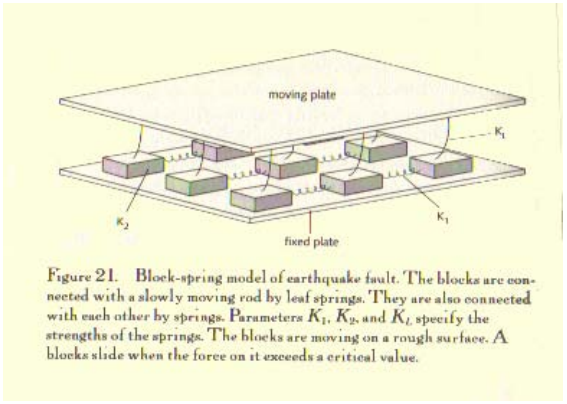
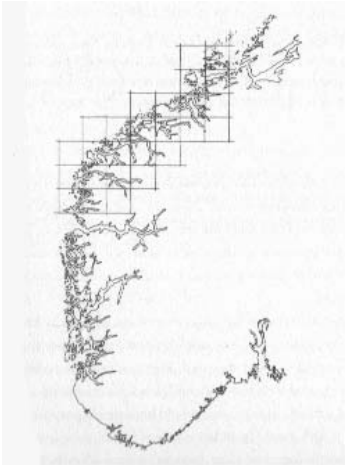
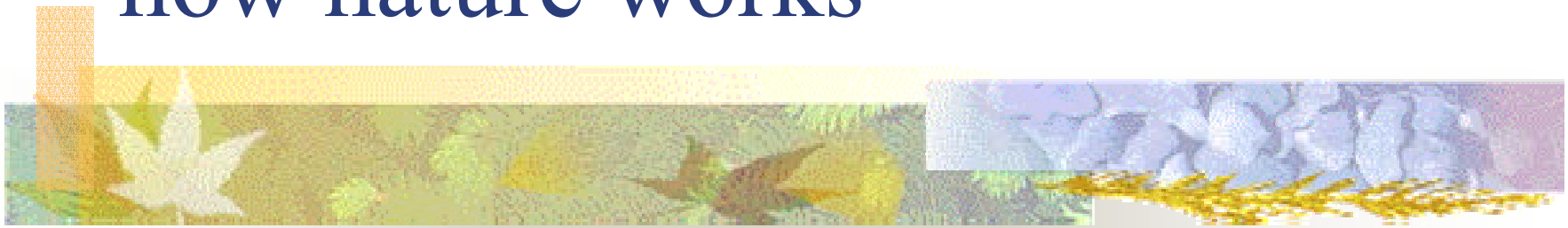


Figure 21. Block-spring model of earthquake fault. The blocks are connected with a slowly moving rod by leaf springs. They are also connected with each other by springs. Parameters K_1, K_2, and K_t specify the strengths of the springs. The blocks are moving on a rough surface. A blocks slide when the force on it exceeds a critical value.



how nature works



the science of self-organized criticality

Per Bak

Presented by Adam Murray

Today's Discussion

I) Introduction

- 1. Theory of self-organized criticality
  - Complexity and Criticality
  - Self-Organized Criticality
- 2. Examples reinforcing the theory
  - Sandpiles
  - Earthquakes
  - Game of Life
  - Economics

II) Observations & Conclusions

III) Questions / Comments

The nature of *how nature works*

- A personable author
- Digestible format
- Dual narratives
  - SOC theory and practice
  - The author's opinions and criticisms of modern science
- Our emphasis and interest is on SOC

Power Laws for Dummies

- A power law comes in the form:

$$N(s) = s^{-t}$$

- The author illustrates his results in the form of a log-log graph of the power rule shown above:

$$\text{Log } N(s) = -t \log s$$

(Straight lines with slope –t)

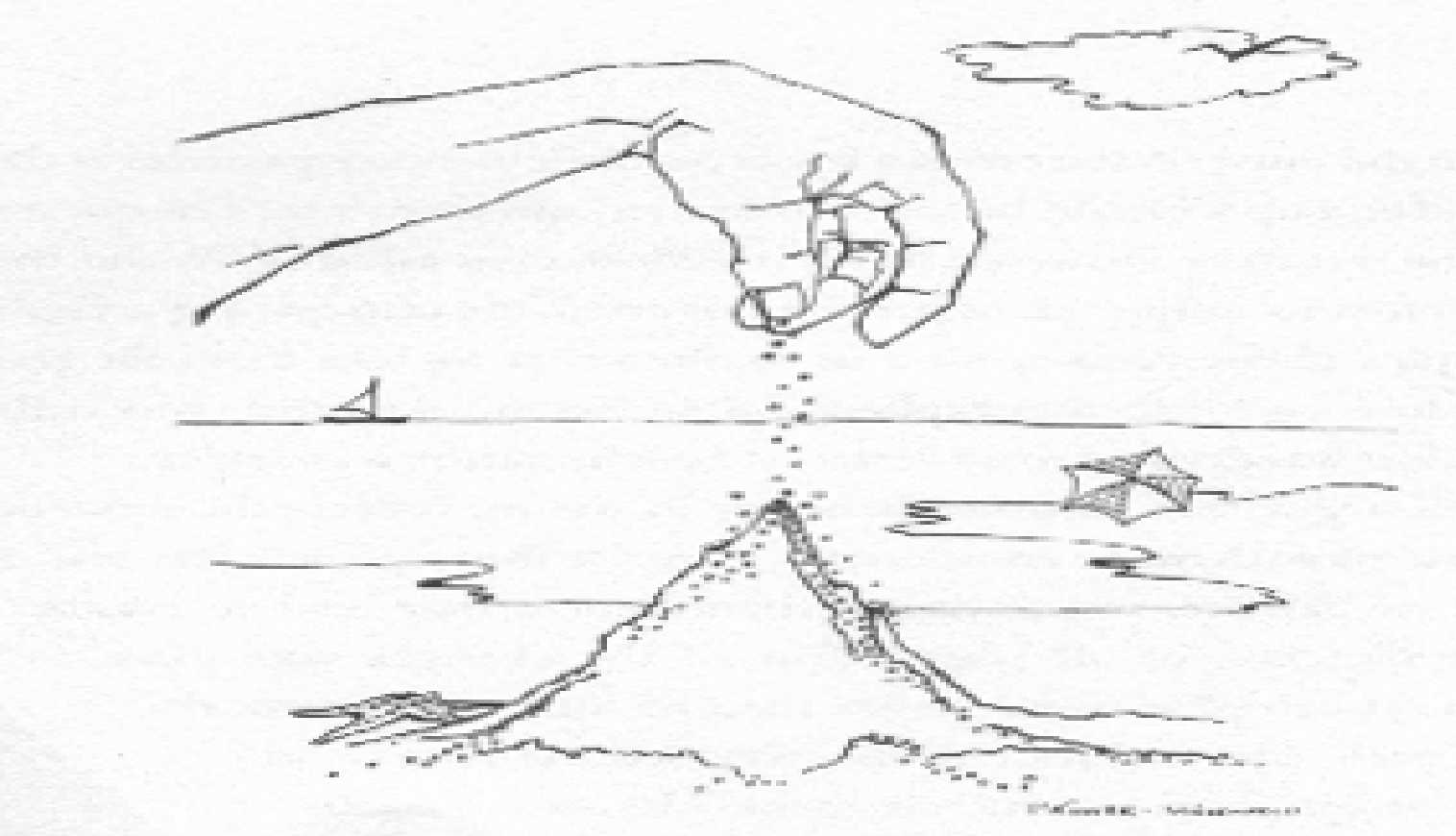
Physics is simple - Nature is complex

- Physics has simple laws, while nature is complex
- Complex behaviour in nature reflects the tendency of large systems with many components to evolve into a critical state

