Penny-pinched: Modeling Aging and Unemployment

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ABSTRACT

This paper investigates the influence of aging on the labor market, specifically focusing on its impact on unemployment and the prices of goods. The analysis is grounded in empirical data from different countries, which reveal a noteworthy trend: retired individuals not only exhibit reduced consumption expenditures compared to working individuals, but also tend to purchase the same goods at lower prices.

KEYWORDS

Agent-based Modeling, Labor Markets, Goods Market, Unemployment, Aging, search intensity

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1 INTRODUCTION

The start of the 21st century saw aging become a serious issue in many countries across the world. In the two decades since 2000, the proportion of the population over 60 years old increased from 16.58 percent to 25.31 percent in Canada. To understand the impact of these trends, we must answer several questions: Is population aging good for economic well-being? What impact will aging have on per capita consumption and income? An equivalent question would be: what impact does aging have on productivity growth, or technological progress? The main work of this paper is to consider the impact of demographic changes on macroeconomic variables, especially the labor market.

A key measurement that characterizes the population's age distribution of a country is its **old-aged dependency ratio**. This is the ratio of the number of retirees to the number of working individuals. A low old-aged dependency ratio means there are many workers in the economy for each retiree; While a high ratio indicates possible economic troubles caused by few workers supporting each retiree. Japan, one of most aged countries in the world, for example, has an old-aged dependency ratio of 0.51 in 2022[1], which means a group of 100 working individuals needs to support 51 retirees who are no longer working. On the other hand, India, a major economy with a younger population, has a dependency ratio of 0.10.

In this paper, we introduce an agent-based model of the economics of an aging population. This paper is divided into an empirical simulation and a comparison of its results with real world data. Our main innovation is the introduction of search intensity into the analysis of unemployment; that is, shoppers in different ages show different time use for even identical goods. The concept can be defined by a value that represents the likelihood of a shopper deciding to conduct further searches after an initial encounter with a seller. For example, a search intensity value of 0.9 indicates that there is a 90% probability that a shopper will engage in a second search for goods after meeting with a seller. A higher value signifies that a shopper is likely to spend more time searching, reflecting a greater degree of search intensity in their purchasing behavior. Our findings provide a new perspective for macroeconomic analysis from an aging perspective. The introduction of unemployment is first and foremost because employment is also one of the core concerns of policymakers in countries with aging populations. Our contribution is to propose a novel agent-based framework to analyze the transfer mechanism between the two.

2 RELATED WORK

Most literature on aging focuses on the implications of demographic trends for social policy, particularly pension balance issues [6, 9, 25, 26]. However, demographic changes will also trigger far-reaching structural changes at the macroeconomic level, which are further reflected in economic activities, such as labor markets, goods and services markets, and so on. Literature on aging of the labor market is also limited, with most of them focusing on how changes in the age structure of the labor force affect the unemployment rate or labor force participation rate of young or old people[3–5, 13]. Instead, we intend here to examine the effect that an aging workforce has on unemployment within the working age population.

Our approach is motivated by Burdett and Judd [8]. They recognize that searching the market for lower prices is time consuming, a limited resource for a worker, but one that is more available for the unemployed or retired. As a result, these segments of the population exhibit increase price sensitivity and are generally better able to purchase goods at lower prices. This is evident when examining data from the Survey of Time Use of Americans (ATUS) data from 2003 to 2016, we find that older adults spend significantly more time shopping. Figure 2 shows the trend of shopping time for different age groups. Krueger and Mueller [15] show that unemployed people spend more time shopping and pay less for their goods than their employed counterparts. Kaplan and Menzio [14] makes use of this assumption in their shopping behavior model.

[1]Data source: https://wdi.worldbank.org/table/2.1

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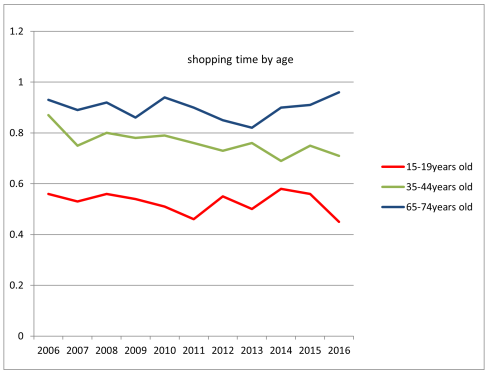

Figure 1: US Time Spent Shopping by Age

2.1 Shopping Behaviors by Age

When establishing a micro-foundation and putting the theory of population aging and unemployment into a unified analytical framework, many literature ignore the fact that the retirees have more time, they have more time to shop around, and they can find products of the same quality but at a lower price, or products of the same price but with higher quality. In other words, the retirees have stronger search intensity and higher bargaining power with manufacturers/sellers. Aguiar and Hurst [1] used the data of the American "AC Nielsen's Homescan Survey" (ACNielsen's Homescan Survey) to find that for the same goods, the average price paid by households whose head is in their 40s is more than 4 percent higher than that of households over 65. The retirees no longer need to participate in the labor market, so they start to have a lot of leisure time, and the cost of time is much lower than before retirement. Reflected in the goods market, they are more willing to use low-cost time to replace prices, and tend to spend more time looking for lower-priced goods. Despite the increasing size of the aging population, research on its consumption behavior has not received due attention. At present, most of the literature is launched from the perspective of marketing to the retirees. Swinyard and Rinne [23] analyzed the reasons why the American "Baby Boomers" prefer to shop in discount stores and the heterogeneity among older consumers through sample questionnaires. Moschis et al. [16] described corporate stereotypes of older consumers: poorer health and dissociated from the general public; Several papers [17, 24] explored the conservative and penny-pinching spending habits of the aged shoppers. Hawes and Lumpkin [12] studied the market behavior of elderly consumers and investigated the impact of retirement on the consumption behavior of the retirees. The findings suggest that age, not retirement, is a key factor in older adults' shopping behavior. Differences between older and younger consumers are developed and suggestions for further research are provided.

