ A_{m+1}, \ldots, not\ A_n$$

  with B_j, A_i atoms, and *not* weak negation as before

  E.g. $P(x) \vee T(x) \longleftarrow R(x), Q(y), not\ T(y), not\ Q(x)$

- Now we have the disjunctive stable model semantics

  (Gelfond & Lifschitz, 1991)

- The semantics is an extension of that for NPs

  Based, as before, on testing candidates to SMs

Start with a candidate set of atoms $S \subseteq HB(\Pi)$

1. Pass from Π to Π_H (grounding)
2. Construct a new ground program Π_H^S:
   (a) Delete from Π_H every rule that has a subgoal *not* A in the body, with $A \in S$
   (b) From the remaining rules, delete the negative subgoals
3. We are left with a ground disjunctive positive program Π_H^S, without negation

   As a "disjunctive Datalog program" it may have several minimal (H-)models (a simple and natural extension of Datalog)

   $MinMod(\Pi_H^S)$ denotes the set of minimal models of Π_H^S
4. If $S \in MinMod(\Pi_H^S)$, we say that S is a stable model of Π

Example:

$$\begin{aligned} p \vee q &\leftarrow s,\ not\ p \\ s &\leftarrow \end{aligned}$$

- For $S_1 = \{s, q\}$, the residual program Π^{S_1} is:

$$\begin{aligned} p \vee q &\leftarrow s \\ s &\leftarrow \end{aligned}$$

  For this positive, disjunctive Datalog program:

  $MinMod(\Pi^{S_1}) = \{\ \{s, p\},\ \ \{s, q\}\}$

  Two minimal models

  $S_1 \in MinMod(\Pi^{S_1})$: is stable model

- For $S_2 = \{s, p\}$, the residual program is:

  Π^{S_2}: $s \leftarrow$

  $MinMod(\Pi^{S_2}) = \{\ \{s\}\ \}$

  $S_2 \notin MinMod(\Pi^{S_2})$: not a stable model

- As before, every stable model of Π is a minimal H-model of Π
- A positive disjunctive program, i.e. without negation, may have several minimal models

  They are also its stable models

- Due to minimality, and unless forced by other rules, a disjunctive rule will pick only one of the disjuncts from the head to make it true

- Example: (3-GC revisited) A disjunctive general program:

$$color(x, green) \lor color(x, blue) \lor color(x, red) \leftarrow country(x)$$
$$\leftarrow edge(x, y), color(x, z), color(y, z)$$

  Unstratified negation is hidden in the program constraint

  The disjunctive head assigns to each country a single color

  This program is equivalent to the one on page 23: they have the same SMs

- Only some disjunctive programs with negation can be rewritten as NPs, i.e. without disjunction
- There is a syntactic class of disjunctive programs, that of the head-cycle free programs, that can be rewritten into equivalent non-disjunctive programs

  By passing, in turns, the (positive) atoms in the head as negative literals in the body

  This is the case of the program for 3-GC above (c.f. page 23)
- Disjunctive logic programs with stable model semantics are very expressive

  More than (non-disjunctive) NPs with stable model semantics

  (under common complexity-theoretic assumptions; more coming ...)

  So, some disjunctive programs cannot be expressed as equivalent NPs
- Disjunctive programs can be used to solve harder combinatorial problems

Complexity Considerations

- For a fixed intentional program Π (i.e. w/o EDB), and an input EDB $\mathcal{D}$ (i.e. data complexity), there are several decision problems:
  1. Model Checking: Given a set of ground atoms $\mathcal{M}$, is it a stable model (of $\Pi \cup \mathcal{D}$)?
  2. Consistency: Does the program have a stable model?
  3. Is a given ground atom a skeptical consequence?
  4. Is a given ground atom a brave consequence?
- They have a high time-complexity; at least *NP*-hard for the last three in the size of $\mathcal{D}$ (more below)

  For NPs, model checking can be done in PTIME
- The "functional problem" of computing a stable model is also hard

- Another difficult problem is Model Counting: How many stable models?