Self-organized and critical

- The out-of-balance critical state leads to avalanches of all sizes
- No outside help to get to critical state
- Most changes are a result of catastrophic events, not a smooth path
- Large catastrophic events occur for the same reasons small ones do

Criticality in a simple sandbox



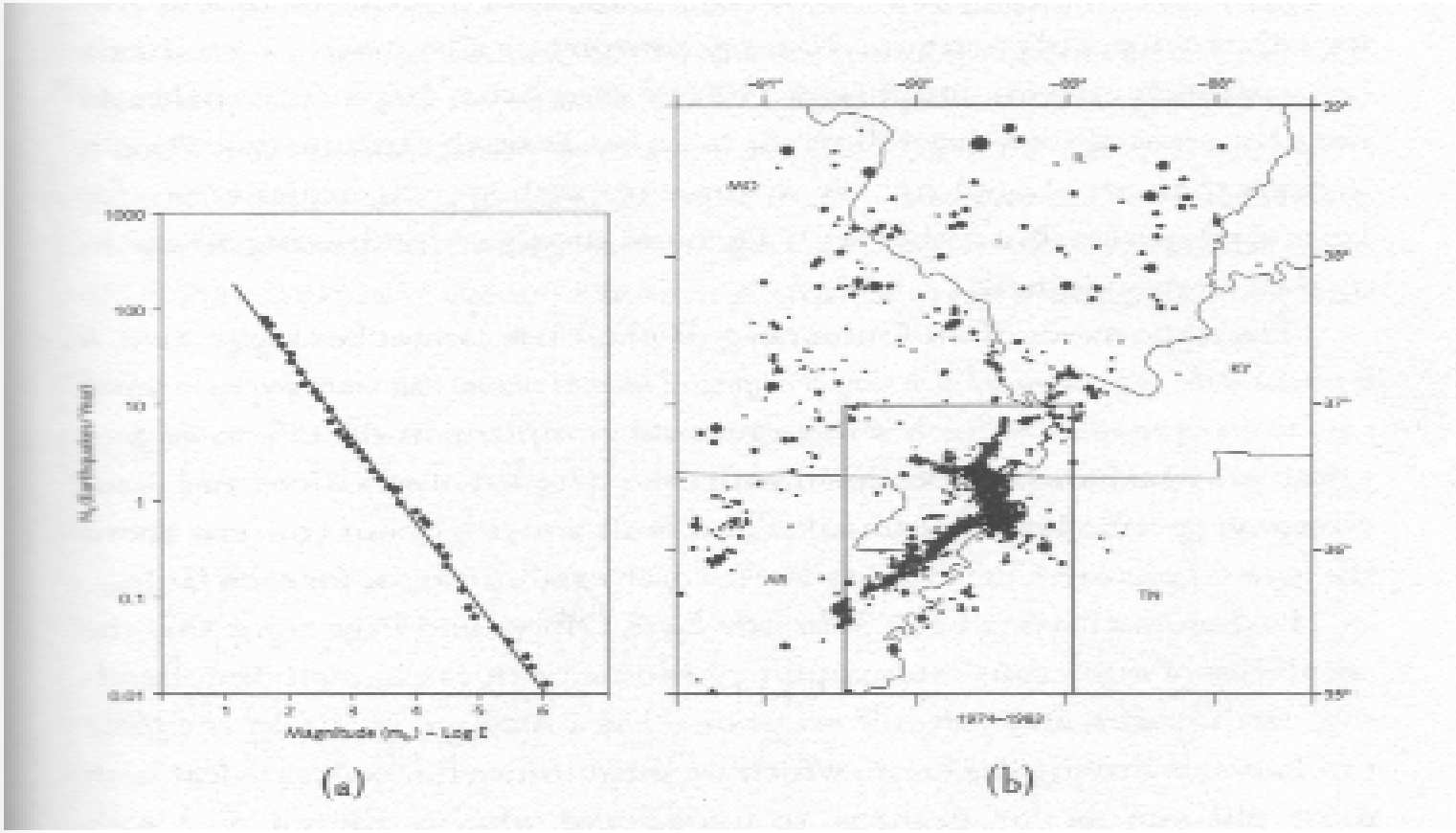
SOC walking a fine line

- Systems in equilibrium do not show signs of SOC
- Chaos theory cannot explain complexity
- Walking with careful balance between equilibrium and chaos
- Perhaps SOC is an underlying principle that can be the law for complex behaviour