2.2 Agent-based Modeling

Agent-Based Modeling (ABM) has emerged as a powerful tool in understanding the complex dynamics of economic systems, particularly in the context of aging populations and their impacts on the goods and labor markets. Studies using ABM have made significant contributions to understanding how aging impacts markets. This highlights the versatility and depth of insights these models offer. Windrum et al. [27] provided an early foundation for using ABM in economic simulations, highlighting its potential to capture individual behaviors and market dynamics in ways traditional models cannot. Their work emphasized ABM's capacity to integrate heterogeneous agents and complex interactions, setting a precedent for subsequent studies on aging and economic dynamics. Silverman et al. [22] explored how aging populations affect labor market dynamics through ABM. They modeled the workforce's aging process and examined the implications for productivity, unemployment, and job matching. Their findings suggested that an aging workforce could lead to increased skill mismatches and longer job search times, illuminating the nuanced ways in which demographic shifts impact labor markets. On the goods market side, Axtell [2] applied ABM to understand how aging populations influence consumption patterns and price dynamics. Their model incorporated consumer heterogeneity, including age-based preferences and income levels, to demonstrate how shifts in the age distribution could alter demand patterns, affecting price adjustments and market equilibrium.

3 METHODOLOGY

Our model simulates the interaction between working and retired force consumers, and goods providers. The simulation proceeds in steps, each representing one year. Agents in the labor force may transition between employment and unemployment within each step. Additionally, after each step, agents age by one year. The model was created by Python, version 3.11. The model is comprised of three components: agents representing persons with ages from 0 to 100, firms, and the goods market. We opted not to include a labor market because the core attributes of a labor market are effectively represented by the latter two classes: namely firms and the goods market in a lump-sum way based on the assumption of constant wage (see Section 3.2).

3.1 Economic Environment

There are two markets in the economy (the labor market and the goods market) and three classes of risk-neutral individuals: economically active workers (either employed or unemployed individuals aged 15 to 64), children under the age of 15, and retirees over the age of 65. Both retiree and children populations do not participate in the labor market. Firms in the market play dual roles: as employers offering job vacancies to the labor force, and as sellers of goods to consumers in firm-worker pairs. All firms produce identical goods but each sets its own prices for those goods. Similarly, workers serve dual functions: they seek employment in the labor market, leading to job matches; and they purchase goods in the goods market after finding and matching with goods. It's important to note, however, that the retiree population exclusively engages in the goods market to purchase goods from firms.

Time is discrete and lasts 300 rounds. Each round has two stages. In the first stage, firms post vacancies and workers search for jobs. In the second stage, the labor market realizes the matching of firms and workers, the workers start to produce and consume. The output of a worker-firm pair will be allocated among the firm and the worker based on the wage determining protocol (see Section 4.3).

The model incorporates a set of parameters that govern agent behaviors and market interactions. The first set of parameters specifies **search intensity**, which dictate how much effort a group of individuals spends in searching the market for the lowest price of a good. Each agent's search intensity for goods varies based on their employment status and age. Retirees tend to search more frequently, employed workers search less, and unemployed workers' search intensity lies between the two[2]. The second set of parameters controls the pricing of goods, which relate to firms' initial pricing decisions and their price adjustments based on market conditions. The third set are labor market parameters that influence firms' hiring and firing decisions, job vacancy costs, job-separation rate and job-finding rate. The last group of parameters mainly govern the transition between population cohorts based on their ages including birth rate and mortality rate. The sources for these parameters come from empirical data, relevant literature, and calibration processes that ensure the model's behavior aligns with observed real-world phenomena. Pseudo-code outlining this framework in provided in the Appendix as Algorithm 1.

The labor market is implemented in a similar way to [20]: labor is traded in a frictional decentralized market. Here friction refers to the various factors that prevent the labor market from clearing; that is, they hinder the efficient matching of job seekers with job vacancies. These frictions can result from information asymmetry between employers and job seekers, geographical mobility issues, and skills mismatch — all of which contribute to the inefficiency of the job matching process. For firms and workers, these inefficiencies translate to additional costs for job creation and job search, respectively.

We assume that the search and matching process in the labor and goods market operate as completely separate processes.

3.1.1 Job Matching in Labor Market. Firms have various job openings. Some positions are filled and production is underway, while others remain open and waiting for the right candidate. We focus on these open positions and the unemployed individuals actively looking for work. Think of it as a matchmaking process where both the job vacancies and unemployed workers are looking for a suitable match. Once they find each other, they can start working together, contributing to the production process. However, unemployment persists due to three main reasons: some jobs are lost due to changes in technology or demand, some workers haven't found the right job yet, and young individuals are entering the job market for the first time. The process of matching jobs and workers depends on the number of available jobs and the probability of a worker finding a job, which in turn influences the labor market's dynamics, including how easy or hard it is for workers to find jobs.

3.1.2 Goods Matching in Goods Market. In the goods market, firms sell goods that they produce. Even though all firms might be selling the same type of product, consumers (which include workers of all employment statuses and retirees) have to search for the products they want to buy. Different firms may offer similar products for different prices, and this influences the consumers' decision purchasing decisions. The search intensity for products varies by the consumer's age and whether they are employed, unemployed, or retired, with retirees often searching most diligently compared to others. This searching and matching process in the goods market is similar to how job vacancies and unemployed workers find each other in the labor market, with the goal of making successful trades.