  Model Counting is crucial in many areas of Computer Science

- We recall several complexity measures: program complexity, query complexity, data complexity, combined complexity (combinations of the previous ones)

- They count the worst-case number of steps to reach the decision in terms of: $|\Pi|$ (leaving, $D, \mathcal{Q}$ as fixed parameters), $|\mathcal{Q}|$, $|D|$, $|\Pi \cup D|$, etc.

- In data management, the most relevant measure is data complexity

  In terms of the size of the underlying EDB, which can be large, while the "intentional part" of the program, short

- For example, the proper formulation of 3. above as a "data problem": $SQA(\Pi, A) := \{D \mid \Pi \cup D \models_{\mathsf{skp}} A\}$

  As a parameterized family of decision problems

  The parameters are the intentional program Π and atom A

  The decision problem is, for an instance D, whether it belongs to $SQA(\Pi, A)$ (do the same with the other problems!)

- Decision problems above, in data complexity, can be placed in the polynomial hierarchy:

  1. Model checking is coNP-complete

     More precisely, there are $\Pi, \mathcal{Q}$ for which $SQA(\Pi, \mathcal{Q})$ is coNP-complete

     For normal programs, i.e. no disjunction, it is always in PTIME

  2. Certain QA becomes Π_2^P-complete

     For normal programs: coNP-complete

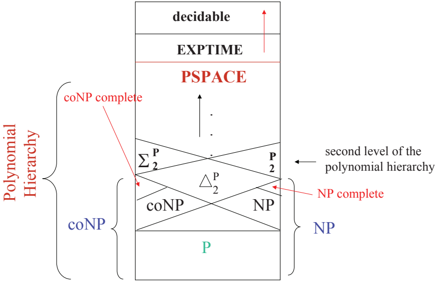
  3. Brave QA becomes: Σ_2^P-complete

     For normal programs: NP-complete

C.f. Dantsin, Eiter, Gottlob, Voronkov. "Complexity and Expressive Power of Logic Programming". ACM Computing Surveys, 2001, 33(3): 374-425

$$
\begin{aligned}
\Pi_0^P := \Sigma_0^P &:= \Delta_0^P := P \\
\Delta_{i+1}^P &:= P^{\Sigma_i^P} \\
\Sigma_{i+1}^P &:= (NP)^{\Sigma_i^P} \\
\Pi_{i+1}^P &:= (coNP)^{\Sigma_i^P}
\end{aligned}
$$

$$
\begin{aligned}
\text{E.g. } \Sigma_1^P &= NP^P = NP \\
\Pi_1^P &= coNP^P = coNP \\
\Sigma_2^P &= NP^{NP} \\
\Pi_2^P &= coNP^{NP} \\
&\ldots
\end{aligned}
$$



- *coNP* is the class of decision problems whose complements are solvable in non-deterministic PTIME
- $(coNP)^{NP}$ is the class of decision problems whose complements can be solved in non-deterministic PTIME with calls to an oracle that solves (in one step) problems in *NP* ETC.

Why "Answer Sets"?

- As opposed to "stable models"
- ASPs may have also "classical negation" in heads and bodies (in addition to weak negation in bodies)

$$\neg A(x) \vee C(y) \leftarrow B(x,y), \neg S(y), \mathit{not}\ P(x),\ \mathit{not}\ \neg M(x,y)$$

- They describe, and have as models, worlds that are only partially represented as *sets of classical literals*
- For example, $\mathcal{M} = \{P(a), P(b), \neg P(c), Q(d), \neg Q(a)\}$ is an incomplete representation of a world
  - $P(a), P(b), Q(d)$ are true
  - $P(c), Q(a)$ are false
  - $P(d), Q(b), Q(c)$ are uncertain

  No application of the CWA
- ASPs have "answer sets" as models, which may be partial worlds

- We expect worlds and models to be "consistent"

  We cannot find something like this in them: $\{\ldots, A, \neg A, \ldots\}$

- ASPs extend disjunctive programs with only weak negation and SMs

- In ASPs, classical and weak negation may coexist

- CWA related to weak negation

  If we want, we may impose (specify) the CWA:

  $\neg A \leftarrow \mathit{not}\ A$ (with A an atom)

  Making false what is not known to be true

- ASPs are also called "extended logic programs"