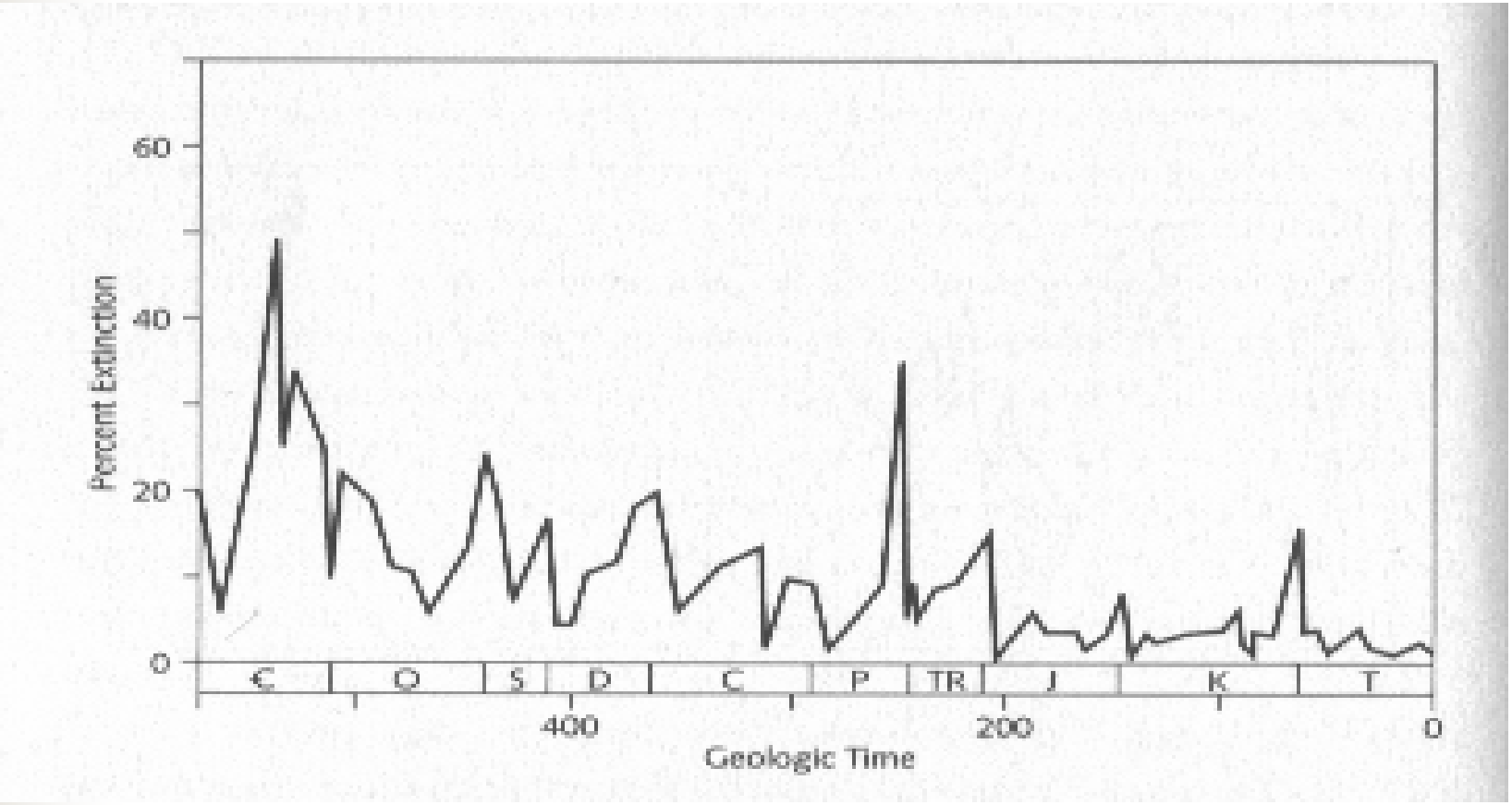
General Empirical Observations

- Gutenberg-Richter power laws
- Fractals
- 1/f signals
- Zipf's law

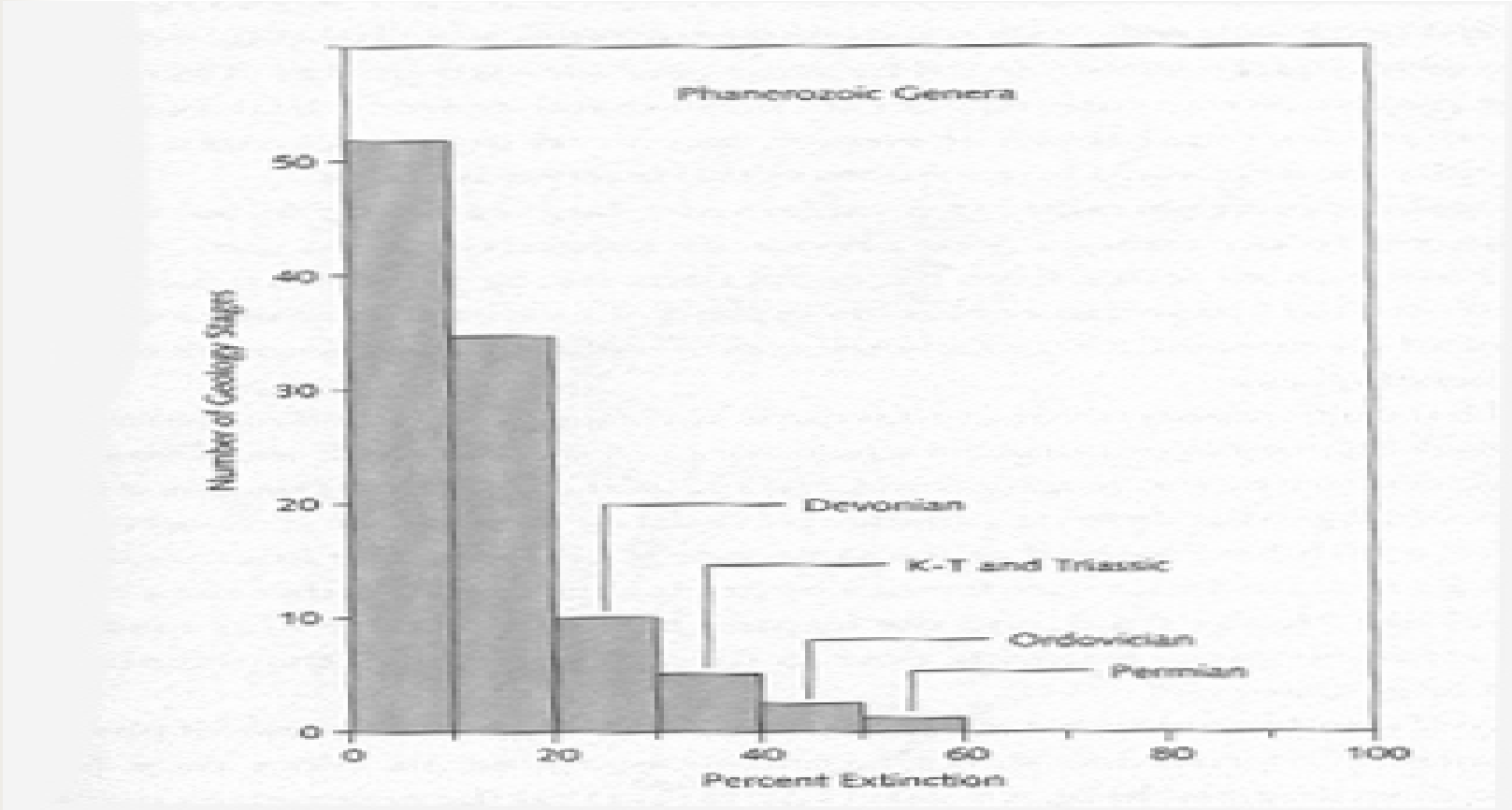
Regularity of earthquakes



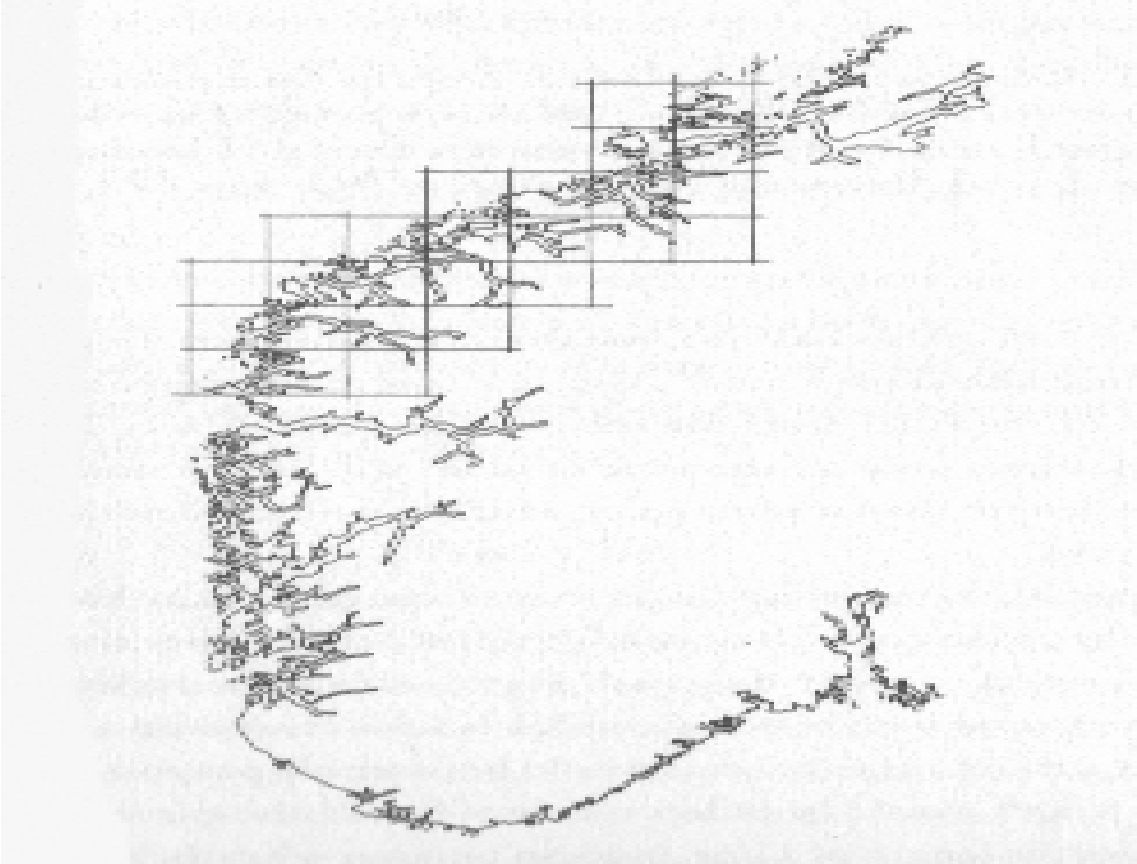
Regularity of Biological Extinctions



Biological Extinctions follow a Power Law



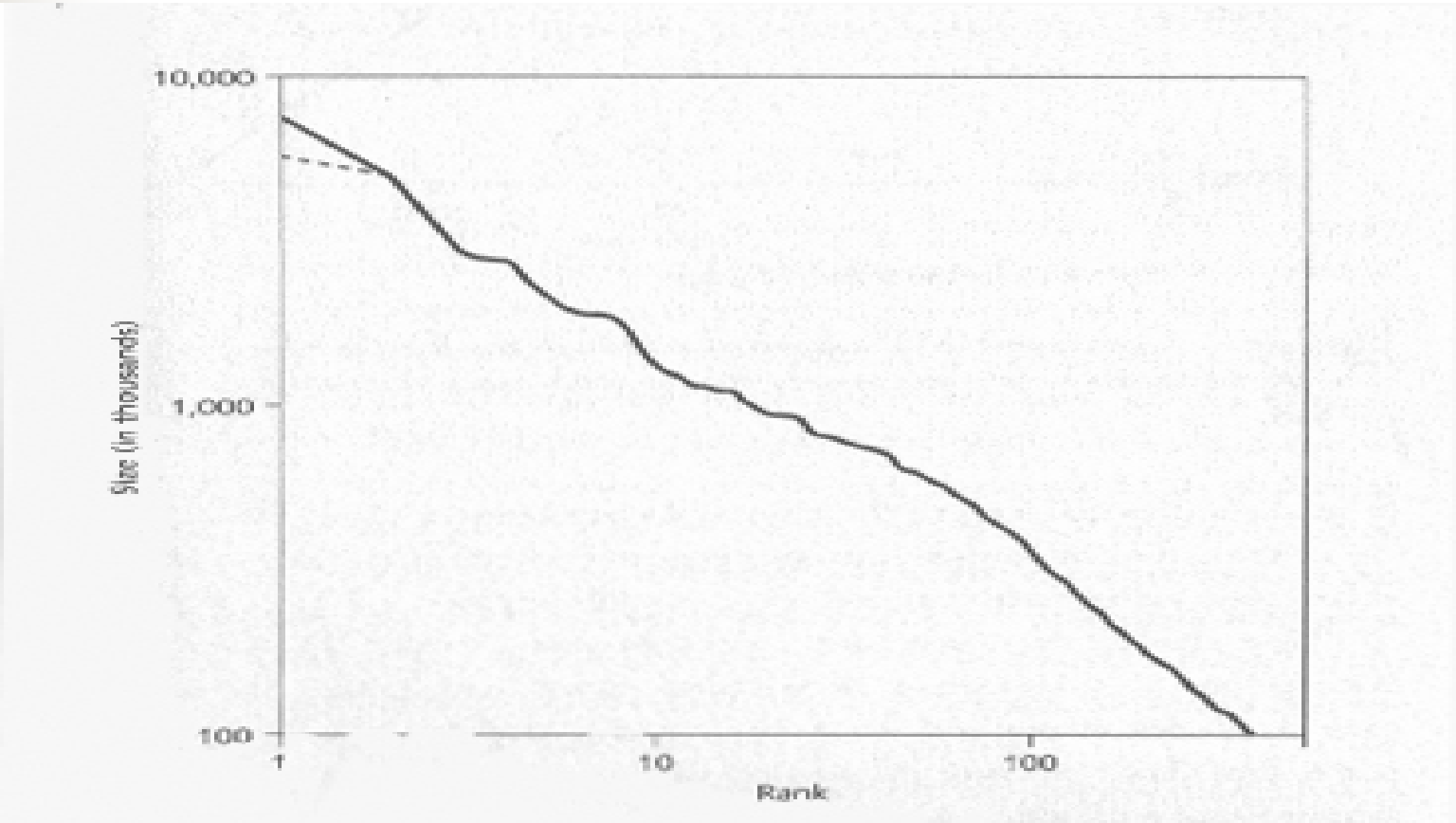
Fractals off the coast of Norway



Zipf's law

- Zipf made interesting observations regarding population distribution in cities, and word distribution in text
- He observed how many cities in the world had more than a given number of inhabitants

City Rankings in 1920



Observed Values: Zipf

- 10^{th} most frequently used word: 2653 times
- 20^{th}: 1311 times
- 20 000^{th}: only once