[2]See Section 2

3.2 Economic Agents

At the core of our model lies the representation of distinct population cohorts, each characterized by age groups and corresponding roles in the economy.

Children Population (Aged 0 to 14): This cohort represents the youngest members of society who are not yet part of the labor force. For simplicity, any individual aged 15 years have a chance to give birth new individuals within the children population. Children do not participate in the work force and do not consume goods. Instead, they age naturally and transition into the working population. This dynamic are crucial for understanding future labor market dynamics as this population defines the shape of the working population in the future.

Working Population (Aged 15 to 64): This cohort encompasses individuals within the prime working-age range. Agents in this group can transition between employed and unemployed states, reflecting labor market fluidity. Modeling this cohort allows us to examine the flows of individuals between employment and unemployment states, shedding light on labor market efficiency. Moreover, the working population also serves as buyers in goods market with their own search intensity.

Retirees: As individuals age beyond the working-age range, they transition into the retiree cohort. Retirees serve as buyers in the goods market. Their higher search intensity means they will spend more effort in securing lower prices for their goods. This is crucial driver of the dynamics in our model.

Firms (Seller Agents): Firms are central players in the goods market, serving as the entities in charge of making and offering products, and providing job opportunities. Firms first start by setting initial selling prices for their products. In labor market, they decide how many workers to hire or fire based on factors such as job vacancy costs and labor market conditions. Firms adjust goods prices according to the average price in the goods market, the number of job vacancies, and the costs of opening new job vacancies.

3.3 Model Parameters

To start, the model begins with a total population of 1000 individuals, consisting of various age groups. The working-age labor force, ranging from 15 to 64 years old, comprises 700 individuals, among whom 35 are unemployed, while 665 are employed. Retirees, aged 65 and over, make up 100 of the initial population. We use search intensities of 0.9, 0.1 and 0.01 for shoppers of retirees, unemployed and employed respectively based on the work of [15] and [14]

Our initial assumptions dictate that only individuals aged 15 years and older can participate in the population's reproduction process, and individuals only experience mortality once they reach retirement age. We have set constant birth rate at 0.011 of the working population[3], and mortality rates at 0.0366 of the retiree

[3]Based on "CIA World Factbook" (https://www.cia.gov/the-world-factbook/countries/world/), the average global birth rate was 0.0181 in 2021

population[4]. This means that at the end of the first year, the model predicts the birth of 8 newborns and the survival of 77 retirees.

Additionally, at the outset, there are 10 firms in the model. These firms have randomly assigned posted prices and job vacancy costs, with values ranging from 0.5 to 1 and 0 to 0.5, respectively. Furthermore, the firms open a random number of job vacancies, varying between 1 and 5. This model is designed to simulate a 300-year duration, allowing us to observe the long-term dynamics and interactions within the system.

3.4 Model Dynamics

3.4.1 Dynamics of the Firms. Firms are characterized by three key variables: the selling price of goods, job vacancies, and the cost of opening a job vacancy (which will be fixed as constant after randomly generated at initialization). The price of goods is set in a competitive market context, according to demand and supply (see Equation 1). Here supply is the number of aggregate available goods in the market produced by all firms. Demand comes from the aggregate purchase power of agents aged 15 years old and over, with their searching intensities taken in account.

$$Price = \frac{Vacancy\_Cost + Aggregate\_Demand}{Num\_Of\_Firms} - \frac{Aggregate\_Supply}{2500} \tag{1}$$

Each firm considers the interplay of aggregate supply and demand in goods market when updating its price every year. If the updated price exceeds the cost of opening a job vacancy, the firm will create new job vacancies. These job openings start out unfilled but can become occupied by new hires, drawn from the unemployed workers. The measure of hired workers in the process is a choice influenced by the job finding rate parameter in the labor market:

$$\left\lfloor \frac{Num\_Of\_Employed \times Job\_Separation\_Rate}{Num\_Of\_Firms} \right\rfloor$$

Conversely, if market conditions turn unfavorable; that is, the selling price falls behind the cost of opening a job vacancy, firms will reduce their workforce by laying off employed workers, causing the worker to become unemployed. The number of workers laid off by a specific firm is moderated by the constant job separation rate parameter in the labor market. This rate represents the likelihood of a firm-worker pair separating or an employed worker losing her job. In essence, firms are central to both the labor and goods markets, making critical decisions that affect employment, production, and pricing dynamics.

3.4.2 Dynamics of the Workers. We define the labor participation rate as the percentage of people between the ages of 15 and 64 who are employed or actively seeking employment. For simplicity, we assume this is 100%; though, the results in the model remains similar when setting the labor participation rate to 62.8%, the rate in August 2023 in US[5] or any other reasonable value.

[4]Base on data from Government of Canada (https://www.osfi-bsif.gc.ca/en/oca/oca-factsheets-other-reports/old-age-security-oas-program-mortality-experience-fact-sheet-april-2022), the mortality rates are 0.0366 and 0.0380 in 2019 and 2020 respectively, far lower than the value we randomly set here

[5]Data source: https://www.bls.gov/charts/employment-situation/civilian-labor-force-participation rate.htm

Actively searching unemployed workers apply to job openings. When a match occurs with a firm, forming a firm-worker pair to produce goods, the worker transitions from the unemployed to the employed category. Then she will participate in the profit distribution along with the firm after selling the goods they have produced, but of course at the cost of giving up her well beings from unemployment, including leisure, home production and unemployment insurance. The wage paid by the firm to the employed worker is assumed to be a constant, still a part of the total net profit though. Conversely, employed workers may experience job loss due to various factors, such as shifts in the firm's cost-benefit structure or technological advancements in the economic environment. In this model, employment transitions are solely influenced by internal firm factors, with external labor market shocks excluded from consideration.

We assume that upon becoming unemployed, a typical worker will immediately start searching for and applying to new jobs with minimal delay. This assumption aligns with the actual situation, as our model operates on time steps on one year. For instance, in Canada as of 2023, a typical jobless worker is eligible to receive unemployment insurance for a maximum of 45 weeks if she loses her job through no fault of her own[6] This means that this working-age person will lose her government income support if she takes a year-long pause from job-hunting. Workers' behavior toward job-searching and shopping also varies depending on their employment status.

Finally, unemployed workers prioritize lower-priced goods in their consumption, and will expend more time searching for lower prices than an employed worker.

3.4.3 Dynamics of the Shoppers. Shoppers play a significant role in the goods market. They are characterized by their age and search intensity. While retirees no longer participate in the labor market as workers, they actively engage in the goods market as buyers. Retiree shoppers have a unique behavioral trait—they exhibit a higher frequency of searching for lower-priced goods.[7] This preference stems from the economic reality that retirees typically have smaller, fixed incomes (typically pensions), and, therefore, seek to maximize the value of their purchases with their relatively abundant leisure time. In contrast, the search intensity of working population shoppers differ based on their employment status, with unemployed workers often prioritizing relatively lower-priced goods given their plentiful time and limit budget compared to their employed counterparts.

3.4.4 Interactions between Agents. The mechanism of our goods market is modeled after real-life market dynamics. First, sellers (i.e. the firms) independently determine and post their respective goods selling prices. Subsequently, buyers engage in the market by searching for potential transactions. Notably, our model assumes that the goods available in the market are essentially identical across sellers, with the sole differentiating factor being the price. Rational buyers will search the market for potential transactions and their search intensities are determined by their employment status and age. This characteristic highlights the significance of price competitiveness in our goods market: buyers will search for the cheapest goods, and,

[6]see https://srv129.services.gc.ca/rbin/eng/rates_cur.aspx

[7]The evidence from [1] is detailed in Section 2.1.

to remain competitive, each firm must adjust its goods' prices to attract a broader customer base and maximize its profit. The market operates within a perfectly competitive environment, wherein buyers and sellers engage in transactions based on mutually agreeable terms. This interplay of buyers and sellers perpetuates the system's annual cycle, effectively mirroring the dynamics of a real-world goods market. The presence of both retirees and the working population as buyers provides additional depth to the goods market, where price trends are shaped by the interactions and preferences of the diverse buyer types.

During each simulation time step, agents update their age and employment status, search potential goods and act as buyers in goods market. Firms create or destroy job vacancies, hire or fire workers, and adjust their prices. Workers search for jobs and transition between employed and unemployed statuses. Buyer agents, both retirees and the working population, participate actively in the goods market, each with their unique shopping behaviors. These interactions and decisions lead to changes in economic variables such as employment rates and price trends. The simulation continues for a specific parameterized number of years.

Figure 2: Effects of old-aged dependency ratio

4 EMPIRICAL RESULTS

Figure 2 depicts the effects of the old-aged dependency ratio (on the x-axis) on several socioeconomic factors (on the y-axes of each subplot): the unemployment rate, labor market tightness (that is, the ratio of number of unfilled job vacancy to unemployed workers), profit, and price respectively. There is a clear upward trend between the old-aged dependency ratio and unemployment rate (top left in Figure 2), which can be confirmed by the downward curve of labor market tightness (top right in Figure 2). That is, lower market tightness means it is less likely for a job seeker to find a job, and therefore increases the unemployment rate. Bottom left curve explains why the unemployment rate goes up as the firms' net profit goes down. As the net profit declines, the firms have to separate the current job position, fire the employed workers and cut the existing job vacancies facing the drop of goods prices. The bottom right curve illustrate why the firms' profits drop: the selling prices of goods decrease.

The model shows a direct correlation between firms' net profit and the unemployment rate. As net profit declines, firms face the imperative to restructure their workforce, leading to the separation of current job positions, layoffs of employed workers, and a reduction in existing job vacancies. This cascade of labor market adjustments contributes to an increase in the unemployment rate, further amplifying this effect.

The simultaneous decrease in firms' profits and goods prices, as depicted in the model, reveals a complex relationship. The downward trajectory in goods prices may stem from firms' strategic choices to attract increasingly discerning buyers in the goods market. As firms lower their selling prices in response to changing consumer preferences or economic conditions, this adjustment impacts their profitability, leading to the observed decline in profits.

While the long term trend is straightforward, an obvious convexity emerges when examining the concept of labor market tightness. The model initially shows a dramatic surge in labor market tightness as the population ages, reflecting a sudden demand for labor that outpaces supply due to the increase of the retirees as consumers. However, this sharp increase is followed by a significant decline. Subsequently, the model reveals a small concave curve, indicative of a unique turning point, after which labor market tightness gradually increases once more. This phenomenon captures the intricate interactions and feedback loops among individual agents, resulting in a more dynamic pattern. The initial surge in labor market tightness can be attributed to the inherent mechanisms governing the behavior of firms and individuals. To explain this phenomenon further, we must delve into the operational principles of the agent-based model.

Labor market tightness is subject to the dynamic interplay of several factors. In the early stages of demographic evolution within the agent-based model, firms exhibit a tendency to open job vacancies at a higher rate than the rate of unemployment. This can be attributed to the dynamic nature of the goods market, which is intersected with labor market dynamics. As the population ages, firms begin to confront an increasing proportion of retirees in the goods market. As the old-aged dependency ratio increases, there's a noticeable rise in consumer numbers, which we call as **consumption effect**. This is because retirees, who only contribute to the demand in the goods market, become a larger portion of the population. Therefore, firms recognize that increasing job openings may be the optimal strategy. This strategic choice is influenced by the dominant role of increasing ratio of retirees.