- logarithmic plot of rank vs. frequency is a straight line

Spherical cows and toy models

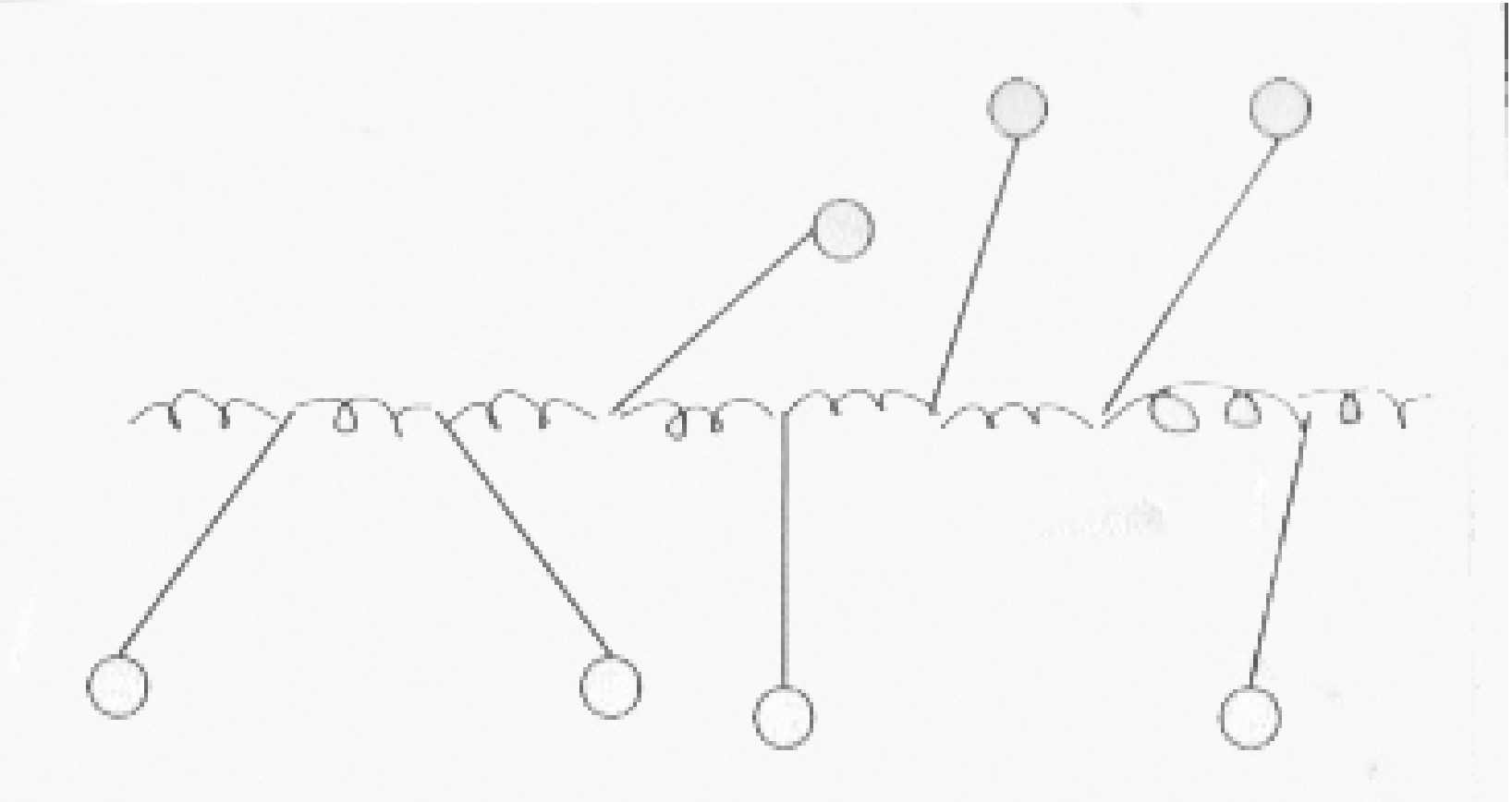
Reading a quotation

Reading a quotation from Mr. Per Bak

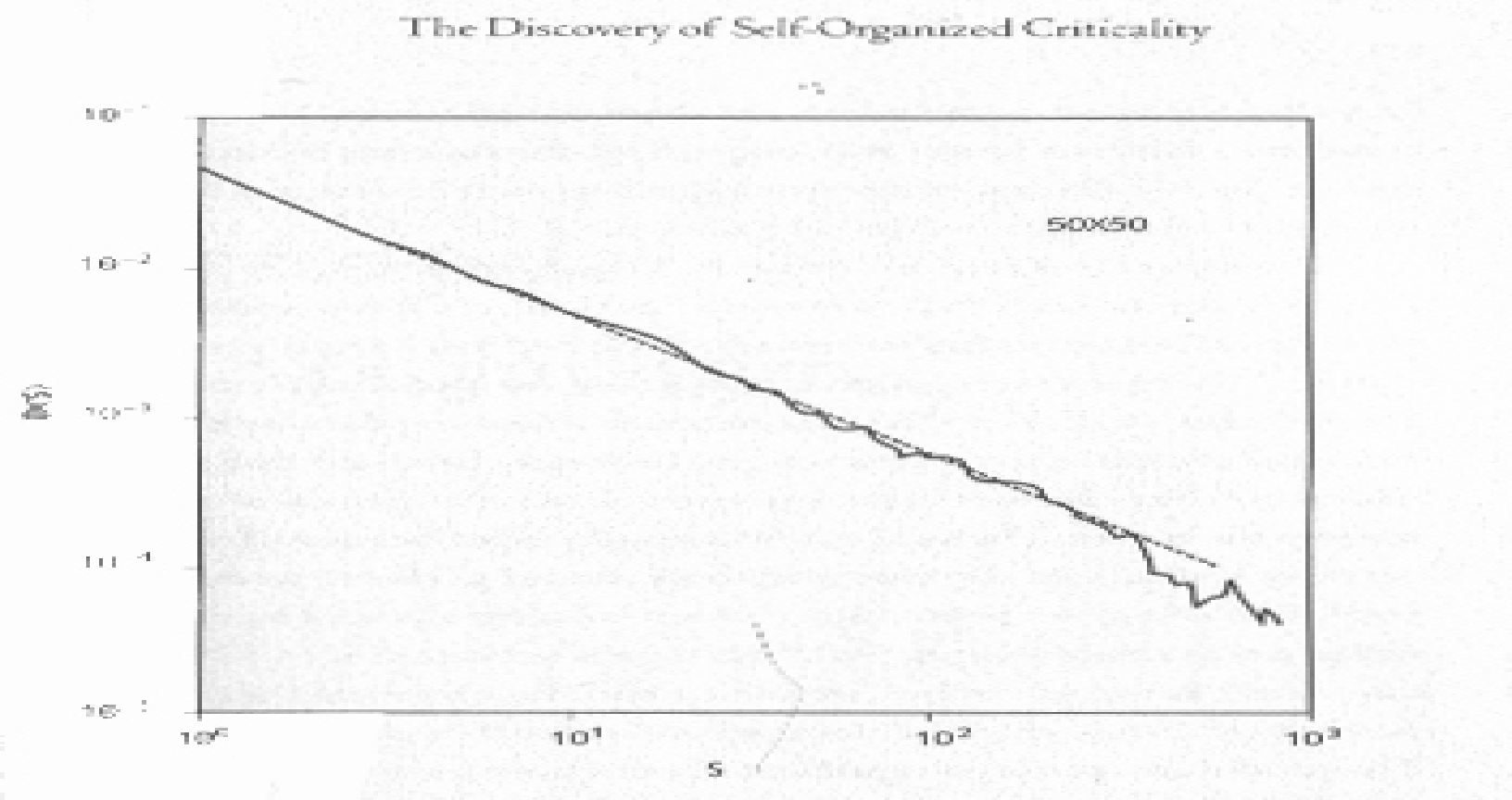
Examples from the text

- Coupled Pendulums
- Sandpiles
- Earthquakes
- Game of Life
- Economics
- And some of the topics are in the book that we do not focus too much on today: Mass extinction, solar flares and black holes, the human brain, traffic jams, and evolutionary biology

Coupled Pendulum Experiment



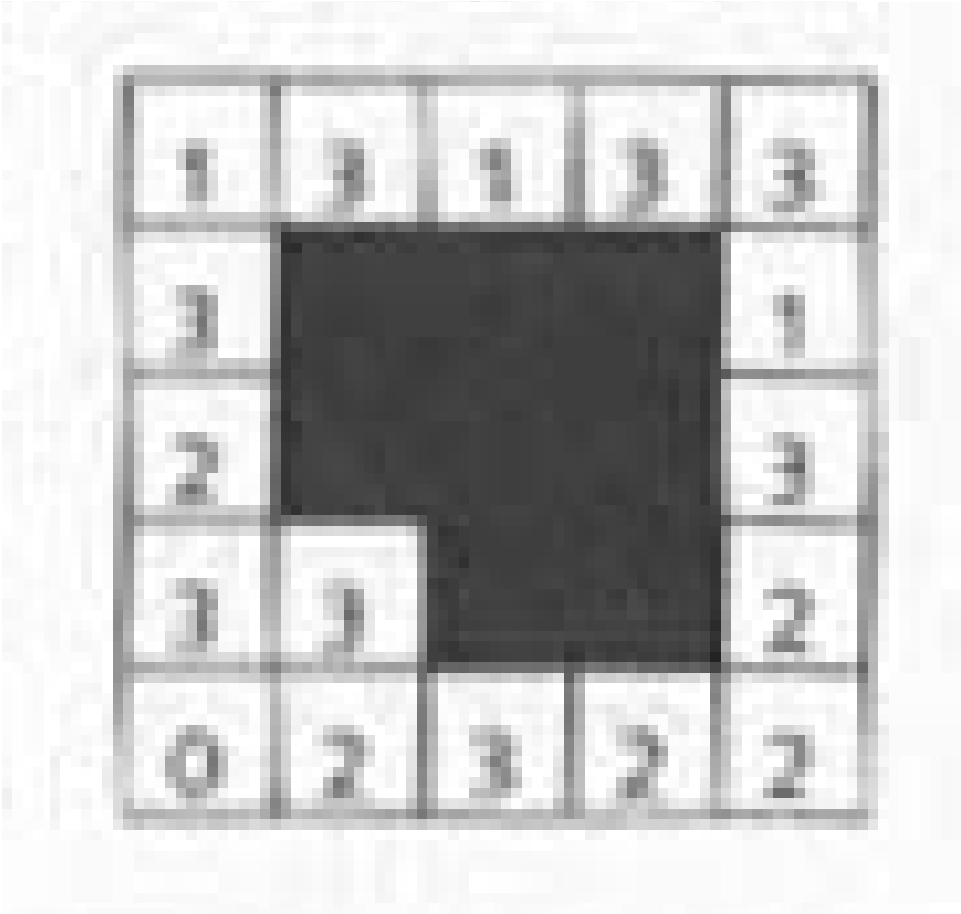
Avalanche Diagram for Pendulums



Better analogy: Sandpile

- Different nomenclature, same math
- The critical state must be robust to modifications
- Experimental representation is identical to coupled pendulums
- An example illustrates the simplicity of the model

Toppling Avalanches in Action



Oversimplification of models

- Real grains have different sizes & shapes
- Instability does not just happen at surface
- So what makes the model acceptable?
  - Essential physics
  - Detachment from details! (I.e. we are not interested in sand)

Why are the results significant?