However, in the second stage of demographic transition — i.e., the concave portion in the middle of the curve reflects a dynamic process of adjustment in job vacancy strategies by firms. As the retirees tend to exhibit more diligent shopping behavior for lower-priced goods, (we call it as **shopping effect**), their impact on firms' strategy of opening job vacancies may show in a nonlinear way, but their influence on goods price and firms' profits is straightforward. At the outset, firms may overestimate the differential effects of the two aforementioned effects brought by the ratio rise of the retirees. Subsequently, they engage in a gradual and cautious re-adjustment of their job vacancy rates to align with the changing demographic transition. This adjustment process reflects the intricate and adaptive nature of labor market dynamics within the agent-based model.

The analysis above can be verified from the evolution in the time series depicted in Fig 3, where the x-axis denotes the time steps over 300 years, and the y-axis measures the old-aged dependency ratio, number of unemployment, labor market tightness, the base-10 logarithm of firms' profit, and the natural logarithm of prices in goods market.

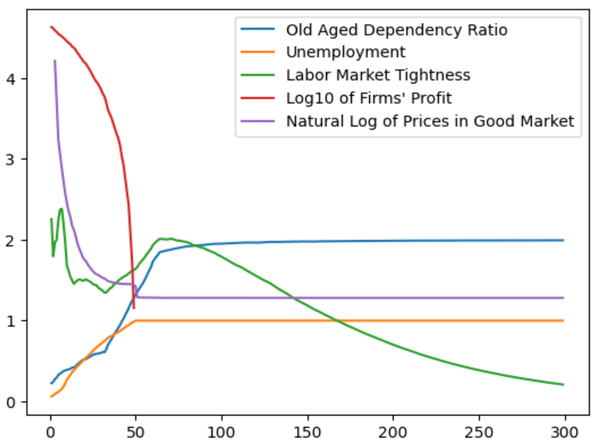

Figure 3: Time Series Results

The unique patterns observed in the model's labor market tightness curve can be attributed to the complex interplay of factors, including the influence of retirees on consumer behavior, the shifting balance between consumption and shopping impact brought by the retirees, and the dynamic job vacancy strategies employed by firms in response to these evolving dynamics.

In conclusion, the agent-based modeling approaches underscores the importance of considering the underlying assumptions and dynamics inherent to the modeling method. These findings encourage further research into the specific mechanisms that give rise to the dynamic patterns observed in the agent-based model, shedding light on the intricate nature of labor market responses to demographic shifts.

5 ANALYSIS ON REAL WORLD DATA

Several quantitative studies have explored the complex relationship between aging and unemployment, revealing that multiple factors can influence this dynamic. The findings from these studies are often inconsistent, lacking a consensus on the impact of aging on unemployment rates. One side believes that aging reduces the unemployment rate [10, 11, 18]. This view is completely contrary to the conclusions reached by [21] based on US data. In this section, we perform a regression analysis on cross-national data data from the United Nations World Populations Prospects 2022 to examine connections between aging and unemployment.

5.1 Statistical Analysis

Before we delve into the regression analysis, let us take a brief look at the population data from the United Nations World Population Prospect 2022 [19]. It comprises the specific age structure information from all members states in the UN over a period of time. The dataset is mainly applied to capture the population aging features across the world. Figure 4 reveals the aging levels globally from 1960 to 2022. As of 2022, the old-age dependency ratio exceeds 0.13 in all large geographic regions[8] around the world, except Africa, which is 0.0618. Based on this, we consider the relationship between the old-age dependency ratio and the unemployment rate from the entire data set.

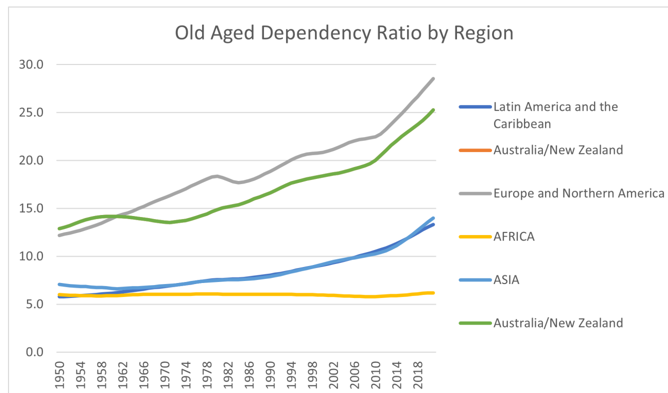

Figure 4: Old-Aged Dependency Ratio by Region

The regression equation is defined as:

$$\begin{aligned} unemp_{it} &= \beta_0 + \beta_1 \cdot oadr + \beta_2 \cdot oadr^2 + \beta_3 \cdot unemp\_lag + \beta_4 \cdot labor \\ &+ \beta_5 \cdot gdppc + \beta_6 \cdot ln\_gdppc + \beta_7 \cdot tax \\ &+ \beta_8 \cdot gdp\_deflator + \Sigma_{j=1,j\neq t}^{62} \gamma_j \cdot y_j + u_i + \varepsilon_{it} \end{aligned} \quad (2)$$

Here the subscript i, t refer to each country and time respectively. u_i is an unobserved time-invariant random variable, which represents the intercept term of individual heterogeneity; that is, individual characteristics that do not change over time, such as the physical geography of a country, core cultural values and traditions, and historical Legacy. The composite disturbance term $u_i + \varepsilon_{it}$, ε_{it} is the time-varying error term. Here the old-age dependency ratio **oadr** is the core explanatory variable, and other macroeconomic indicators such as GDP growth rate, deflator, labor participation rate, etc. are control variables. Descriptions of all variables are listed in Table 1.

5.1.1 Statistical Result Analysis. To first set a benchmark, we assume there is no unobservable country-specific effect; this implies all countries in the dataset are assumed to share the same regression equation. Consequently, u_i is assumed to be constant across countries in Equation 2 and there is no correlation within individual country groups. This means, for example, in country A, person 1 might have a high income while person 2 might have a low income,

[8]Based on the UN definition of "region" or "subregion"

Table 1: Variable Descriptions.

Variable Name	Economic Variable	Description
unemp	unemployment rate	# of unemployment / (# of unemployment + # of employment)
oadr	old-aged dependency ratio	# of population 65 years and over / # of population between 15 and 64 years old
labor	labor participation rate	(# of unemployment + # of employment) / # of population between 15 and 64 years old
gdppc	gross domestic product per capita	gross domestic product / population
ln_gdppc	GDP per capita annual growth	growth rate of gross domestic product / population
tax	tax rate on income, profits and capital gains	income, profits and capital gains / total revenues
gdp_deflator	deflator	nominal (or current-price) GDP to the real (or base year's prices) measure of GDP
unemp_lag	lag term of unemployment rate	
y_j	Year dummy variable	

Table 2: Regression Results[10]

	(1) Pooled OLS	(2) FE FGLS
oadr	.0402*	.0881*
	(.0218)	(.0508)
$oadr^2$	-.0011**	-.00259***
	(.00054)	(.00095)
unemp_lag	.9458***	.8505***
	(.0089)	(.0115)
labor	-.0049	.0130
	(.0046)	(.0129)
gdppc	.0000047***	-.000001
	(.0000017)	(.0000078)
ln_gdppc	-.1176***	-.1202***
	(.02362)	(.0087)
tax	-.0026	-.0197**
	(.0026)	(.0070)
*gdp_*deflator	-.0001	.0002
	(.0001)	(.0002)
y_{2017}	-.9886***	-.6089***
	(.3355)	(.2229)
y_{2020}	-.5544	-.2741
	(.3800)	(.2417)
constant	1.5800	.7350
	(.4046)	(1.0476)
		F Test: All $u_i = 0$: F(117,2061) = 2.36 P-value=0.000
σ_u		1.1684
σ_e		1.3216
$corr(u_i, X)$		0.6757
Observation Values	2218	2218
Number of Groups	118	118
R-Squared	0.9342	0.9304

but their incomes are not related to each other. The regression results are shown in column labelled (1) in Table 2. The effect of the old-age dependency ratio is found to be positive. Column (2)-(4) use the equation 2 with the existence of u assuming $\sigma_u \neq 0$. Specifically, column (2) represents the estimate in the presence of fixed effects (i.e. there exists correlations between the explanatory variables and the unobserved time-invariant country effect $E(x_{it}, u_i) \neq 0$). Columns (3) uses a random effects model to represent estimates conditional on the conditional expectation to zero[9].

It's good that the old-aged dependency ratio variable is positive for Pooled OLS. But more importantly, after Pooled OLS, we have to test for the presence of an unobserved effect u_i in the panel data, which reflects the heteroskedasticity across countries. If there is no such effect, then Pooled OLS is efficient and the associated statistics are asymptotically valid. Otherwise, Fixed or Random Effect models should be taken into consideration. There are 3 approaches to verify the existence of time-invariant unobserved effect u_i. First is the F test in column (2) of Table 2 with the Null Hypothesis that All $u_i = 0$. After estimating both models, we perform an F-test to evaluates whether the coefficients of the time-invariant variables are jointly zero. As the p-value of the F-test is 0.0000, it suggests that the Fixed Effects model is more appropriate than pooled OLS, indicating the presence of significant time-invariant omitted variables.

Second approach is the Least-squares Dummy Variable(LSDV) model, we essentially want to assess whether incorporating panel-specific effects (via dummy variables in the LSDV model) provides a better fit to the data compared to a simple pooled model. The LSDV approach is closely related to the fixed effects (FE) model and is a practical way to see the impact of omitting individual-specific effects, which takes robust standard error into account, that is, the assumption of uniform variance or homoscedasticity in a regression model is violated, this means the variance is not constant across time and countries. Here all the features have the same meaning as in 2 except y_j, which is the dummy variable taking binary values 0 and 1 as we take 63 years from 1960 to 2022 for each country. It turns out the coefficient of oadr from LSDV model is identical with that from fixed effect models which is shown in column 5 of Table 2 (Due to space limitations, only part of the country dummy variables are provided).

Breusch-Pagan Lagrange Multiplier (LM) test [7] is an alternative approach to check for the presence of heteroskedasticity, i.e., whether the variance of the errors from the regression model is

[9] See Appendix A for detail statistical explanation of the Pooled OLS, FE and RE FGLS.
[10] In all tables here, the values in parentheses are robust standard deviations, and ***, **, and * indicate significance at the 1%, 5%, and 10% confidence levels respectively.