- No organization is suggested, but in reality the pile has organized in a highly orchestrated, susceptible state.
- Plotting the log # of avalanches of a given magnitude vs. that magnitude reveals the GR power law
- Fractal geometry for the profile of sandpile
- Sandpile dynamics obey Zipf's law

On real sandpiles, and landscape formation

- Testing on real sand is tedious
  - Sandpiles must be large to test power law behaviour
  - We do not have the patience nature has, nor the space nature has.
- Not everything in this world is SOC!
- Long-grain rice approach seems most indicative of SOC behaviour

Some Experiment Layouts (1)

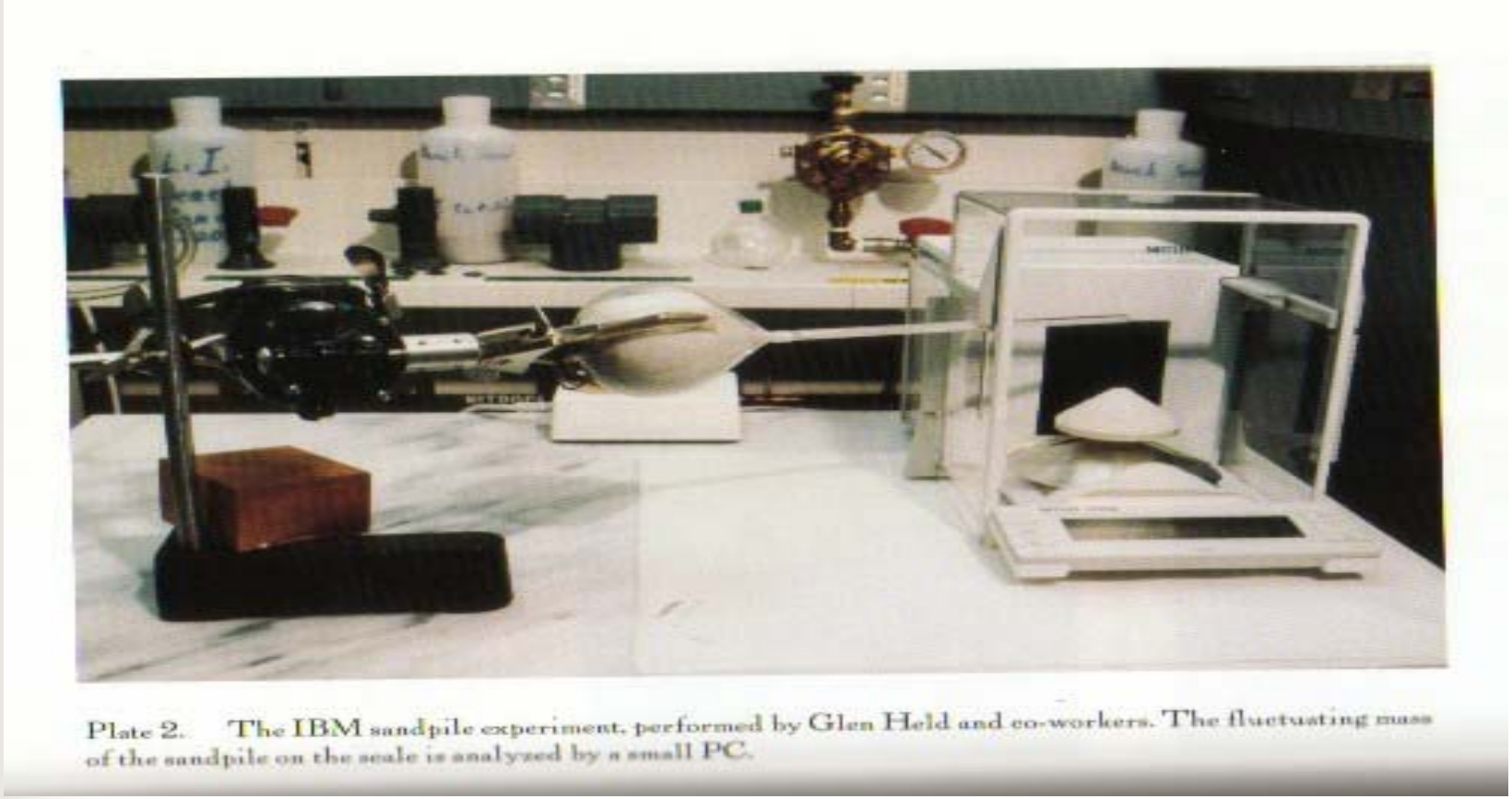
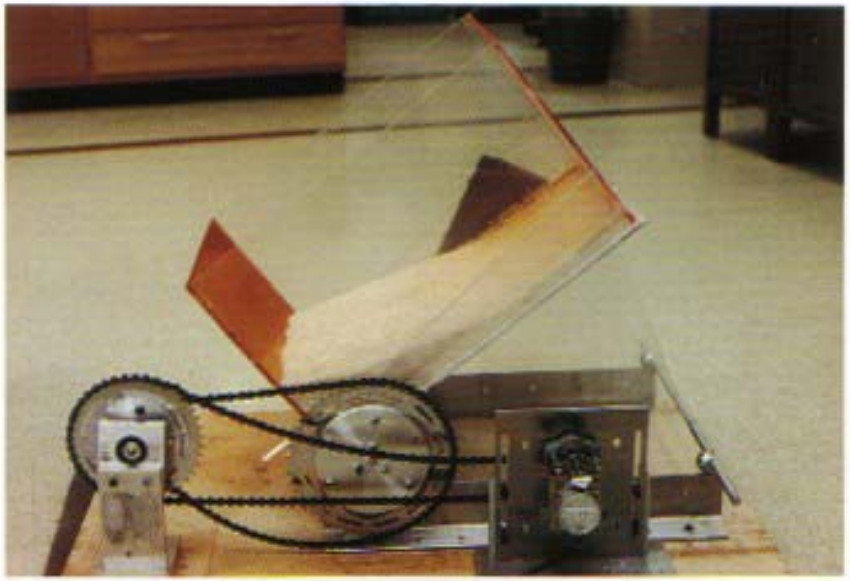


Plate 2. The IBM sandpile experiment, performed by Glen Held and co-workers. The fluctuating mass of the sandpile on the scale is analyzed by a small PC.