	(3) RE FGLS	(4) LSDV
oadr	.0560**	.0881*
	(.0229)	(.0508)
oadr^2	-.0016**	-.00259***
	(.0006)	(.00095)
unemp_lag	.9230***	.8505***
	(.0078)	(.0115)
labor	-0.0037	.0130
	(.0034)	(.0129)
gdppc	-.0000059**	0.000001
	(.0000028)	(.0000078)
ln_gdppc	-.1208***	.1202***
	(.0085)	(.0087)
tax	-.0037	-.0197***
	(.0034)	(.0070)
*gdp*_deflator	-.0001	-.0002
	(.0002)	(.0002)
y_{2017}	-.6219***	-.6089***
	(.2206)	(.2229)
y_{2020}	-.2466	-.2741
	(.2426)	(.2417)
constant	1.4245	.7350
	(.4474)	(1.0476) F test of absorbed indicators: F(117,2061)=2.359 P-value=0.000
$corr(u_i, X)$	0	
Observation Values	2218	2218
Number of Groups	118	118
R Squared	0.9420	0.9420

Table 3: Breusch-Pagan Lagrange Test for Heteroskedasticity

Assumption: Normal error terms
Variables: All independent variables $H_0 : \sigma_u^2 = 0$ $\chi^2(01) = 1053.65$ Prob > $\chi^2 = 0.0000$

constant, which is statistically the same as $H_0 : \sigma_u^2 = 0$. We can see the Chi-square statistic, that is, the value indicating the test statistic of the Breusch-Pagan LM test, is 1053.65, and the P-value is 0.0000, less than 1% significance level, so we reject the Null Hypothesis of homoskedasticity (constant variance of errors), suggesting that our model's errors are heteroskedastic. The LM test result is given as in Table 3.

It turns out the P-value is 0.0000, which means the Null Hypothesis should be rejected. Therefore, there exists unobserved effect and the Random or Fixed Effect models are more appropriate than Pooled OLS. This can also be confirmed by the F test in column (2) of table 2 which tests whether there are omitted variables that do not change over time, with the Null Hypothesis of "H_0: All $u_i = 0$". The P-value of the F test is 0.0000, so we reject the Null Hypothesis, that is, the fixed effects model is superior than the Pooled regression. It can also be seen from table 3 that individual country dummy variable is significant for most of countries, which means there exist individual effects. Therefore, Pooled regression should not be used. From Random and Fixed Effect models, it's clear that the impact of old-aged dependency ratio on unemployment is positive and large, and the estimator is highly statistically significant.

In summary, after adjusting for the influence of time and other key macroeconomic variables traditionally considered to influence the unemployment rate, through the application of Pooled OLS, Fixed Effect, and Random Effect Feasible Generalized Least Squares, we discovered that the relationship between the unemployment rate (r_u in short) and the old-age dependency ratio (*oadr* in short) appears to in quadratic form. Specifically, The formula is expressed as $r_u = a \cdot oadr^2 + b \cdot oadr + c$, where $a < 0$ and $b > 0$. Given that the calculated axis of symmetry, determined by $-\frac{b}{2a}$, surpasses 40 in all four models — far exceeding the real-world old-age dependency ratios, such as Japan's 0.51 in 2022 — the portion of the curve of interest is the ascending segment to the left of the axis of symmetry. Furthermore, we employed three different methods to test the existence of heteroskedasticity across countries. These tests reveal that the outcomes derived from either the Fixed Effect or Random Effect models are more reliable than those obtained from the Pooled OLS ones. Finally, these results confirm a positive impact of the old-age dependency ratio on the unemployment rate in real-world data, which aligns with the findings from our agent-based model.

6 CONCLUSIONS

In conclusion, this paper sheds light on the dynamics between aging populations and labor markets, particularly highlighting the impact on goods prices adjustment. Through empirical observations and the development of an agent-based model, we unveil the dual effects of aging: while retirees contribute to increased net demand, fostering employment opportunities, their distinct price sensitivity poses challenges for manufacturers, leading to competitive pricing strategies. Our study shows that changes due to more older people can lead to businesses making less money and more people being out of work. This key finding points out how changes in the population and the economy are connected. It suggests we need to think again about how we approach market strategies and policies. This interplay between demand stimulation and price pressure underscores the complexity of the relationship between aging and key economic indicators, offering valuable insights for policymakers and stakeholders navigating the evolving landscape of labor markets in aging societies.

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A APPENDIX: ECONOMIC TERMINOLOGY EXPLANATIONS

A.1 Introduction to Panel Data Analysis Methods

The following discussion are mainly from the text of Wooldridge [28], with the exception of the detailed mathematical derivation.

Panel data, also known as longitudinal or cross-sectional time-series data, provide valuable insights into economic phenomena by capturing observations across multiple entities over time. In this Appendix, we introduce three common methods for analyzing panel data: pooled ordinary least squares (POLS), random effects, and fixed effects generalized least squares (GLS).

The Pooled OLS model employs the Ordinary Least Squares (OLS) approach to analyze panel data, and treats all countries and time periods equally, assuming constant coefficients. That is, under Equation 2, the parameter u_i is considered to be constant across countries, and there is no interdependence observed within individual countries. Therefore, we can rewrite Equation 2 in simple form as

$$y_{it} = \beta_0 + x_{it}\beta_1 + \varepsilon_{it}, t = 1, 2, ..., T, i = 1, 2, ..., N. \tag{3}$$

To consistently estimate the coefficient β, three assumptions are usually sufficient:

Assumption POLS.1: Exogeneity: The independent variable x_{it} is uncorrelated with the error term ε_{it}: $\mathbf{E}(\varepsilon_{it}|x_{it}) = 0, t = 1, 2, ..., T.$

Assumption POLS.2: Independence and Identically Distributed (i.i.d): Independent variables x_{it} are i.i.d across countries:

Assumption POLS.3: Homoscedasticity: The error term ε_{it} has constant variance across all countries and over time.