Some Experiment Layouts (2)

b

Plate 3. Sandpile experiments by the University of Michigan group led by Michael Bretz and Franco Nori. (a) Tilted sandpile. (b) Conical pile. The sandpile shown is the digital image from the video recorder.



a

Some Experiment Layouts (3)

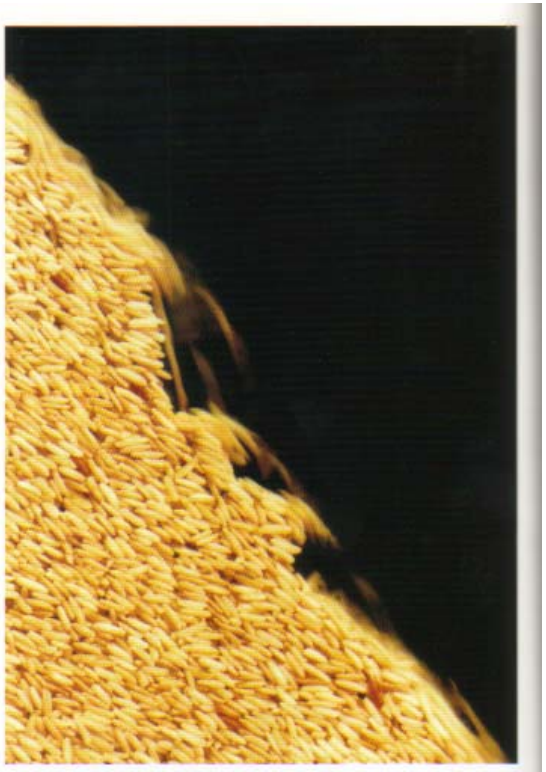


Plate 4. Rice pile profile in the self-organized critical state. (Frette et al., 1995.)

Earthquake Modeling

- The model is mathematically identical to CP & sandpiles. If you study one, you study them all
- Tectonic plate motion == tilting sandpile
- Toppling grains == ruptures leading to other ruptures
- The physical model is quite similar to CP

Earthquake Experiment Display

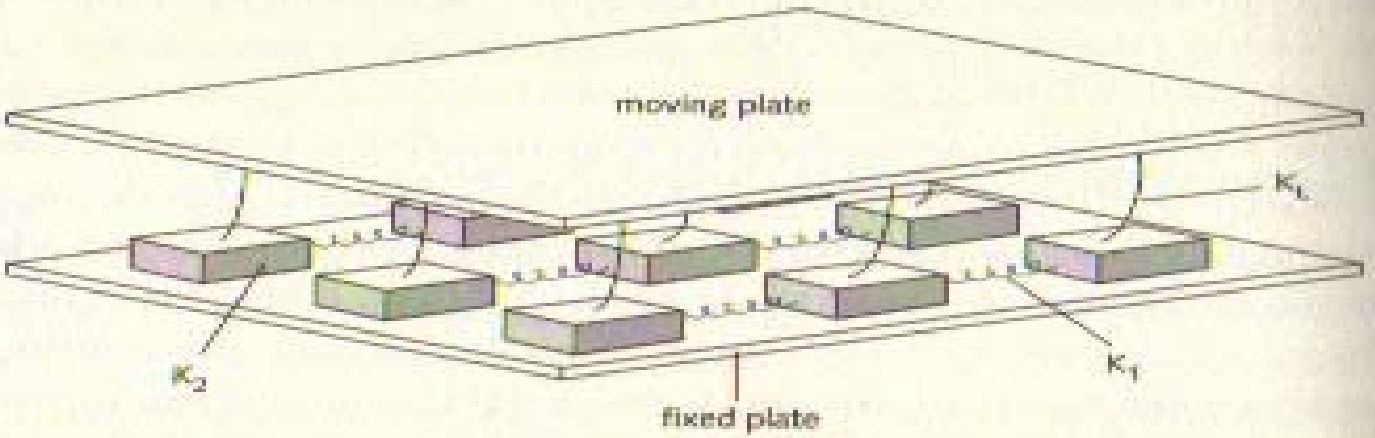


Figure 21. Block-spring model of earthquake fault. The blocks are connected with a slowly moving rod by leaf springs. They are also connected with each other by springs. Parameters K_1, K_2, and K_L specify the strengths of the springs. The blocks are moving on a rough surface. A blocks slide when the force on it exceeds a critical value.

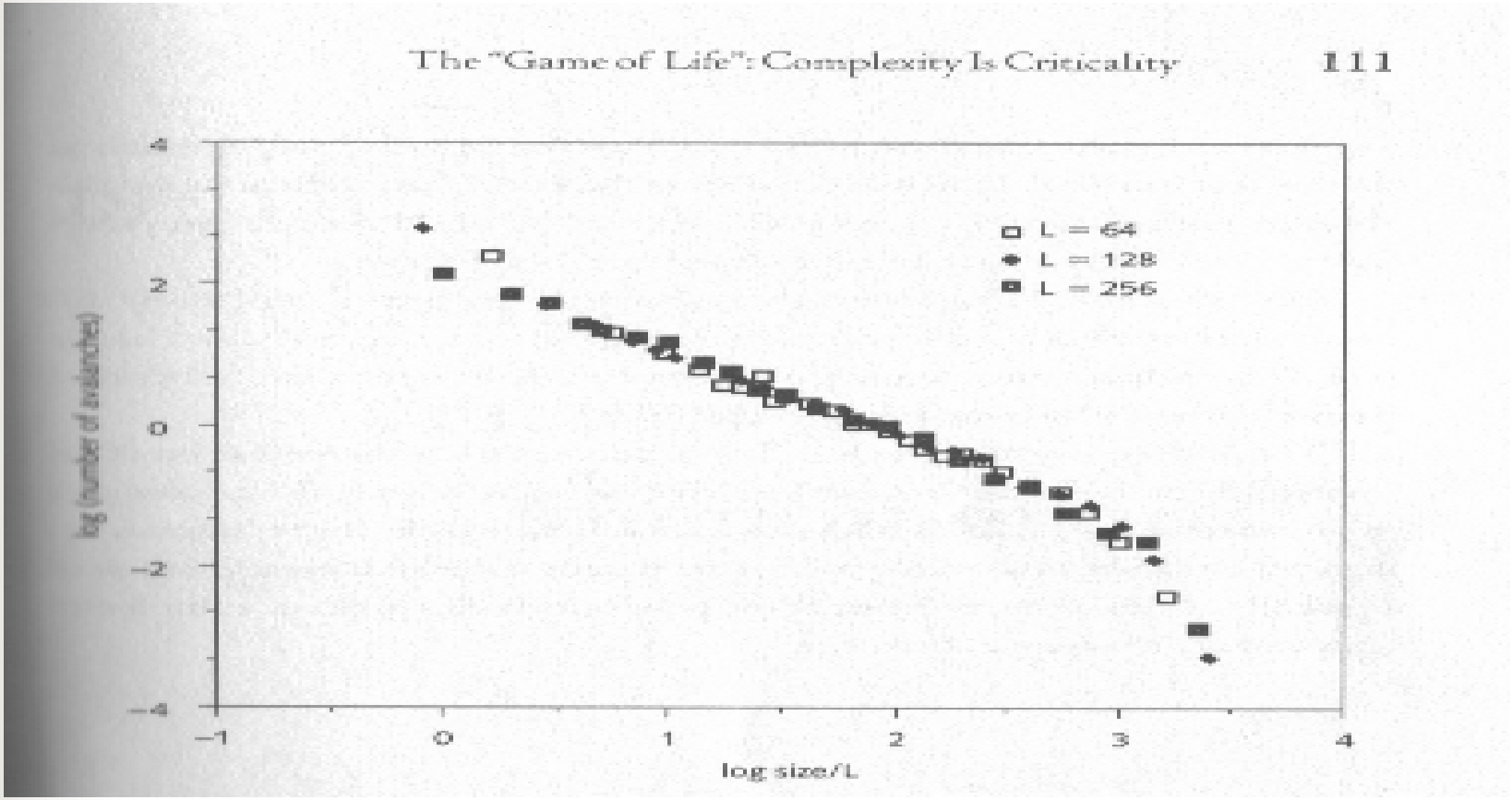
There are greater implications

- The GR Power law is the fingerprint that the crust of the earth has self-organized to the critical state
- Because of robustness, the criticality does not depend on choice of model
- The crust of the Earth has organized itself into a critical state

The game of life: complexity is criticality

- The game of life is a toy model of the formation of organized, complex, societies
- Complex phenomena can be generated from simple local rules
- 0's and 1's across a grid, at each time step all numbers are updated by a simple rule

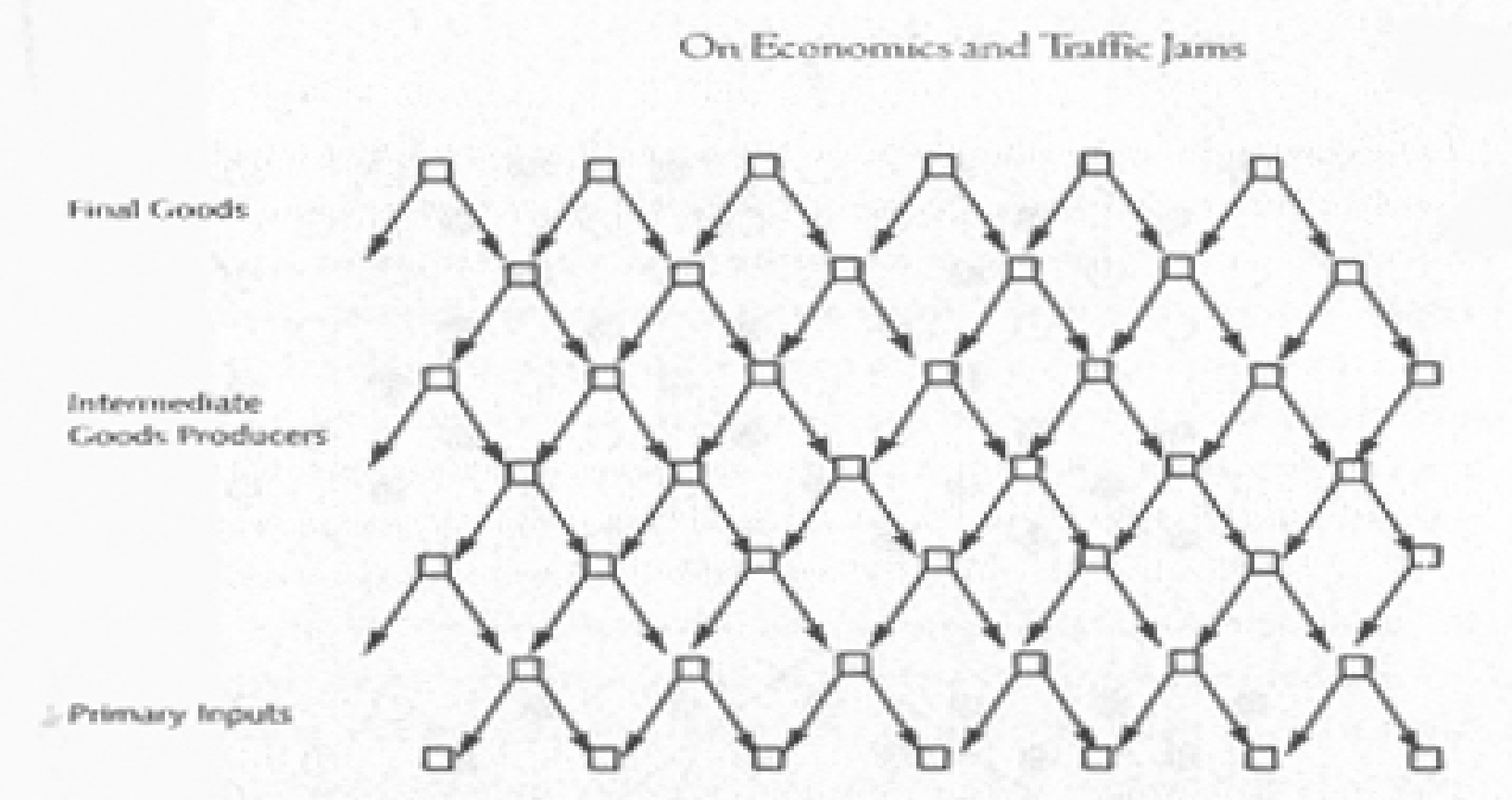
Game of Life Avalanches



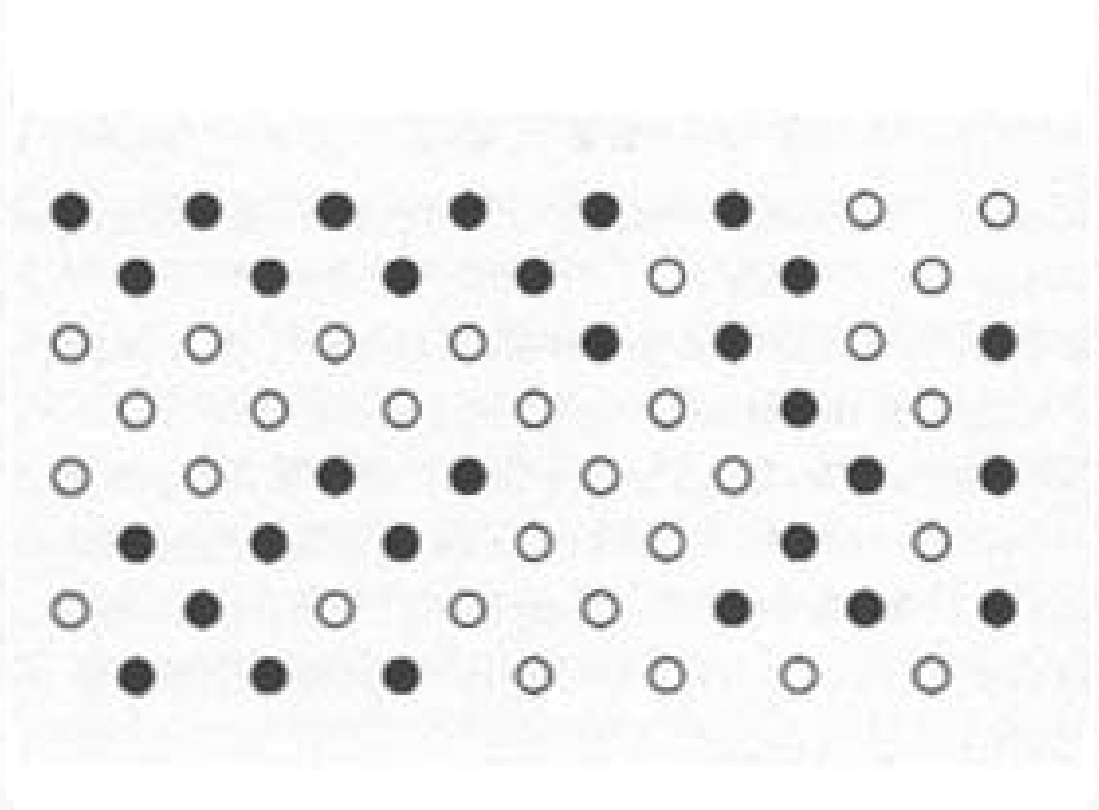
Economic Systems

- In equilibrium systems, everything adds up nicely and linearly
- Traditional economics is not realistic
- Real economics is like sand
- A histogram of (1960) monthly variations of cotton prices displays a “Levy Distribution”, which has a power law for large events
- So Per Bak builds a toy model…

Economic Model Pre-Action



Economic Model in Action



Observations & Conclusions

- Complexity is a consequence of criticality
- Real-life operates at a critical point between order and chaos
- SOC is a theoretical foundation for catastrophism, and explaining complexity
- Large fluctuations cannot be prevented by local manipulations
- Any small behaviour in the critical state eventually affects everything in the system
- Harder Conclusions would have been nice, such as the application of the said theory

References

- Per Bak, *How Nature Works* (Springer, New York, 1996).

Questions / Comments?

- Thank you for your attention!