(a) $\mathbf{E}(\varepsilon_{it}^2|x_{it}) = \sigma^2, t = 1, 2, ..., T, i = 1, 2, ..., N;$

(b) $\mathbf{E}(\varepsilon_{it}\varepsilon_{is}|x_{it}, x_{is}) = 0, t \neq s, t, s = 1, ..., T, i = 1, 2, ..., N.$

Here **Assumption POLS.3** guarantees a homoskedasticity variance matrix. But based on the results of the LM test as well as F test we've done in Section 5, this assumption could not hold as there exists an unobserved effect across countries, which means the existence of either heteroskedasticity: $\mathbf{Var}(\varepsilon|X) = \Omega \neq \sigma^2 I_n$ or serial correlation: $Cov(\varepsilon_t \varepsilon_s|X_t, X_s) \neq 0)$. To ensure the estimators are efficient, we transform the model 3 by left multiplying $\mathbf{w}$ on both sides,

$$\mathbf{w}y_t = \mathbf{w}X_t\beta + \mathbf{w}\varepsilon_t$$

where $\mathbf{w}$ satisfies $\mathbf{w}^T\mathbf{w} = \Omega^{-1}$, a **Cholesky decomposition** of Ω^{-1}. An issue arises here as it's hard to know the matrix Ω, let alone determine its singularity and compute its inverse if it's nonsingular. So here is the trick of Feasible Generalized Least-square method.

(1) Pooled OLS is applied to the data, and then residuals ($\hat{\varepsilon_t}$) are computed.
(2) Variance matrix $\hat{\Omega} = var(\hat{\varepsilon_t})$ is estimated based on the Pooled model (Here explains the meaning of "feasible").
(3) The coefficient $\hat{\beta}_{FGLS}$ is estimated as $(X^T\hat{\Omega}^{-1}X)^{-1}(X^T\hat{\Omega}^{-1}y)$ along with its covariance matrix.
(4) This process can be iterated until $\hat{\beta}_{FGLS}$ converges.

One significant advantage of fixed or random effect FGLS models is their ability to account for all time-invariant unmeasured (or latent) variables that impact the dependent variable, that is, u_i in Equation 2, regardless of whether these factors are recognized

Algorithm 1 Simulation of Aging Impact on Labor Market and Goods Price

Input: Initial parameters are population size, birth rate, death rate, job finding rate, job separation rate, firm characteristics (price, vacancy, vacancy cost),and simulation duration.

1: Create initial population and firms:

1.1: Generate persons with attributes (age, employment status).

Initially, the population consists of 1,000 individuals, categorized into three distinct cohorts: a working labor force of 700 with a 10% unemployment rate, 100 seniors, and 200 individuals ranging from newborns to 14 years old. The birth and death rates are set at 0.011 and 0.23014, respectively.

1.2: Generate firms with attributes (price, vacancy, vacancy cost).

In both markets, there are 10 firms. Each is randomly assigned a selling price for its product ranging from 0.5 to 1, opens a random number of job positions between 1 and 5, and incurs a random job opening cost between 0 and 0.5.

2: for each year in simulation years do:

2.1: Age update: Increment the age of all persons.

2.2: Employment status update: Update employment status based on labor market dynamics. Employed workers risk being laid off if their firm's product sales falter, whereas firms experiencing robust sales are likely to hire unemployed workers. The rates for both job separation and job finding are established at 0.2.

2.3: Births and Deaths:

Individuals aged 15 years and older can give birth, with a birth rate of 0.011, while mortality is restricted to those aged 65 and above, at a death rate of 0.23014.

2.3.1: Calculate and simulate births based on birth rate.

2.3.2: Calculate and simulate deaths based on death rate.

2.4: Agents search in goods market and make deal with sellers.

Individuals aged 15 years and older participate in the goods market as buyers regardless of their age or employment status.

2.5: Firm dynamics:

2.5.1: for each firm do:

2.5.1.1: Make hiring or firing decisions.

Firms experiencing strong sales will increase job vacancies and hire additional workers, while those with lower sales will reduce job vacancies and lay off employees.

2.5.1.2: Update goods prices based on demand and supply.

2.6: Market interactions:

2.6.1: Calculate total demand in the goods market.

2.6.2: Update unemployment rates and labor market tightness.

2.6.3: Collect data for analysis: Store relevant data points for later analysis (e.g., unemployment rates, firm profits).

2.7: Repeat at Step 2 until year > 300.

3: Post-simulation analysis: analyze collected data and generate insights.

4: End of simulation

Table 4: Parameters of the Agent-based Model

Parameters	*Value*
birth rate	0.011
mortality rate	0.23014
initial population	1000
initial unemployment rate	0.05
job finding rate	0.2
job separation rate	0.2
wage	0.6
output per firm-worker pair	1
unemployment insurance	0.6
pension	0.4
search intensity of retirees	0.9
search intensity of unemployed	0.1
search intensity of employed	0.01

or not by the researcher. This is particularly valuable given the probable existence of these unaccounted variables. The Random Effects Model assumes that these unobserved, time-invariant variables do not have any correlation with the included time-varying variables([28]), that is, $\mathbf{cov}(x_{it}, u_i) = 0$ where x_{it} means all the explanatory variables in Equation 2 including oadr, labor, etc. On the other hand, the Fixed Effects Model (FEM) permits these variables to exhibit correlations.

A.2 Least-squares Dummy Variable (LSDV)

The Least-squares Dummy Variable model is actually the Ordinary Least-squares approach with addition of dummy variables. It has the advantage that we don't need to adjust the variance matrix, just like what we have done in Feasible Generalized Least-squares (FGLS). The result from LSDV estimation is equvalent to Fixed-effects within group FGLS, which you can see from column 3 of Table 2 and Table 3.